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Case Report

The Model Design for Education Fund Investment Issues

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Abstract: In order to help the Goodgrant Foundation Education Fund to invest on higher education of American, this report makes the following scheme. Based on minimal risk principle, we employ statistical analysis and Python programming, to screen the 93 from about 3,000 schools, which are necessary to be invested. Structure Entropy and Factor Analysis are used to select the key indicators closely related investment returns. Then we design four-investment strategies, based upon the ratio of faculty and student. We consider the risk factors and total revenue, and then establish the investment return and risk model. According to investment benefit of first year, we make the investment strategy of the next few years. The next few years are with rule that returns on investments over the last year. This report will effectively help solve the Goodgrant Foundation Education Fund investment issues.

Keywords: Structure Entropy, Factor Analysis Optimization Model, FAHP Model, Investment Return and Risk Model, Investment Strategy

1. Introduction

Goodgrant Foundation is committed to helping schools to improve educational performance, but how to investment is an important issue. It should be taken into consideration that Schools in the area, the nature of the school, the proportion of teachers and students, graduation rates and income, and so on. The problem belongs to the education investment; the aim of investment return is to improve the performance of the investment before and after the investment. It should solve that the investment amount per school, the return on that investment, and the time duration that the organization's money should be provided to have the highest likelihood of producing a strong positive effect on educational performance.

Then we will make a certain analysis of the investment problem, according to the analysis of the result, we establish the corresponding models and give the corresponding investment strategy.

2. Analysis of the Problem

The Goodgrant Foundation is a charitable organization that wants to help improve educational performance of undergraduates attending colleges and universities in the United States. To do this, the Foundation intends to donate a total of $100,000,000 (US100 million) to an appropriate group of schools per year, for five years, starting July 2016. This is an education investment problem and the aim of investment return is to improve the educational performance. It should solve the problem that the investment amount per school, the return on that investment, and the time duration.

According to the indicators provided, to select the best candidates is a complex decision-making problem, because there are many factors we should take into account. For example, schools in the area, the nature of the school, the proportion of teachers and students, graduation rates and income, these factors need to be taking into consideration. We built models and make investment strategy to decide which school more needs to be funded and the investment amount.

3. Selection of Indicators

3.1. Indicator Screening

According to the *College Score card Data Dictionary*, we select 7 preliminarily factors that affect the return on investment and improve the whole efficiency of investment.

Table 1. *Interpretations of Factors.*

Factors	Interpretations
Income of family	Income weighted average of each layer of the family income
Race of student	A summary of the proportion of schools of different ethnic groups
Admission score	SAT and ACT results summary
Number of subject	Discipline of the number of graduates, the ratio of 0%as there is no such discipline
Student-to-faculty ratio	Ratio of teachers and students in school (Tagging: the use of http://nces.ed.gov/collegenavigator through the website Python access)

3.2. Data Screening

In order to improve the effectiveness of indicators and avoid the risk of investment, we firstly make a preliminary screening of extreme data.

Table 2. *Interpretations of Factors.*

Index	Risk aversion interval
HCM2	Financial transparency does not make contributions to consider
LOCALE	The busy regional universities do not give priority to consideration
Number of subjects	The number of subjects is not a priority
UGDS	Enrollment of fewer than 100 of the school is not a priority
CURROPER	Not currently certified as an operating institution
NPT4_PRIV	The amount of school fees ranked in the top 10% is not a priority
Graduate retention rate	Graduation rate of less than 30% is not a priority
PCTFLOAN	The federal loan ratio above 80% is not a priority
md_earn_wne_p10	10 years after graduation, the average wage of more than 5000 dollars is not a priority
gt_25k_p6	After 6 years of graduation, the annual salary of more than 2500 US dollars ratio is greater than 70% is not a priority
NPT4_PRIV, NPT4_PUB	The average tuition is over $15000. The school is not a priority
PPTUG_EF	A part-time ratio of 0 is not a priority
Relevant financial indicators	Fiscal transparency is not a priority
Information provision	The information provided is too few to give priority to (the number of NULL in more than half)

4. Optimization Model Based on Structure Entropy and Factor Analysis

4.1. Model for Selecting

Assuming that there are k participants to participate in the survey, the number of recycling questionnaire is k, so each questionnaire corresponds to a set of indicators, denoted by U, $U = \{u_1, u_2, \cdots, u_{in}\}$. The corresponding "structural entropy" denoted by $\{a_{i1}, a_{i2}, \cdots, a_{in}\}$, k pieces of questionnaire form the "structural entropy matrix", denoted by$A = \{a_{\text{ij}}\}_{k\times n}$, the means of a_{ij} is that the evaluation of the participants No. i to the index No. j, i= l, 2,…, k; j =1, 2,…, n. Based on entropy theory to construct the structure entropy model, the standard entropy model is shown as follows:

$$X(I) = -\lambda p_n(I) Inp_n(I) \quad (1)$$

in the expression (1), let $p_n(I) = \frac{m-I}{m-1}, \lambda = \frac{1}{\ln(m-1)}$,
so (1) became this,

$$X(I) = -\frac{1}{\ln(m-1)}(\frac{m-I}{m-1})\ln(\frac{m-I}{m-1}) \quad (2)$$

simplify it, get:

$$X(I) = -\frac{(m-I)\ln(m-I)}{(m-1)\ln(m-1)} + (\frac{m-I}{m-1}) \quad (3)$$

Both sides divided by $\frac{m-I}{m-1}$, and let

$$\frac{X\ (I)}{\frac{(m-I)}{(m-1)}-1} = \mu(I) \quad (4)$$

So, the structural entropy model is

$$\mu(I) = -\frac{\ln(m-I)}{\ln(m-1)} = -\ln(p_n(I)) \quad (5)$$

in the expression (3) and expression (5), $\mu(I)$ is subordinative function value corresponding a_{ij} , I, m are transformed parameters. According to the theory of structure entropy, let *I*=*q*+1, *m*=*q*+2, so we get it:

$$p_n(I) = \frac{(q+2)-(1+1)}{(q+2)-1} = \frac{1}{q+1} \quad (6)$$

in expression (6), q is sort number of participants give to the indicators. The greater impact of an investment indicator, the number of its sort before the more, vice versa. If the indicator a_{ij} have "significant effect", the value of q is 1; if the indicator a_{ij} have "greater impact", the value of q is 2, and so on.

According to expression (5) and expression (6), we can get:

$$\mu(q) = -\ln(\frac{1}{q+1}) \quad (7)$$

Put a_{ij} into (7), get Shannon entropy values of a_{ij} is $b_{\text{ij}}(b_{\text{ij}} = u\ (a_{\text{ij}}))$ and form the "Structure entropy matrix", denoted by $b_{\text{ij}} = (a_{\text{ij}})_{k\times n}$.

Suppose sorted results of every participant are equally important, so calculate the average recognition degree. denoted by b_j,

$$b_j = (b_{1j} + b_{2j} + b_{3j} + \cdots + b_{nj})/k \quad (8)$$

Then calculate the uncertainty of every participant's cognition, denoted by Q_j,

$$Q_j = [\{|\max(b_{1j}, b_{2j}, \cdots, b_{ij}) - b_j + \min(b_{1j}, b_{2j}, b_{ij}) - b_j)|\} / 2] \quad (9)$$

According to the average awareness and blindness awareness of participants, calculate the general cognition, denoted by x_j, it can form the general cognition of all participants to all indicators, denoted by $X = (x_1, x_2, \dots, x_n)$. According to the general cognition.

$$X_j = b_j(1 - Q_j), x_j > 0 \quad (10)$$

4.2. Results and Analysis

(1) The important of indicator preliminary selection

According to the results of the questionnaire survey, the formation of the dynamic evaluation of the investment efficiency of the primary key Indicator "typical sort matrix" by removal of noise processing we get "structure entropy matrix".

We get the dynamic evaluation of the key Indicator to sort the results shown in table 3.

Table 3. *Test results of Structure Entropy of dynamical key indicators.*

Indicator Numbers	Relationship Indicator	Overall cognition	Rank
1	Family income	0.642	2
2	Admission score	0.617	1
3	Student race	0.708	3
4	Income after graduation	0.712	4
5	Women only	0.966	7
6	Local	0.770	6
7	Discipline Quantity	0.767	5

(2) The optimization of the key Indicator in the primary election

Using SPSS software, the standard χ^2 values is 175.160 (136 degrees of freedom), reaching a significant level (ρ =0.000<0.001). This indicates that the set of standardized data is suitable for factor analysis. Similarly, the principal component analysis method, the relevant data to determine primary 18 dynamic evaluation of key indicator fate of variance of Varimax orthogonal rotation orthogonal is obtained after the common factors explain the total variance ability of 3-4 as shown in the following table.

Table 4. *The ability of common factor of dynamical key indicators to explain the total variance.*

Common factor	Initial eigenvalue		
	Characteristic root	Cumulative interpretation %	Explained variance %
1	3.471	33.420	33.420
2	2.493	26.666	60.086
3	2.107	22.395	82.481
4	1.295	11.973	94.454

Furthermore, the dynamic evaluation of the primary Key Indicator factor load matrix is analyzed, as shown in table 5.

Table 5. *Factor loading matrix and optimization results of dynamical primary key indicators.*

Indicator	Rotation factor load matrix				Indicator
	Common factor 1	Common factor 2	Common factor 3	Common factor 4	Selection result
Family income	0.606	0.030	0.275	-0.280	Retain
Student race	0.558	0.275	0.109	0.300	Retain
Income after graduation	-0.105	0.646	0.261	0.137	Retain
Admission score	0.067	0.579	0.115	0.290	Retain
Women only	-0.020	0.135	0.039	0.227	Delete
Discipline quantity	0.228	0.490	0.167	0.175	Retain
Local	0.283	0.176	0.407	-0.056	Retain

Based on, table 5 before and after the adjustment of factor loading matrix values were standardized, then It use Amos structural equation modeling software number extraction volume calculation of the two sets of standardized data, composite reliability (CR) and the average variation (AVE), in order to determine optimization adjustment of the results of the rationality.

(3) Indicator optimization results

In summary, based on the structure factor entropy- analysis optimization model, and concludes that the key indicator selection optimization results as specified in table 5 shows.

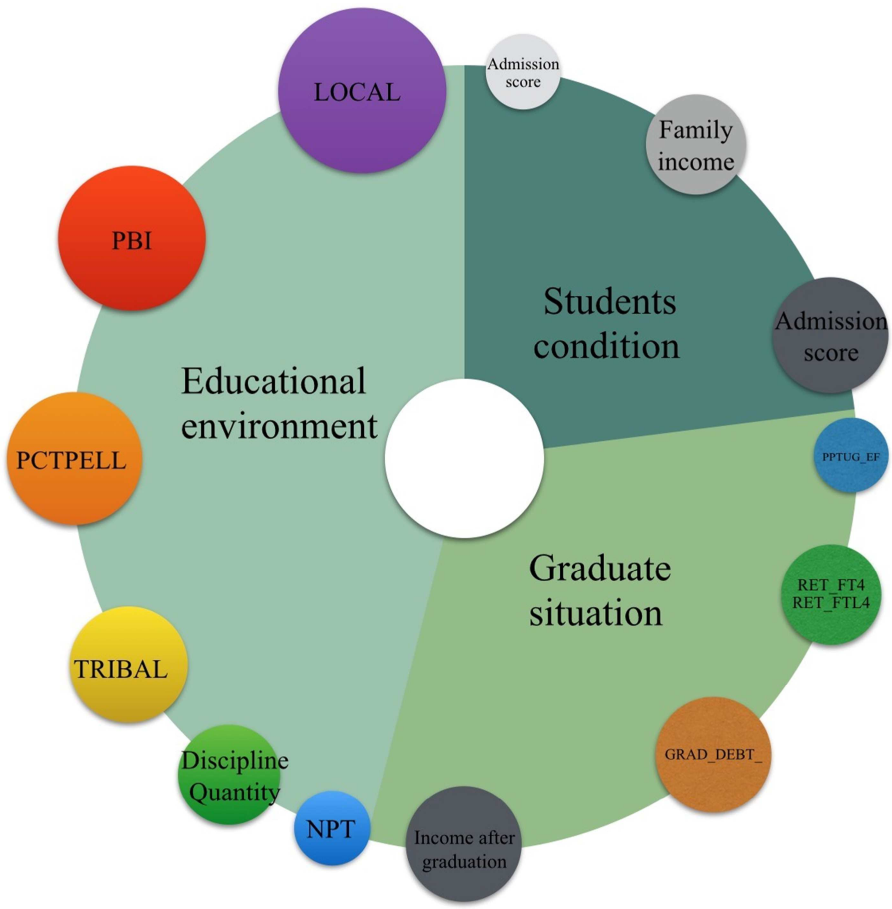

Figure 1. *Selecting results of key indicators of investment benefit.*

5. Model for Classify and Ranking

5.1. Fuzzy Analytic Hierarchy Process

Because many indicators weight involved in this paper is determined by experts or individuals, with people's subjective judgment and the numerical value of judgment factors to will be difference, so we choose the FAHP as the way to solve this problem.

(1) Built the hierarchy structure

Table 6. *The four-hierarchy structure of model.*

Goal	Criteria	Components	Object
		Admission score	School 1
	Condition of students	Family income	School 2
Education investment scheme		Student race	School 3
		Discipline quantity	.
	Educational environment	Local	.
		Income after graduation	School n

(2) Obtain the index weights, built fuzzy judgment matrix

According to the hierarchy of evaluation index system, structure comparison judgment matrix.

$$A = a_{ij}\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{pmatrix}$$

(3) Calculate the eigenvalues and eigenvectors

For the completely consistent comparison matrix, the maximum eigenvalue is equal to the dimension n, so the feature vector is obtained accurately. The greatest eigenvalue denoted by λ_{max}, the corresponding eigenvector denoted by $u = (u_1, u_2, \cdots, u_n)^T$.

$$\lambda_{max} = \sum_{j=1}^{n} \frac{u_i}{u_j}$$

(4) Do a consistency check

$$R.I. = \frac{\lambda_{max} - n}{n-1}$$

The indicator of consistency is

$$CI = \frac{\lambda - n}{n-1}$$

When the consistency ratio $C.R. = \frac{C.I.}{R.I.} < 0.1$, think the consistency of judgment matrix A is meet the requirements.

(5) According to the weight built the fuzzy evaluation matrix

The higher the score of indictors, the more need to be funded.

$$R_i = \{r_1, r_2, \cdots, r_n\}$$

(6) Form flexible comprehensive evaluation index system

$$Q_1 = W_q R_q$$

5.2. Results and Analysis

Based on the above analysis, we obtain the following results, taking the indicator of A as an example, the other processes are the same:

(1) Judging matrix

$$A = \begin{bmatrix} 1 & 4/5 & 2/5 \\ 5/4 & 1 & 1 \\ 5/2 & 1 & 1 \end{bmatrix}$$

(2) Weight vector of criteria level:

$$W_a = (0.21, 0.26, 0.53)$$

(3) Fuzzy evaluation matrix

$$R_a = \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0 & 0.5 & 0.5 \\ 0.2 & 0.8 & 0 \end{bmatrix}$$

(4) Flexible comprehensive evaluation index system

$$A_1 = W_a R_a = (0.21, 0.26, 0.53) \begin{bmatrix} 0.2 & 0.7 & 0.1 \\ 0 & 0.5 & 0.5 \\ 0.2 & 0.8 & 0 \end{bmatrix} = (0.15, 0.7, 0.15)$$

i=1, 2,…, n, represent the school number. In the same way, get flexible comprehensive evaluation index system of B and C and other schools':

$$B_1 = (0.27, 046, 0.27)$$

$$C_1 = (0.33, 0.59, 0.8)$$

(5) Creating remark collective

$V = \{v_1, v_2, v_3\}$ corresponds to $V = \{high, medium, low\}$, value it to $V = \{[90-100), [80-90), [70-80)\}$.

According to the principle of maximum degree of membership, the school of "conduction of students", "educational environment", "graduate situation" belonging in the "medium", "medium ", "medium" grade, as represented by Figure 2.

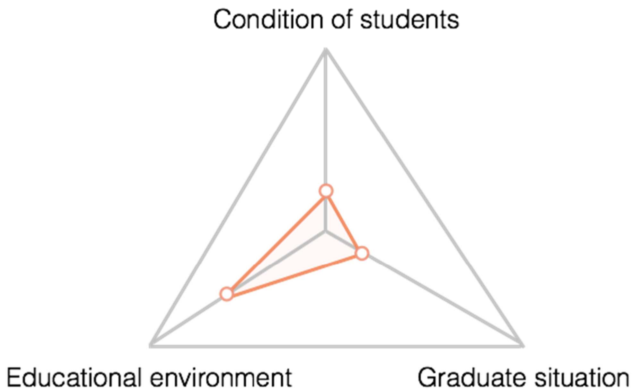

Figure 2. *Evaluation System.*

5.3. Funding Strategies

Similarly, for each candidate school, we use the same method of calculation. According to the situation of the three indicators, we propose four funding strategies, corresponding to four different situations.

Table 7. *Investment grade table.*

Type of indicators	**Funding strategy**
There are two indicators of membership level is "high" at least	A
There is a indicator of membership level is "high", at least one indicator membership level to "Medium"	B
There are at least two membership level to "Medium"	C
There is a membership index rating of "medium" or three indicators are all "low"	D

According to the *problem C*, the foundation of existing $100000000 will be investment in 93 schools in a year. According to the three indicators above, we determine the coverage ratio of each funding strategy as represented by Figure 3.

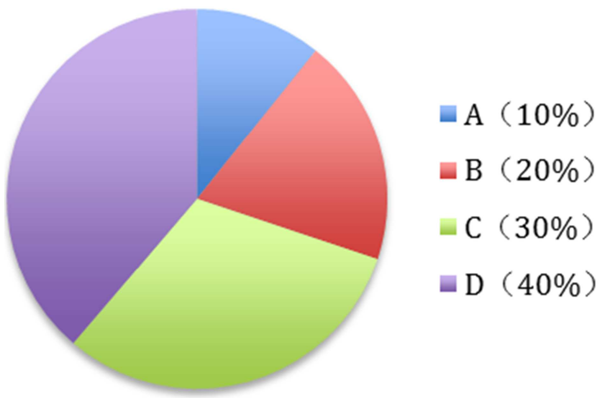

Figure 3. *Investment rating scale.*

Because the number and quality of teachers determine the quality of schools and students in a large extent, so we decided to determine the amount of funding according to the Ratio of teachers and students. According to the ratio, we divide the schools into four categories and different types of investment are different.

Table 8. *Investment strategy.*

Policy type	Investment strategy	
	Student ($/ person)	Teacher ($/ person)
A	500	1200
B	400	1000
C	200	800
D	100	600

In order to achieve the greatest benefit of investment, in the first year, the amount of funding according to the indictors, from the list of schools to select the most need to invest in 93 schools, investment amount mainly based on teacher-student ratio and other indicators in the first year.

From the second year, funded mainly depend on the school's output efficiency, which is evaluated by the investment - return model. Under the circumstances of limited funds and time, according to the assessment results of each school grants to adjust the investment amount to achieve the maximum investment benefit. The process of building investment - risk assessment model in the No. 6 part.

Table 9. *Funding strategy.*

Year	Funding strategy
1	According to the school indicators, establish the model to select the most need to invest in 93 schools, the teacher-student ratio as the main basis to determine the amount of funding
2, 3, 4, 5	According to the benefit of the school in the output, establish the investment - risk assessment model, and adjust the proportion of each school's funding to achieve the best benefit.

Base on the investment criteria, we used the PYTHON programming from t*he National Center for education statistics* to search the teacher-student ratio of the 93 schools, calculated the funding amount of each school to accept. Due to limited space, a part of investment strategies are listed here.

Table 10. *Investment of every school in first year.*

Policy type	INSTNM	Investment ($)
A	Community College of Rhode Island	6556731.6
A	Butte College	4468671.4
A	Chief Dull Knife College	48568.7
A	John C Calhoun State Community College	3343476.5
…	…	…
B	Pasco-Hernando Community College	1201341.2
B	Kalamazoo Valley Community College	2474930.6
B	Passaic County Community College	2424707.04
B	Blackfeet Community College	118126.08
…	…	…
C	Pearl River Community College	1500519.2
C	Northeast State Community College	1378114.56
C	Walters State Community College	1369001.12
C	Los Angeles Southwest College	1702282.6
…	…	…
D	Richland Community College	472777.12
D	Lake Michigan College	505804.8
D	Maysville Community and Technical College	551049.54
D	University of Hawaii Maui College	255296

6. Establishing Investment Returns and Risk Model

6.1. Analysis

First, according to the thinking of the educational funding evaluation indicators system and the principle of setting indicators system, combined with the characteristics of philanthropic investments to determine the preliminary evaluation indicators system. Second, based on the rough set theory and method, design the scientific Evaluation system. Then, combined with the indicators, set up the investment income and risk model the best and the investment portfolio is obtained.

(1) Select scientific evaluation indicators

After investment, we need to assess the return. Of course, many changes are difficult to quantify, but there are still some indicators to measure, these indicators are important basis for the establishment of the assessment model.

We chose a number of indicators to assess the effectiveness of the funding, see table 11 below:

Table 11. *Indicators choice.*

No.	Indicator
1	Income after graduation
2	Graduation rate of students
3	Number of students enrolling
4	The impact of school in society
5	Proportion of school teachers and students
6	Academic research in schools

Of course, we can also add more evaluation indicators, the by professionals to evaluate the performance of schools in funding before and after.

(2) The determination of risk indicators

Any investment activity has its risk. The purpose is to maximize the investment efficiency, all of the risk factors are also very important to consider. The importance of risk indicators also need to be determined, the following is a model to consider some of the risk factors:

- Funded schools will not accept the amount of subsidy cost risk in education;
- Funded schools will not accept funding amount spent in probability to maximize the benefit of the local;
- Accepts funded schools although subsidy cost but the effect is not obvious in the probability of education;

6.2. Modeling

(1) Hypothesis

- All school data can be collected;
- Funded schools in the process of funding will normally operate;
- The weight of each index is set as close as possible to the actual level;

(2) Symbol description

I: *i*=1, funding strategy I; *i*=2, funding strategy II; *i*=3, funding strategy III;

j: *j*=1, 2, 3,…, *n*, the number of school;

s_{ij}: The amount of money received by the *j* School of the *i* class strategy.

CA_i: income of student after graduation before funding;

CF_i: income of student after graduation after funding;

RA_i: Graduation rate of school before funding;

RF_i: Graduation rate of school after funding;

SA_i: Enrollment quantity before funding;

SF_i: Enrollment quantity after funding;

FA_i: The impact degree of school in society before funding;

FF_i: The impact degree of school in society after funding;

TA_i: Ratio of teachers and students before funding;

TF_i: Ratio of teachers and students after funding;

AA_i: Academic research situation before funding (quantify the number of published papers, to apply for a patent);

AF_i: Academic research situation after funding (quantify the number of published papers, to apply for a patent);

M: The risk of the funding school accepted does not spent on education;

p: The risk of the funding school accepted does not spent on the benefit maximization;

q: The risk of the funding school accepted spent on education but the effect is not apparent;

(3) Establish the model of investment return and risk

$$\max_{1\to n} MAX(RIO) = \sum_{\substack{i=1,2,3 \\ j=1}}^{j\to n} s_{ij} \times (\frac{CA_{ij}}{CF_{ij}} \times \frac{RA_{ij}}{RF_{ij}} \times \frac{SA_{ij}}{SF_{ij}} \times \frac{FA_{ij}}{FF_{ij}} \times \frac{TA_{ij}}{TF_{ij}} \times \frac{AA_{ij}}{AF_{ij}}) \times (\frac{m_{ij}+p_{ij}+q_{ij}}{3})$$

$$\sum_{j=1}^{n} s_j = US100million$$

According to the formula, we can draw a variety of different investment strategies. Select RIO that can make the largest investment portfolio in strategies to achieve the largest investment benefit and the least risk.

(4) Inspection

Through the calculation of the last model, we can get the most benefit of the investment portfolio. There is no need to specialized test.

7. Conclusion

To help solve the Goodgrant Foundation Education Fund investment issues, the report above makes the Investment program. First of all, on the basis of the investment risk minimization principle, this paper screens the 93 schools which are necessary to be invested. And, the Structure Entropy and Factor Analysis optimization model is established to select the key indicators closely related investment returns. Then, based on the selected key indicators, the FAHP model is built to sort the 93 schools, and combine with the ABC analysis, the amount of investment in each school will be determined. Finally, by establishing the investment return and risk model, the optimal portfolio strategy is made. Overall, this report will effectively help solve the Goodgrant Foundation Education Fund investment issues.

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