



# Energy Levels Calculations of $^{24}\text{Al}$ and $^{25}\text{Al}$ Isotopes

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**Abstract:** In this article, the energy levels of Aluminum isotopes  $^{24}\text{Al}$  and  $^{25}\text{Al}$  are calculated using OXBASH shell model code. The calculations were carried out in the SD model space with the W and CW Hamiltonian [1, 2] using the code OXBASH for windows PC [3]. This code which is based on one of the most applicable nuclear models, the shell model, deals with evaluating energy levels in nuclei. Using this code to calculate the energy levels of an isotope, produces several files that contains a set of data. The ground state energy of a nucleus and also the probable energies of excitation levels can be calculated by OXBASH. Programs will be reliable only when results meet experimental procedures. A compilation of SD-shell energy levels calculated with the USD Hamiltonian was published around 1988. A comparison had been made between our results and the available experimental data to test theoretical shell model description of nuclear structure in Aluminum isotopes. The calculated energy spectrum is in good agreement with the available experimental data.

**Keywords:** Aluminum Isotopes, OXBASH Code, Shell Model Structure, USD Interaction

## 1. Introduction

Obtaining the nuclear structure and energy levels of nuclei is one of the criteria to improve investigations of nuclei properties. Nuclear models have the property to help us to better understanding of nuclear structure which contains main physical properties of nuclei, and shell-model is one of the most prominent and successful nuclear models [4-11]. This model can be compared with the electron shell model for atoms. As atomic behavior and properties can be described with valance electrons which exist out of a closed shell, similarly, valance nucleons (protons or neutrons) in a nucleus which are placed out of close shells (with magic numbers 2,8,20,28,50,82 and 126) play important roles in determining nuclear properties. Nuclei with magic numbers are very stable and have completely different properties comparing with their neighbors.

Existence of spatial levels is determined by Pauli Exclusion Principle. By knowing nuclear potentials for all nucleons in a nucleus, one can calculate energy levels.

For light nuclei, there are several “standard” effective interactions for the  $p$  and  $SD$  shells, respectively [12, 13]. Analysis of neutron-rich SD nuclei has been of high interest in recent years as they present new aspects of nuclear structure [14]. Traditional shell-model studies have recently

received a renewed interest through large scale shell-model computing in no-core calculations for light nuclei, the  $1s0d$  shell, the  $1p0f$  shell and the  $3s2d1g_{7/2}$  shell with the inclusion of the  $0h_{11/2}$  intruder state as well. It is now therefore fully possible to work to large-scale shell-model data and study the excitation levels for large systems. In these systems, inter core is assumed and space is determined by considering shell gaps. Figure 1 shows the PF and SD model spaces according to shell model theory.

### The valence space of two major shells

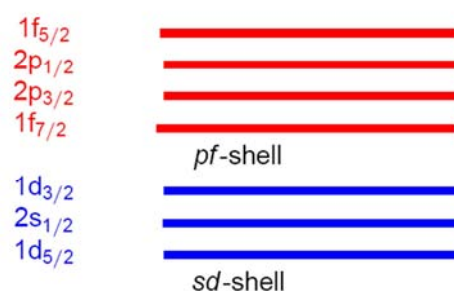


Figure 1. Sd and pf shell model spaces.

## 2. Theory and Calculation

In order to calculate the nuclear structure properties of both ground and excited states based on the nuclear shell model one needs to have wave functions of those states. These wave functions are obtained by using the shell-model code OXBASH [12]. OXBASH (Oxford- Buenos Aires Shell Model Code) is a powerful computer code to calculate the energy levels of light and medium nuclei. By using it, we can measure the energy levels of the nucleus and compare it with experimental data. One of the most important challenges in nuclear physics is to calculate and measure energy levels of medium and heavy nuclei by using shell model theory. As a result, it is important to check computer codes in this area. There are many codes written for this purpose such as: NutShell, REDSTICK, ANTOINE, OXBASH, etc. In this paper we are using OXBASH code, a code which has been developed in this field for many years.

Experimental studies on the stability of some nuclei show that nuclei with protons or neutrons numbers (2, 8, 20, 50, 82, etc.) are more numerous and stable. In other words, the existence of these magic numbers suggests shell model structure in nuclear physics similar to atomic physics. In this paper, the energy levels of Na and Mg isotopes have been measured using the code OXBASH. The program includes a set of computational code which is based on the ability to measure the energy levels by forming ground state matrices with dimensions up to 2000000 and JT matrix with dimension up to 100,000. The version of this code is 2005-8 which can be installed and used on any operating system without using any other additional software.

### Space model SD

Considering the number of valence nucleons for sodium and magnesium isotopes, SD model space is the suitable model for these calculations. This model consists of  $2s_{1/2}, 1d_{3/2}, 1d_{5/2}$  valence orbitals. The code OXBASH for Windows has been used to calculate the nuclear structure for Aluminum nucleus, by employing the SD (independent charges) and SDPN (depending charges) model space with three effective interactions [3]. The first interaction for the lower part of the SD-shell is Chung-Wildenthal particle interaction (CW), secondly, the Universal SD-shell Hamiltonian (USD interaction). In the third interaction the New Universal SD-shell Hamiltonian (USDAPN) is used. Richter *et al* [15] used this shell model successfully in the p-shell, and fp-shell [16], [17] and Wildenthal [2] and Brown *et al* [18] in the SD-shell to describe the systematic observed in the spectra and transition strengths. In the present work, we focus our attention on the description of energy levels of SD shell of Aluminum isotopes  $^{24}\text{Al}$  and  $^{25}\text{Al}$  which have configurations  $0d_{5/2}, 1s_{1/2}$  and  $0d_{3/2}$ .

The basis of Hamiltonian in shell model calculations is normalized Green matrix. For SD model space in OXBASH code, 13 different interaction potentials to perform computations on different nuclei is included [3].

The first normalized G matrix was introduced in 1960 which its results were in good agreement with experimental data for  $A = 18$  to  $A = 38$  nuclei [17, 18], however, with

increasing nucleon numbers, solving this matrix was very difficult [19]. In 1976 for the bottom layer of SD model space, the Hamiltonian CWH [3, 17] was introduced and in 1984 the potential W was introduced for the entire layer SD [21].

OXBASH code is a computer program that is described with a set of model spaces and interactions to apply in shell model calculations with high dimensions.

In order to use this code one should specify the model space and interaction. In other words after choosing appropriate model space which is chosen considering valance nucleons, this code constructs a set of possible ground states and then makes JT matrix based on linear combination of ground states which give suitable T and J values. Finally by choosing the desirable interaction potential it constructs the Hamiltonian of the problem and carries out the calculations and as a default gives 10 lowest energies.

The applied model space illustrates the orbitals which are considered in calculations using main shells in shell-model theory.

In the second line of approach the two body matrix elements are treated as parameters, and their values are obtained from best fit to experimental data [18].

Brown and coworkers [19] have carried extensive studies of energy level and spectroscopic properties of SD-shell nuclei in terms of a unified Hamiltonian applied in full SD-shell model space. The universal Hamiltonian was obtained from a least square fit of 380 energy data with experimental errors of 0.2MeV or less from 66 nuclei. The USD Hamiltonian is defined by 63 SD-shell two body matrix element and their single particle energies. In more recent work Brown and co-workers have modified USD type Hamiltonian to USDA and USDB based on updated set of binding energy and energy levels of O, F, Ne, Na, Mg and P isotopes [22].

## 3. Results

The isotopes of Aluminum  $^{24}\text{Al}$  and  $^{25}\text{Al}$  provide a unique work frame for examining the foundation of SD shell model calculations. The nucleons of the core of these isotopes are 8 protons and 8 neutrons which are in the  $(1s_{1/2}, 0p_{3/2}, 0p_{1/2})J=0, T=0$  ground state configuration of  $^{16}\text{O}$  and the remaining nucleons are distributed over all possible combinations of the  $0d_{5/2}, 1s_{1/2}$  and  $0d_{3/2}$  orbits according to Pauli Exclusion Principle. The package of program called "SHELL" was used to generate the One Body Density Matrix Element (OBDME), and the package of program called "LPE" is used to calculate the wave functions and energy levels.

We present here some results concerning ground and excitation energies of the Al isotopes for which recent data has been reported in the literature. Table 1 and figure 2 shows data for  $^{24}\text{Al}$  isotope and table 2 and figure 3 shows data for  $^{25}\text{Al}$  isotope. E (OXBASH) are energies calculated in this work and E(exp) are experimental data.

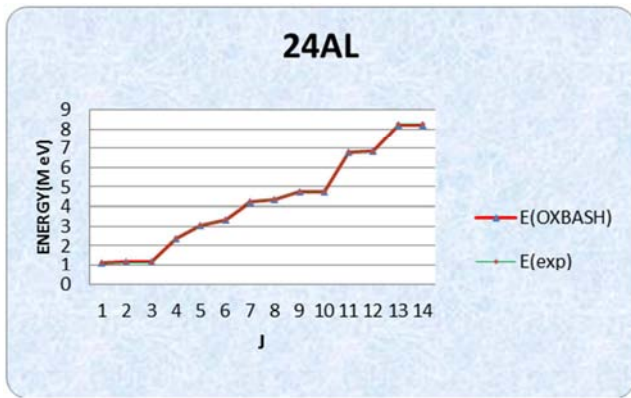


Figure 2. Comparison Calculated data with Experimental data for  $^{24}\text{Al}$ .

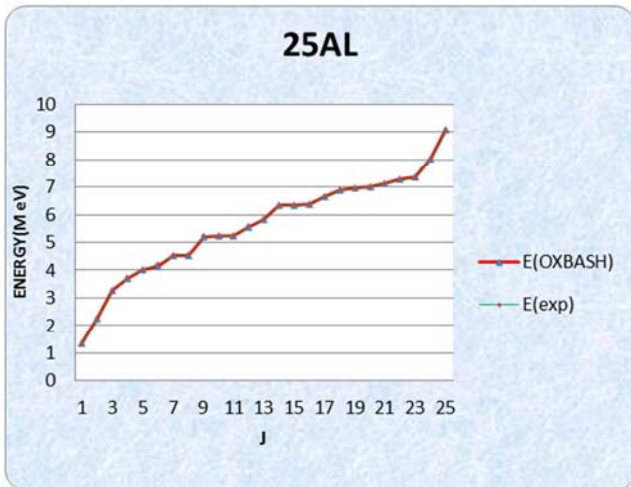


Figure 3. Comparison Calculated data with Experimental data for  $^{25}\text{Al}$ .

Table 1. Data for  $^{24}\text{Al}$ . All energies are in MeV.

E(exp)	E(OXBASH)	J/AL24
1.110	1.155	1→J=2
1.130	1.155	2→J=2
2.380	2.378	3→J=2
4.340	4.369	4→J=2
3.060	3.031	5→J=3
4.770	4.736	6→J=3
4.258	4.259	7→J=4
4.770	4.728	8→J=4
1.110	1.101	9→J=5
6.810	6.822	10→J=5
6.810	6.808	11→J=6
8.250	8.205	12→J=7
8.250	8.183	13→J=8
3.350	3.331	14→J=3

## 4. Conclusions

As seen from all figures, very good agreement is obtained for most of energy levels of Aluminium isotopes, and the ordering of levels is correctly reproduced. Unfortunately, not enough experimental data were available for  $^{24}\text{Al}$ ; regarding the USD data we can judge that almost all calculations reasonably reproduce the observed level structure.

Table 2. Data for  $^{25}\text{Al}$ . All energies are in MeV.

E(exp)	E(OXBASH)	J/AL25
4.514	4.546	1→J=1/2
4.583	4.546	2→J=1/2
8.026	8.025	3→J=1/2
9.065	9.078	4→J=1/2
3.744	3.715	5→J=3/2
6.288	6.330	6→J=3/2
6.327	6.330	7→J=3/2
7.300	7.344	8→J=3/2
1.306	1.352	9→J=5/2
4.196	4.129	10→J=5/2
5.232	5.200	11→J=5/2
5.232	5.247	12→J=5/2
6.909	6.969	13→J=5/2
2.236	2.237	14→J=7/2
5.232	5.244	15→J=7/2
6.327	6.393	16→J=7/2
6.881	6.883	17→J=7/2
7.022	7.015	18→J=7/2
7.112	7.139	19→J=7/2
3.251	3.254	20→J=9/2
4.026	4.034	21→J=9/2
5.582	5.563	22→J=9/2
5.809	5.809	23→J=9/2
6.645	6.637	24→J=9/2
7.240	7.261	25→J=9/2

In general the best and most complete results are found with the largest model space while calculations in an infinite space are not possible and the computation time increases exponentially with model space size so some truncation is required. Also the interaction used must be appropriate for the model space. The empirical interactions are (usually) better determined for smaller model spaces. The model space in OXBASH is defined by the active valance nucleon orbits and our calculated results are reasonably consistent with experimental data, although the structure of odd-even nuclei is much more complicated than their odd-odd neighbours.

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