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Non oscillatory nonlinear differential systems with slowly varying coefficients in presence on certain damping forces

Pinakee Dey

Department of Mathematics, Mawlana Bhashani Science and Technology University, Santosh, Tangail-1902, Bangladesh

Email address:
pinakee_68@yahoo.com (P. Dey)

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Abstract: Krylov-Bogoliubov-Mitropolskii method is modified and applied to certain damped nonlinear systems with slowly varying coefficients. The results obtained by this method show excellent coincidence with those obtained by numerical method. The method is illustrated by an example.

Keywords: Perturbation Methods, Varying Coefficient, Unperturbed Equation, Nonlinear Differential Systems, Damped System

1. Introduction

The Krylov-Bogoliubov-Mitropolskii (KBM) method [1-3] is well known in the theory of nonlinear oscillations. The method was originally developed by Krylov and Bogoliubov [1] for obtaining the periodic solution of non linear systems with small nonlinearities. Then the method was amplified and justified by Bogoliubov and Mitropolskii [2]. Mitropolskii [3] has extended the method to nonlinear differential system with slowly varying coefficients. On the other hand, Popov [4] extended the method to nonlinear damped oscillatory systems with constant coefficients. Murty, Deekshatulu, and Krisna [5] investigated an over damped nonlinear system using Bogoliubov's method. Murty [6] presented a unified KBM method for solving second order nonlinear systems which cover the un-damped, damped and over-damped cases. Shamsul [7] has presented a unified method for solving an *n*-th order differential equation (autonomous) characterized by oscillatory, damped oscillatory and non-oscillatory processes. Hung and Wu [8] obtained an exact solution of a differential system in terms of Bessel's functions where the coefficients varying with time in an exponential order. Roy and Shamsul [9] found an asymptotic solution of a differential system in which the coefficient changes in an exponential order of slowly varying time. Bojadziev and Edwards [10] studied some damped oscillatory and purely non-oscillatory systems with slowly varying coefficients. Recently Pinakee *et.al* [11-12] has presented extended KBM method (by Popov [4]) for solving nonlinear problems in which the coefficients change slowly and periodically with time. In accordance to Pinakee *et.al* [11-12] observation, [10]'s solution is not useful for strong damping effects. The aim of this article is to find an approximate solution for strong damping effect with varying coefficients in where [10]'s solution is unable to give desired results.

2. Materials and Method

Let us consider the nonlinear differential system

$$\ddot{x} + 2k(\tau)\dot{x} + \omega^2(\tau)x = -\varepsilon f(\dot{x}, x, \tau), \quad \tau = \varepsilon t \quad (1)$$

where the over-dots denote differentiation with respect to t, ε is a small parameter, $k = O(\varepsilon)$, f is a given nonlinear function and $\omega(\tau)$ is known as frequency. For $\varepsilon = 0$ and $\tau = \tau_0$ = constant, $k(\tau_0) \pm i\omega(\tau_0)$ are two eigen values of the unperturbed equation of (1) and has the solution

$$x(t,0) = \alpha_0 e^{-k(\tau_0)t} \cos(\omega_0(\tau_0)t + \varphi_0), \quad (2)$$

where α_0 and φ_0 are arbitrary constant and τ_0 represents the value of τ when $\varepsilon = 0$.

When $\varepsilon \neq 0$ we seek a solution in accordance with the KBM method, of the form

$$x(t,\varepsilon) = \alpha\cos\varphi + \varepsilon u_1(\alpha,\varphi,\tau) + \varepsilon^2 \quad (3)$$

where α and ϕ satisfy the equations

$$\begin{aligned}\dot{\alpha} &= -k(\tau) + \varepsilon A_1(\alpha,\phi,\tau) + \varepsilon^2 ...,\\ \dot{\phi} &= \omega_0(\tau) + \varepsilon B_2(\alpha,\phi,\tau) + \varepsilon^2 ...,\end{aligned} \quad (4)$$

Confining attention to the first few term 1, 2…m in the series expansion of (3) and (4), we evaluate functions u_1, A_1 and B_1 such that $\dot{\alpha}$ and $\dot{\phi}$ appearing in (3) and (4) satisfy (1) with an accuracy of ε^{m+1}. In order to determine these unknown functions it was early assumed by Murty [6], Shamsul [7] that the functions u_1 exclude all fundamental terms, since these are included in the series expansion (3) at order ε^0.

Now differentiating (6) twice with respect to t, substituting for the derivatives $\ddot{x}$, $\dot{x}$ and x in (1), utilizing relation (4) and comparing the coefficients of ε, we obtain

$$\begin{aligned}&\omega_0\alpha\sin\varphi\begin{pmatrix}k\alpha\frac{\partial}{\partial\alpha}A_1 - kA_1\\ -2\omega_0\alpha B_1\end{pmatrix}\cos\varphi\\ &-\sin\varphi\begin{pmatrix}-k\alpha^2\frac{\partial}{\partial\alpha}B_1\\ +2\omega_0 A_1\end{pmatrix}+\\ &\left(-k\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\varphi}\right)^2 u_1 + 2k\begin{pmatrix}-k\alpha\frac{\partial}{\partial\alpha}\\ +\frac{\partial}{\partial\varphi}+\omega^2\end{pmatrix}u_1\\ &= -f^{(0)}(\alpha,\varphi,\tau),\end{aligned} \quad (5)$$

where $f^{(0)} = f(x_0,\dot{x}_0)$ and $x_0 = \alpha\cos\phi$

It is assumed that both $f^{(0)}$ can be expanded in Fourier series [7]

$$f^{(0)} = \sum_{n=0}^{\infty} F_n(\alpha)\cos n\phi + G_n(\alpha)\sin n\phi\ , \quad (6)$$

and

$$\begin{aligned}&u_1(\alpha,\varphi) = U_0(\alpha) + \sum_{n=2}^{\infty} U_n(\alpha)\cos n\varphi\\ &+V_n(\alpha)\sin n\varphi\end{aligned} \quad (7)$$

Substituting the expression for $f^{(0)}$ and u_1 in (8), we obtain the following equations for A_1, B_1 and u_1 as

$$k'\alpha - k\alpha\frac{\partial A_1}{\partial\alpha} + kA_1 - 2\omega_0\alpha B_1 = -F_1 \quad (8)$$

$$-\omega_0'\alpha - 2\omega_0 A_1 + k\alpha^2\frac{\partial B_1}{\partial\alpha} = -G_1 \quad (9)$$

and

$$\begin{aligned}&\left(\left(-k\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\varphi}+k\right)^2 + \omega_0^2\right)u_1\\ &= -F_0 - \sum_{n=2}^{\infty} F_n(\alpha)\cos n\varphi + G_n(\alpha)\sin n\varphi\end{aligned} \quad (10)$$

The particular solution of (8)-(10) gives three unknown functions A_1, B_1 and u_1 which complete the determination of the first order Bojadziev and Edwards [10] solution of (1). It is clear that both functions A_1 and B_1 is independent of phase variable ϕ and u_1 excludes all first harmonic terms. In accordance to [7] assumption A_1, B_1 and u_1 satisfy the following equations (instead of (8)-(10))

$$k'\alpha - k\alpha\frac{\partial A_1}{\partial\alpha} + kA_1 - 2\omega_0\alpha B_1 = -F_1\cos^2(\phi-\omega t) \quad (11)$$

$$-\omega_0'\alpha - 2\omega_0 A_1 + k\alpha^2\frac{\partial B_1}{\partial\alpha} = -G_1\cos^2(\phi-\omega t) \quad (12)$$

$$\begin{aligned}&\left(\left(-k\alpha\frac{\partial}{\partial\alpha}+\frac{\partial}{\partial\varphi}+k\right)^2 + \omega_0^2\right)u_1\\ &= -F_0 - (F_1\cos\varphi + G_1\sin\varphi)\sin^2(\varphi-\omega t) -\\ &\sum_{n=2}^{\infty} F_n(\alpha)\cos n\varphi + G_n(\alpha)\sin n\varphi\end{aligned} \quad (13)$$

The particular solution of (11)-(13) gives three unknown functions A_1, B_1 *and* u_1. Thus the determination of the first order solution is clear. In this case A_1 and B_1 depend on both α and ϕ and u_1 is not independent of first harmonic terms.

Example: We consider a second order nonlinear system with constant and slowly varying coefficient

$$\ddot{x} + 2k(\tau)\dot{x} + \omega(\tau)x = -\varepsilon x^3, \quad (14)$$

Here $f = x^3$, $f^{(0)} = \alpha^3\cos^3\alpha = \frac{1}{4}\alpha^3(3\cos\phi + \cos 3\phi)$; so that non-zero coefficients are $F_1 = \frac{3}{4}, F_3 = \frac{1}{4}$. Substituting the values pf F_1 and F_3 into (11)-(13) and solving them, we obtain

$$\begin{aligned}A_1 &= -\frac{k'\alpha}{2\omega_0} + \frac{3k\alpha^3\cos^2\varphi_0}{8\omega^2},\\ B_1 &= \frac{-\omega_0'}{2\omega_0} + \frac{3\omega_0\alpha^2\cos^2\varphi_0}{8\omega^2}\end{aligned} \quad (15)$$

and

$$u_1 = \frac{3\alpha^3(-\cos\phi + \omega_0 \sin\phi / k)\sin^2\phi_0}{16\,\omega^2} + \frac{\alpha^3(-(k^2 - 2\omega_0^2)\cos 3\phi + 3k\omega_0 \sin 3\phi}{16\,\omega^2(k^2 + 4\omega_0^2)} \tag{16}$$

$$A_1 = -\frac{k'\alpha}{2\omega_0} + \frac{3k\alpha^3}{8\omega^2}, \quad B_1 = \frac{-\omega_0'}{2\omega_0} + \frac{3\omega_0\alpha^2}{8\omega^2} \tag{17}$$

and

$$u_1 = \frac{\alpha^3(-(k^2 - 2\omega_0^2)\cos 3\phi + 3k\omega_0 \sin 3\phi}{16\omega^2(k^2 + 4\omega_0^2)} \tag{18}$$

Now substituting the functional values of A_1, B_1 from (15) into (4), we obtain

$$\dot{\alpha} = -k(\tau)\alpha - \frac{k'\alpha}{2\omega_0} + \frac{3k\alpha^3\cos^2\phi_0}{8\omega^2}, \quad \dot{\phi} = \omega(\tau) - \frac{\omega_0'}{2\omega_0} + \frac{3\omega_0\alpha^2\cos^2\phi_0}{8\omega^2} \tag{19}$$

Therefore, the first order solution of the equation (14) is

$$x(t,\varepsilon) = \alpha\cos\phi + \varepsilon u_1 \tag{20}$$

where α and ϕ are the solution of the equation (19) u_1 is given by (16). Substituting the values of A_1, B_1 from (17) into (4) and solving them, [10] found the solution of (4) similar to (19). In this paper, we have used the Runge-Kutta (fourth order) method. Numerically, it is advantageous; a large step size can be used in the integration (see [13] for detail).

3. Results and Discussions

In order to test the accuracy of an approximate solution obtained by a certain perturbation method, one compares the approximate solution to the numerical solution (considered to be exact). With regard to such a comparison concerning the presented KBM method (by Popov [4]) of this article, we refer to the works of Murty [6] and Shamsul [7]. In our present paper, for different damping forces, we compare solution (20) and [10]'s solution.

First of all, for damping force $k(\tau) = 0.5e^{-\frac{.1}{2}\tau}$ and for $\varepsilon = .1$, x is calculated by (20) with initial conditions [$x(0) = 1.00000$ $\dot{x} = -0.00000$]. Then $x(\varepsilon,t)$ has been computed by [10]'s solution. Finally the numerical solution has been obtained and percentage error has been calculated. All the results are shown in Table 1. From Table 1 it is seen that errors of the results obtained by solution (20) and [10]'s solution are less than 1% and on an average percentage errors of [10]'s solution are less than those computed by solution (20). Secondly, for damping force $k(\tau) = .707e^{-\frac{.1}{2}\tau}$ and for $\varepsilon = .1$, x is calculated by (20) with initial conditions [$x(0) = 1.0000($ $\dot{x} = -0.00000$]. The results are given in Table 2. Table 2 shows that percentage errors of solution (20) occur in an order of 1% (except at ($t = 3$ and $t = 8$), while errors of [10]'s are many times greater than 1% and almost twice of those obtained by solution (20). If the damping force is increased and strong, solution (20) shows a good coincidence with the numerical solution. Contrary, errors of [10]'s solution increase.

Table 1

t	x	x_{nu}	$E\%$	x_{BE}	$E_{BE}\%$
0.0	1.000000	1.00000	0.0000	1.000000	0.0000
1.0	0.642193	0.641101	0.1703	0.641957	0.1335
2.0	0.136156	0.134311	1.3744	0.134611	0.2241
3.0	-0.140639	-0.141801	-0.8195	-0.142327	0.3709
4.0	-0.192790	-0.192868	-0.0404	-0.193747	0.4558
5.0	-0.131988	-0.131299	0.5248	-0.132070	0.5872
6.0	-.050690	-0.049836	1.7136	-0.050253	0.8367
8.0	0.033573	0.033807	-0.6922	0.033981	0.5147
10	0.027473	0.027256	0.7962	0.027513	0.9429

Table 2

t	x	x_{nu}	$E\%$	x_{BE}	$E_{BE}\%$
0.0	1.000000	1.000000	0.00000	1.000000	0.00000
1.0	0.678614	0.677989	0.0922	0.684010	0.8881
2.0	0.265293	0.263317	0.7504	0.268888	2.1157
3.0	0.03068	0.031519	4.9145	0.034209	8.5345
4.0	-0.049626	-0.050305	-1.3498	-0.049713	-1.1768
5.0	-0.057106	-0.057181	-0.1311	-0.057548	0.6418
6.0	-0.039592	-0.039411	0.4593	-0.039994	1.4793
8.0	-0.007227	-0.007075	2.1484	-0.007350	3.8869
10	0.002924	0.002941	-0.5780	0.002932	-0.3060

4. Conclusion

Based on the works of extended KBM method an approximate solution of a second order nonlinear deferential system with slowly varying coefficients has been found. In this paper solution (20) gives satisfactory results for the strong damping effect but Bojadzeiv and Edwards's [10] solution fail to give desire result.

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