

Constant Stress Partially Accelerated Life Tests for Extended Generalized log Logistic Distribution Based on Type I Censored Competing Risks Data

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Abstract: As a result of technology improvement getting information about products and materials lifetimes under usual conditions. Therefore accelerated life testing or partially accelerated life testing usually are used to truncate the tests survives. The test items under accelerated life testing run under accelerated conditions and partially life tests run under both accelerated and use conditions. The main idea of accelerated life testing that the acceleration element is not unknown or the mathematical model relating the lifetime of the unit and the stress is known or can be assumed. In some cases, neither acceleration factor nor life-stress relations are not unknown. This paper concerned with studying and discussed the constant-stress partially accelerated life test (CPALT) under type I censored (T.I.C) competing risks data. Failure times resulting from T.I.C competing risks data are assumed to follow the Extended generalized log logistic (EGLL) distribution because this model is completely flexible to study positive data. This distribution is applied in various fields, for example lifetime studies, economics, finance and insurance. The maximum likelihood (ML) method is used to estimate the parameters under TIC competing risks data. The simulation algorithm is performed to assess the theoretical results of the maximum likelihood estimates based on TIC competing risks data.

Keywords: Constant Stress Partially Accelerated Life Test, Type I Censored Data, Cot Ending Failure Causes, Maximum Likelihood Method

1. Introduction

In experiments ALT is applied to reduce time and cost. There are different methods of acceleration. Among these methods, the constant stress in which stress on the units remains constant, the progressive stress in which the stress applied on the test units increases with time and the step stress in which the test conditions change for a given time or for a given number of failures, see DS. Bai, SW. [1]

In PALT the test items are run at both accelerated and normal conditions. PALT is suitable when the acceleration failure are the mathematical model is unknown, see Abdel-Hamid and Al-Hussaini [2], Hassan et.al.[3], Hassan et.al.[4], Abu-Zinadah and Ahmed [5], Ismail [6], A.A. Ismail, A.A. Al-babtain [7], Ismail and Al Tamim [8], Ismail and Al Harbi [9], Li and Zheng [10], Zarrin, et al.[11], Fawzy [12],

The EGLL was first introduced by Lima and Corderio [13].

This new model is very flexible for modeling various types of data. In this paper, the MLES of the EGLL parameters are obtained under constant stress partially accelerated life test.

The rest of this paper is organized as follows: Competing risks schemes and model description are presented in Section 2. The ML estimators under T.I.C competing risks data is illustrated in Section 3. The simulation study is performed to assess the theoretical results in Section 4

2. Competing Risks Plans and Model Description

In Survival study, the items failure might be credit ready to more reason simultaneously. These “causes” would contending for the test unit failure. In the statistical literature, this issue is common as those contending risks pattern. The information of competing risks study, the data comprises of a

time failure and the related cause of its failure. It is assumed that the failure reasons are independent or reliant. In this study, we expect the suppressed failure time experiment, as proposed by Cox [14], that the times failure are individually circulated. where the failure is caused because of many reasons of failure, see Crowder [15].

Regarding experiment lifetime with i.i.d units n are drawn from N with i.i.d. Random variables (RVs) Z_1, \dots, Z_n . Without loss of generality; accept that there are just two reasons of failure. We have

$$T_i = \min(Z_{1i}, Z_{2i}) \text{ for } i = 1, \dots, n,$$

where Z_{1i}, Z_{2i} indicates the underlying time failure of the i -th unit in 1st and 2nd failure reasons, separately. It is expected that the underlying time failure Z_{1i} and Z_{2i} are not dependent, the sets (Z_{1i}, Z_{2i}) are i.i.d. The observed time failure is indicated through the RV, $= \min(Z_{1i}, Z_{2i})$.

The reliability function (SF) of the RV T is defined as

$$\bar{T}(z) = P(T > z) = P(T > z_1) P(T > z_2) = \bar{F}(z) \bar{F}(z),$$

Where $\bar{F}(\cdot) = 1 - F(\cdot)$, and the SF is the reliability function of EGLL appropriation.

Model Description and Its Assumptions

In this section, we show the fundamental assumptions for the life test of the product in CPALT competing failure of the model. Additionally, the procedures of test in CS_PALT dependent on T.I.C plans that the competing failures lifetimes are expected to be EGLL distribution are clarified.

$$f(x_j) = \beta_1 \beta b a (\beta_1 x_j)^{b-1} \left(1 + (\beta_1 x_j)^b\right)^{\beta-1} e^{-a \left[(1 + (\beta_1 x_j)^b)^{\beta} - 1\right]}, x_j, \beta, \alpha, b > 0, \beta_1 > 1, \quad (3)$$

And

$$F(x_j) = 1 - e^{-a \left[(1 + (\beta_1 x_j)^b)^{\beta} - 1\right]}, \quad (4)$$

where the observed ordered failure times are $x_{(1)} < \dots < x_{(na)} < \tau$ and na is the failed units number in accelerated situations under T.I.C.

Basic Assumption

- 1) The lifetimes $T_i, i = 1, 2, \dots, n(1 - \pi)$, of units approached at typical conditions are i.i.d RVs
- 2) The lifetimes $X_j|_{(j=1, 2, \dots, n\pi)}$ of items designed at accelerated conditions are i.i.d RVs
- 3) The lifetimes T_i and X_j are commonly independent.

3. ML Estimators Based on T.I.C Competing Risks Data

Assume that the observed estimations of the whole lifetime T of size $n(1 - \pi)$ at typical states are $t_{(1)}, t_{(2)}, \dots, t_{(n(1-\pi))}$, and the observed number of the whole lifetime X of size $n\pi$ at accelerated situation are $x_{(1)}, x_{(2)}, \dots, x_{(n\pi)}$. Let δ_{ui} and δ_{aj} showed the indicators of the failure are

$$\delta_{ui} = \begin{cases} 1, & t_i > \tau \\ 0, & \text{otherwise} \end{cases} \quad i = 1, 2, \dots, n(1 - \pi),$$

and

$$\delta_{aj} = \begin{cases} 1, & x_j > \tau \\ 0, & \text{otherwise} \end{cases} \quad j = 1, 2, \dots, n\pi,$$

The procedure of test in CPALT is recognized as:

- 1) Whole n units are split into two sets:
- 2) Group 1 (G.1) consists of $n_1 = n(1 - \pi)$, $(1 - \pi)$ is sample part units shared to typical circumstances.
- 3) Group 2 (G.2) consists of $n_2 = n\pi$ residual units are exposed to accelerated conditions.
- 4) All items in G.1 and G.2 are run at steady stress level till the test ends after the time of censoring τ in the case of T.I.C is reached.
- 5) The lifetimes $T_i, i = 1, 2, \dots, n(1 - \pi)$, of units designated in ordinary states are EGLL distribution with shape parameter β, b , scale parameter a and its probability density function (pdf) and cumulative distribution function (cdf) are given as follows:

$$f(t_i) = \beta b a t_i^{b-1} (1 + t_i^b)^{\beta-1} e^{-a \left[(1 + t_i^b)^{\beta} - 1\right]}, t_i, \beta, \alpha, b > 0, \quad (1)$$

and

$$F(t_i) = 1 - e^{-a \left[(1 + t_i^b)^{\beta} - 1\right]}, \quad (2)$$

where the observed ordered failure times are $t_{(1)} < \dots < t_{(nu)} < \tau$ under T.I.C and nu is the failed units number at ordinary situations.

- 6) The lifetimes $X_j|_{(j=1, 2, \dots, n\pi)}$ of units assigned at accelerated conditions are a EGLL distribution with shape parameter β, b and scale parameter a and its the pdf and cdf are given by:

The probability work for T.I.C contending risks of data at when time the reason of failure is known at ordinary circumstances as follows:

$$L \propto \prod_{i=1}^{n\bar{\pi}} [f_1(t_i) \bar{F}_2(t_i)]^{I(\delta_i=1)} [f_2(t_i) \bar{F}_1(t_i)]^{I(\delta_i=2)} [\bar{F}_1(\tau) \bar{F}_2(\tau)]^{\bar{\delta}_i}, \quad (5)$$

where, $t_i = t_{(i)}$, and $\bar{\pi} = (1 - \pi)$. By substituting (1), (2), (3) and (4) in (5), then

$$L_{1(ui)} \propto \prod_{i=1}^{n\bar{\pi}} \left[\beta_1 b_1 a_1 t_i^{b_1-1} (1 + t_i^{b_1})^{\beta_1-1} e^{-\{a_1[(1+t_i^{b_1})^{\beta_1}-1] + a_2[(1+t_i^{b_2})^{\beta_2}-1]\}} \right]^{\delta_{1ui}} \left[\beta_2 b_2 a_2 t_i^{b_2-1} (1 + t_i^{b_2})^{\beta_2-1} e^{-\{a_1[(1+t_i^{b_1})^{\beta_1}-1] + a_2[(1+t_i^{b_2})^{\beta_2}-1]\}} \right]^{\bar{\delta}_{ui}}$$

Additionally, the T.I.C. likelihood function contending risks information at what time the failure reason of failure is known at accelerated conditions is given by

$$L_{(aj)} \propto \prod_{j=1}^{n\bar{\pi}} \left[\beta_1 \beta_{11} b_1 a_1 (\beta_{11} x_j)^{b_1-1} (1 + (\beta_{11} x_j)^{b_1})^{\beta_1-1} e^{-\{a_1[(1+(\beta_{11} x_j)^{b_1})^{\beta_1}-1] + a_2[(1+(\beta_{12} x_j)^{b_2})^{\beta_2}-1]\}} \right]^{\delta_{1aj}} \left[\beta_2 \beta_{12} b_2 a_2 (\beta_{12} x_j)^{b_2-1} (1 + (\beta_{12} x_j)^{b_2})^{\beta_2-1} e^{-\{a_1[(1+(\beta_{11} x_j)^{b_1})^{\beta_1}-1] + a_2[(1+(\beta_{12} x_j)^{b_2})^{\beta_2}-1]\}} \right]^{\bar{\delta}_{aj}}$$

Since the lifetimes of t_1, \dots, t_{nu} and x_1, \dots, x_{na} are i.i.d then the whole T.I.C likelihood function competing risks data after the failure cause is known at typical and accelerated cases ($t_1; \delta_{u1}; \dots, t_{n\bar{\pi}}; \delta_{un\bar{\pi}}; x_1; \delta_{a1}; \dots, x_{n\pi}; \delta_{an\pi}$) is given by

$$L_i \propto L_{(ui)} L_{1(aj)} \propto \prod_{i=1}^{n\bar{\pi}} \left[\beta_1 b_1 a_1 t_i^{b_1-1} (1 + t_i^{b_1})^{\beta_1-1} e^{-\{a_1[(1+t_i^{b_1})^{\beta_1}-1] + a_2[(1+t_i^{b_2})^{\beta_2}-1]\}} \right]^{\delta_{1ui}} \left[\beta_2 b_2 a_2 t_i^{b_2-1} (1 + t_i^{b_2})^{\beta_2-1} e^{-\{a_1[(1+t_i^{b_1})^{\beta_1}-1] + a_2[(1+t_i^{b_2})^{\beta_2}-1]\}} \right]^{\bar{\delta}_{ui}} \prod_{j=1}^{n\bar{\pi}} \left[\beta_1 \beta_{11} b_1 a_1 (\beta_{11} x_j)^{b_1-1} (1 + (\beta_{11} x_j)^{b_1})^{\beta_1-1} e^{-\{a_1[(1+(\beta_{11} x_j)^{b_1})^{\beta_1}-1] + a_2[(1+(\beta_{12} x_j)^{b_2})^{\beta_2}-1]\}} \right]^{\delta_{1aj}} \left[\beta_2 \beta_{12} b_2 a_2 (\beta_{12} x_j)^{b_2-1} (1 + (\beta_{12} x_j)^{b_2})^{\beta_2-1} e^{-\{a_1[(1+(\beta_{11} x_j)^{b_1})^{\beta_1}-1] + a_2[(1+(\beta_{12} x_j)^{b_2})^{\beta_2}-1]\}} \right]^{\bar{\delta}_{aj}} \quad (6)$$

where, $\bar{\delta}_{ui} = 1 - \delta_{ui}$ and $\bar{\delta}_{aj} = 1 - \delta_{aj}$.

The ML assessors $\hat{\beta}_1, \hat{b}_1, \hat{a}_1, \hat{\beta}_2, \hat{b}_2, \hat{a}_2, \hat{\beta}_{11}$ and $\hat{\beta}_{12}$ of the parameters and acceleration factors $\beta_1, b_1, a_1, \beta_2, b_2, a_2, \beta_{11}$ and β_{12} are the values when boosts the likelihood function. The logarithm of the likelihood function $\ell = \ln L_i$ is given by:

$$\ell \propto n\bar{\pi} \ln \beta_1 + n\bar{\pi} \ln b_1 + n\bar{\pi} \ln a_1 + (b_1 - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{1ui} t_i + (\beta_1 - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{1ui} (1 + t_i^{b_1}) - a_1 \sum_{i=1}^{n\bar{\pi}} \delta_{1ui} [(1 + t_i^{b_1})^{\beta_1} - 1] - a_2 \sum_{i=1}^{n\bar{\pi}} \delta_{1ui} [(1 + t_i^{b_2})^{\beta_2} - 1] + n\bar{\pi} \ln \beta_2 + n\bar{\pi} \ln b_2 + n\bar{\pi} \ln a_2 + (b_2 - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{2ui} t_i + (\beta_2 - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{2ui} (1 + t_i^{b_2}) - a_2 \sum_{i=1}^{n\bar{\pi}} \delta_{2ui} [(1 + t_i^{b_2})^{\beta_2} - 1] - a_1 \sum_{i=1}^{n\bar{\pi}} \delta_{2ui} [(1 + t_i^{b_1})^{\beta_1} - 1] + n\pi \ln \beta_1 + n\pi \ln b_1 + n\pi \ln a_1 + (b_1 - 1) \sum_{j=1}^{n\pi} \ln \delta_{1aj} x_j + (\beta_1 - 1) \sum_{j=1}^{n\pi} \ln \delta_{1aj} (1 + x_j^{b_1}) - a_1 \sum_{j=1}^{n\pi} \delta_{1aj} [(1 + x_j^{b_1})^{\beta_1} - 1] - a_2 \sum_{j=1}^{n\pi} \delta_{1aj} [(1 + x_j^{b_2})^{\beta_2} - 1] + n\pi \ln \beta_2 + n\pi \ln b_2 + n\pi \ln a_2 + (b_2 - 1) \sum_{j=1}^{n\pi} \ln \delta_{2aj} x_j + (\beta_2 - 1) \sum_{j=1}^{n\pi} \ln \delta_{2aj} (1 + x_j^{b_2}) - a_2 \sum_{j=1}^{n\pi} \delta_{2aj} [(1 + x_j^{b_2})^{\beta_2} - 1] - a_1 \sum_{j=1}^{n\pi} \delta_{2aj} [(1 + x_j^{b_1})^{\beta_1} - 1] - \bar{\delta}_{aj} \{a_1[(1 + (\lambda_k \tau)^{b_1})^{\beta_1} - 1] + a_2[(1 + (\lambda_k \tau)^{b_2})^{\beta_2} - 1]\} \quad (7)$$

The first derivatives of the logarithm of the likelihood function (7) concerned to $\beta_k, b_k, a_k, \lambda_k, k=1,2$, are given by

$$\frac{\partial \ell}{\partial \beta_k} = \frac{n\bar{\pi}}{\beta_k} + \sum_{i=1}^{n\bar{\pi}} \ln \delta_{kui} (1 + t_i^{b_k}) - a_k \sum_{j=1}^{n\pi} \delta_{kaj} [(1 + x_j^{b_k})^{\beta_k}] \ln(1 + x_j^{b_k}) + \frac{n\pi}{\beta_k} + \sum_{i=1}^{n\pi} \ln \delta_{kui} (1 + t_i^{b_k}) - a_k \sum_{j=1}^{n\pi} \delta_{kaj} [(1 + x_j^{b_k})^{\beta_k}] \ln(1 + x_j^{b_k}) - \bar{\delta}_{aj} \{a_k[(1 + (\lambda_k \tau)^{b_k})^{\beta_k} - 1] \ln(1 + (\lambda_k \tau)^{b_k})\}, \quad (8)$$

$$\frac{\partial \ell}{\partial b_k} = \frac{n\bar{\pi}}{b_k} + \sum_{i=1}^{n\bar{\pi}} \ln \delta_{kui} t_i + (\beta_k - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{kui} (1 + t_i^{b_k}) \ln t_i - a_k \sum_{i=1}^{n\bar{\pi}} \delta_{kui} \ln t_i [(1 + t_i^{b_k})^{\beta_k} - 1]$$

$$\frac{n\pi}{b_k} + \sum_{i=1}^{n\bar{\pi}} \ln \delta_{kui} t_i + (\beta_k - 1) \sum_{i=1}^{n\bar{\pi}} \ln \delta_{kui} (1 + t_i^{b_k}) \ln t_i - a_k \sum_{i=1}^{n\bar{\pi}} \delta_{kui} \ln t_i [(1 + t_i^{b_k})^{\beta_k} - 1] - \bar{\delta}_{aj} \{a_k [(1 + (\lambda_k \tau)^{b_k})^{\beta_k} - 1]\}, \quad (9)$$

$$\frac{\partial \ell}{\partial a_k} = \frac{n\bar{\pi}}{a_k} + \sum_{i=1}^{n\bar{\pi}} \delta_{kui} [(1 + t_i^{b_k})^{\beta_k} - 1] + \frac{n\pi}{a_k} - \sum_{i=1}^{n\bar{\pi}} \delta_{kui} \ln t_i [(1 + t_i^{b_k})^{\beta_k} - 1] - \bar{\delta}_{aj} [(1 + (\lambda_k \tau)^{b_k})^{\beta_k} - 1], \quad (10)$$

and

$$\frac{\partial \ell}{\partial \lambda_k} = \bar{\delta}_{aj} b_k \{a_k [(1 + (\lambda_k \tau)^{b_k})^{\beta_k} - 1]\} (\lambda_k \tau)^{b_k - 1}. \quad (11)$$

Setting Eqs. (8)-(11) by zeros. The arrangement of these nonlinear conditions can't be solved analytically. Thus, numerical solution via iterative techniques is applied to obtain the ML estimators.

4. Simulation Study

A simulation study is used to assess the estimates performance. The assessments of the acceleration factor (λ_1, λ_2) and population parameters $\beta_1, b_1, a_1, \beta_2, b_2, a_2$ are evaluated regarding the mean squared errors (MSEs) and biases. The numerical technique is approached as follows:

- 1) A random sample size $n_1 = n(1 - \pi)$, where $\pi = 0.4$ is the ratio and n is the whole sample size, is generated under typical circumstances. So, samples are generated from $W_1 \sim ELL(n_1, \beta_1, b_1, a_1)$ and $W_2 \sim ELL(n_2, \beta_2, b_2, a_2)$. Taking into account two generated samples generate new samples $t_1 = t_{(1)}, t_{(2)}, t_{(3)}, \dots, t_{(n_1)}$ where

$$T = \min(W_1, W_2).$$

- 2) A random sample size $n_2 = n\pi$ is deduced under

accelerated cases. Thus, the two samples from $W_1 \sim ELL(n_1, \beta_1, b_1, a_1)$ and $W_2 \sim ELL(n_2, \beta_2, b_2, a_2)$ are generated. The two generated samples generate new samples

- 3) $X = x_{(1)}, x_{(2)}, x_{(3)}, \dots, x_{(n_2)}$ where

$$X = \min(W_1, W_2).$$

- 4) In T.I.C, let $\tau = 1, 1.5, 2$ for sample sizes 30, 60, 100, 200 and 500, for some selected the unknown parameters and factor of accelerated, this process is iterated 1000 times at various estimations of population $\beta_1 = 1.5, b_1 = 2, a_1 = 2.5, \beta_2 = 3, b_2 = 4, a_2 = 2$, $\lambda_1 = 1.5, \lambda_2 = 3$

- 5) The average estimations of biases (Abias) and MSEs are processed.

The mathematical results are reported in Table 1. The outcomes can be found as follows:

- 1) The Abias and MSEs decrease as n increase under T.I.C data.
- 2) It is noticed that the Abias and MSEs decrease, censoring time τ decrease.
- 3) Clearly, the acceleration of the experiment is useful to get outcomes and data quickly, yet the most consequences of normal condition are more exact that speeding up condition.

Table 1. Abias and MSEs of MLE based on T.I.C at $\tau = 1, 1.5, 2$ and $\pi = 0.4$.

n	MLE	$\tau = 2$		$\tau = 1.5$		$\tau = 1$	
		Abias	MSE	Abias	MSE	Abias	MSE
30	$\hat{\beta}_1$	0.0923	0.0101	0.7760	0.0073	0.0456	0.0089
	\hat{b}_1	0.0853	0.0088	0.0706	0.0062	0.0686	0.0079
	\hat{a}_1	0.1894	0.0375	0.1747	0.0315	0.1427	0.0330
	$\hat{\lambda}_1$	0.1174	0.0154	0.1027	0.0116	0.1005	0.0134
	$\hat{\beta}_2$	0.1836	0.0353	0.1689	0.0298	0.1669	0.0310
	\hat{b}_2	0.1642	0.0286	0.1495	0.0236	0.1475	0.0250
	\hat{a}_2	0.1507	0.0243	0.1360	0.0193	0.1340	0.0212
	$\hat{\lambda}_2$	0.1054	0.0127	0.0907	0.0005	0.0887	0.0111
	$\hat{\beta}_1$	0.0462	0.0037	0.0315	0.0022	0.0295	0.0022
	\hat{b}_1	0.0426	0.0034	0.0279	0.0020	0.0258	0.0019
60	\hat{a}_1	0.0532	0.0044	0.0385	0.0027	0.0365	0.0025
	$\hat{\lambda}_1$	0.0476	0.0038	0.0329	0.0023	0.0309	0.0021
	$\hat{\beta}_2$	0.0612	0.0053	0.0465	0.0034	0.0445	0.0020
	\hat{b}_2	0.0547	0.0046	0.0400	0.0028	0.0380	0.0027
	\hat{a}_2	0.0754	0.0073	0.0607	0.0049	0.0587	0.0036
	$\hat{\lambda}_2$	0.0624	0.0034	0.0270	0.0019	0.0250	0.0017
	$\hat{\beta}_1$	0.0131	0.0018	0.0016	0.0012	0.0036	0.0032
	\hat{b}_1	0.0135	0.0019	0.0012	0.0013	0.0032	0.0023
	\hat{a}_1	0.0163	0.0019	0.0015	0.0012	0.0004	0.0032
	$\hat{\lambda}_1$	0.0104	0.0017	0.0043	0.0013	0.0063	0.0025
100	$\hat{\beta}_2$	0.0046	0.0016	0.0101	0.0013	0.0121	0.0034
	\hat{b}_2	0.0099	0.0016	0.0048	0.00125	0.0068	0.0033
	\hat{a}_2	0.0165	0.0019	0.0018	0.00135	0.0021	0.0031
	$\hat{\lambda}_2$	0.0111	0.0017	0.0036	0.00124	0.0056	0.0034

n	MLE	$\tau = 2$		$\tau = 1.5$		$\tau = 1$	
		Abias	MSE	Abias	MSE	Abias	MSE
200	$\hat{\beta}_1$	0.0065	0.00004	0.0052	2.74×10^{-5}	0.00393	1.54×10^{-5}
	\hat{b}_1	0.0079	6.65×10^{-5}	0.0064	2.92×10^{-5}	0.0041	1.64×10^{-5}
	\hat{a}_1	0.0081	6.64×10^{-5}	0.0065	4.25×10^{-5}	0.0049	2.39×10^{-5}
	$\hat{\lambda}_1$	0.0052	2.7×10^{-5}	0.0042	1.73×10^{-5}	0.0031	9.37×10^{-6}
	$\hat{\beta}_2$	0.0023	5×10^{-6}	0.0018	3.38×10^{-6}	0.0014	1.90×10^{-6}
	\hat{b}_2	0.0049	2.45×10^{-5}	0.0040	1.57×10^{-5}	0.0030	8.82×10^{-6}
	\hat{a}_2	0.0083	6.80×10^{-5}	0.0066	4.36×10^{-5}	0.0049	2.45×10^{-5}
	$\hat{\lambda}_2$	0.0055	3.08×10^{-5}	0.0044	1.97×10^{-5}	0.0033	1.11×10^{-5}

Table 1. Continued.

n	MLE	Abias		MSE		Abias	
		Abias	MSE	Abias	MSE	Abias	MSE
500	$\hat{\beta}_1$	0.0016	2.64×10^{-6}	0.00032	1.54×10^{-6}	0.00028	8.09×10^{-8}
	\hat{b}_1	0.0019	3.90×10^{-6}	0.00039	3.15×10^{-6}	3.56×10^{-4}	1.17×10^{-7}
	\hat{a}_1	0.0020	4.10×10^{-6}	0.00040	3.83×10^{-6}	2.49×10^{-4}	1.26×10^{-7}
	$\hat{\lambda}_1$	0.0013	1.69×10^{-6}	0.00026	1.21×10^{-6}	0.00227	5.18×10^{-8}
	$\hat{\beta}_2$	0.0006	3.31×10^{-7}	0.00012	2.27×10^{-7}	1.01×10^{-4}	1.16×10^{-8}
	\hat{b}_2	0.0013	1.50×10^{-6}	0.00025	1.51×10^{-6}	2.05×10^{-4}	4.60×10^{-8}
	\hat{a}_2	0.0021	4.31×10^{-6}	0.00045	3.73×10^{-6}	3.13×10^{-4}	1.32×10^{-7}
	$\hat{\lambda}_2$	0.0014	1.89×10^{-6}	0.00025	1.89×10^{-6}	2.41×10^{-5}	5.79×10^{-8}

5. Conclusion

In this study, the CPALT under T.I.C data is discussed assuming that failure times are EGLL distribution. The ML method is applied to estimate the unknown parameters under TIC competing risks. To assess the theoretical results of the ML method for CPALT based on T.I.C data, the simulation algorithm is performed. It is noted that the Abias and MSEs decrease as n increase under T.I.C data and censoring time τ decrease. The acceleration of the experiment is valuable helpful to get outcomes and data quickly, yet the most consequences of normal condition are more exact that speeding up condition.

Conflicts of Interest

The authors declare no conflict of interest.

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