Mathematical Modelling and Applications
2021; 6(4): 101-106
http://www.sciencepublishinggroup.com/j/mma
doi: 10.11648/j.mma.20210604.13
ISSN: 2575-1786 (Print); ISSN: 2575-1794 (Online)

Novel Method for Solving Balance and Unbalance Assignment Problem Using New Approach for Ant Colony Algorithm

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To cite this article:
Ekanayake E. M. U. S. B. Novel Method for Solving Balance and Unbalance Assignment Problem Using New Approach for Ant Colony Algorithm. *Mathematical Modelling and Applications*. Vol. 6, No. 4, 2021, pp. 101-106. doi: 10.11648/j.mma.20210604.13

Received: October 11, 2021; **Accepted**: November 4, 2021; **Published**: December 24, 2021

Abstract: Assignment Problem (AP) is the fundamental application of TP studied in the area of Operations research. The AP is also an essential problem in the field of optimization, where the goal is to optimize production time or cost by allocating one task to one machine, one machine to one job, one destination to one source, or one source to one destination. Various solutions for resolving the assignment problem have been offered in the literature. Excellent response to task problems is obtained using the Hungarian technique. In this study, however, we are discussing a novel alternative strategy that is the almost best performance. Here, we are looking at another technique to resolve algorithm and solution steps AP. In addition, we present an innovative alternative strategy that provides the best performance of APs by utilizing a modified ant colony optimization (ACO) algorithm. A few revisions to the ant colony algorithm (Transition Rule and Pheromone Update Rule) are created, resulting in a solution that is extremely close to the ideal solution. This method is also to be noticed that, requires the least number of steps to reach optimality as compare the obtained results with other well-known meta-heuristic algorithms. Finally, a numerical example is provided to demonstrate this procedure.

Keywords: Optimal Solution, Assignment Problem, Feasible Solution, Alternate Method, Hungarian Method

1. Introduction

The AP represents a subset of the transportation problem. It can be found in a wide variety of decision-making situations. The personnel AP differentiates itself by assigning only one worker to one task. In general, the AP consists of n tasks that must be assigned to n workers, each of whom has different competencies in completing each task [15]. AP develops in a variety of decision-making situations when a collection of jobs must be assigned to a group of machines or employees. And so that the overall resource consumption to execute all duties can be minimized. The best resource could be the cost of an assignment, time spent on doing tasks, mileage, and so on. To solve the AP, various strategies have been suggested in the literature. Kuhn's [19] Hungarian technique was one of the better methods available in the literature for determining the optimal AP solution. In the honor of two Hungarian mathematicians Dénes Konig and Jeno Egerváry who worked for this method, he gave the name of method "Hungarian method". Kurtzberg [20] provided a strategy for approximating the large-scale assignment problem.

Many scholars have investigated variants of traditional AP, such as the Semi-Assignment problem [3], the bottleneck assignment (BAP) [24], generalized AP [22]. Sreeja [26] and Dhanashri et al. [7] proposed innovative approaches to the AP. K. Ghadle and colleagues [16, 17] proposed a novel approach for solving both balanced and unbalanced AP. Sasaki [25] also devised an innovative approach for dealing with one-sided assignment problems. Maxon and Bhadury [21] proposed a problem of repetitive work assignments and attempted to incorporate a human factor into the research. Bogomolnaia and Moulin [5] created a straightforward random assignment problem with a unique solution. Sreeja [28] described assignment problems and comparative studies, while Votaw [30] was created specifically for assignment difficulties. To handle scheduling difficulties with irregular cost functions, Sourd [27] investigated the continuous assignment problem. Odior et al. [23] investigated the efficacy of feasible solutions to assignment problems.

Furthermore, how ants can find the shortest paths between food sources and their home. These concepts are based on the typical behavior of ants. This concept's growth is dependent on the probabilistic method known as finding good pathways across graphs. This is known as the Ant Colony Optimization algorithm (ACO), and it was proposed by Marco Dorigo [11-13]. While traveling in this manner, the ants store a chemical component that aids in communication among themselves. When finding their shortest paths between food sources and their home, waymarked by strong pheromone focuses to guide them. Because ants can detect pheromones and choose the optimal path. Dorigo, Maniezzo, and Colorni proposed the ant system in the literature [9, 10]. Dorigo devised the ant algorithm with elitist ants [14]. Following that, many writers researched ACO, such as the max-min ant system (Stützle&Hoos, 2000), the ant algorithm with additional reinforcement (Fidanova, 2002), and the best-worst ant system (Cordón, Fernàndez de Viana, & Herrera, 2002), solving budget constrained and unconstrained dynamic facility layout problems [2], Ant Colony System and Genetic Algorithm for TSPs [8], among others. Many optimization problems, including assignment problems, have been solved using an ant colony algorithm.

A few solutions for addressing assignment problems in an Ant colony environment are proposed in the literature, such as ACO for the cell assignment problem in PCS networks (2006), Improved Ant Colony Algorithm for Solving Assignment Problem [6], and so on. In this paper, we research the balanced and unbalanced assignment problems, with a few modifications of the ACO algorithm are proposed to find the best performance.

2. Mathematical Formulation of Assignment Problem

A cost table is connected with each assignment problem [4, 18]. In general, the row comprises the objects or people we wish to assign, and the column comprises the jobs or tasks we want them assigned to. Consider the challenge of allocating n resources to m activities in such a way that the overall cost or time is minimized while each resource is associated with one and only one job. The cost table (k_{ij}) is as follows:

Table 1. *Assignment problem cost table.*

		Activity				
		A_1	A_2	…	A_n	*Available*
Resource	R_1	k_{11}	k_{12}	…	k_{1n}	1
	R_2	k_{21}	k_{22}	…	k_{2n}	1
	⋮	⋮	⋮	⋮	⋮	⋮
	R_n	k_{n1}	k_{n2}	…	k_{n3}	1
	Required	1	1	…	1	

The cost table is the same as that of a transportation problem except for that availability at each of the resources and the requirement at each of the destinations is unity.

Let x_{ij} denote the assignment of ith resource to jth activity, such that|:

$$x_{ij} = \begin{cases} 1; \text{if resource } i \text{ is assigned to activity } j \\ 0; otherwise \end{cases}$$

The mathematical model of the assignment problem for parallel processing can be described by an objective function minimizing the total cost of realizing processes:

$$Min\ z = \sum_{i=1}^{n} \sum_{j=1}^{n} k_{ij}\, x_{ij} \tag{1}$$

and the following constraints:

$$\sum_{i=1}^{n} x_{ij} = 1, j = 1,2, \dots, n \tag{2}$$

$$\sum_{j=1}^{n} x_{ij} = 1, i = 1,2, \dots, n \tag{3}$$

$$x_{ij} \in \{0,1\}, i = 1,2, \dots, n, j = 1,2, \dots, n \tag{4}$$

where:

x_{ij} – a binary variable for modeling a decision of selecting processes' modes; the variable assumes the value of 1 if the activity i is to be executed by the resource (crew) j, and equals 0 in the other case. According to equations (2) and (3), considering equal numbers of crews and activities, each crew may be assigned only to one process and each process may be realized by only one crew.

3. Improvement Model of Modified Ant Colony Algorithm

To explore a new way of solving assignment problems. We propose an improved ant colony algorithm for the assignment problem.

Ants communicate to each other by leaving pheromones along their path, so the area where ants head inside and around their underground insect settlement is a stigmergic (significant stimuli) system. In numerous insect species, ants walk around or to a food source store on the ground a substance called pheromone. Various ants can detect this pheromone, and its composition influences their behavior, i.e., they will generally pursue strong pheromone obsessions. The pheromone that is deposited on the ground creates a pheromone trail, which helps the ants to discover new sources of food that have recently been discovered by other ants. The ants will leave the home, find food, and return to the home using unexpected walks and pheromones inside the ground including one home and one food source. After some time, the ants' path will converge on the most constrained path.

When ants migrate from source to objective, they leave a chemical, pheromone, to mark their paths. This makes the going with ants find the way for their associates as they recognize pheromone and pick, in probability, ways having a progressively imperative concentration of pheromone. The algorithm works by changing the pheromone on routes at each node in real-time. The selection of this node is guided by a probability-based approach. Here the new path based on the probability from the place i to place j for the k-th ant as shown in Equation (5)

$$P_{ij} = \frac{\Omega^{\vartheta}{}_{ij}\mathcal{L}_{ij}{}^{\omega}}{\sum_k \Omega^{\vartheta}{}_{ik}\mathcal{L}_{ik}{}^{\omega}} \tag{5}$$

where $\mathcal{L}_{ij}$, the heuristic information or visibility of arc *(i, j),* is the inverse of the distance between city i and city j. Where, Ω_{ij} is the values of the move's pheromone trail level and some heuristic information, respectively, which correspond to the link (i, j). During component selection ϑ, and ω are both parameters and are used to control the importance of the pheromone trail and heuristic information during component selection.

Pheromone Update Rule.

Following the completion of all ants' tours, the pheromone trails' local update rule is applied to each route, as described in [14],

$$\Omega_{ij} \leftarrow (1-\rho)\Omega_{ij} + \sum_{k=1}^{m} \Delta\Omega^{k}{}_{ij} \tag{6}$$

Then, use the global pheromone update rule, which adds the appropriate amount of pheromone to the best, lowest-cost path. This rule is defined in (7). First let's look at it mathematically:

$$\Delta Q^{k}{}_{ij} = \frac{\zeta}{Y^k}; \text{ if component } (i,j) \text{ was used by ant (best route)} \tag{7}$$

=0; Otherwise

Here, $Y^k$is the distance of the best route. ζ is simply a parameter to adjust the amount of pheromone deposited, typically it would be set to 1. We sum $\frac{\zeta}{Y^k}$for every solution which used component (i, j), then that value becomes the amount of pheromone to be deposited on component (i, j). In our case, the continuous ACO is based on both the global and local search towards the elitist. The local ants have the capability of moving to the latent region with the best solution, according to transition probability:

$$P_{ij} = \frac{1/k_i}{\sum_{i=1}^{g} 1/k_i} \tag{8}$$

where k_i is the unit cost, and g is the number of global ants (number of columns). Therefore, the better the region is the more attraction to the successive ants it has. The novel method transforms Assignment unit cost (Table 1) into a probability table (Table 2) using formula (8).

Table 2. *AP Probability Table.*

p_{11}	p_{12}	p_{12}	p_{14}
p_{21}	p_{22}	p_{23}	p_{24}
p_{31}	p_{32}	p_{33}	p_{34}
p_{41}	p_{42}	p_{43}	p_{44}

4. New Approach for Solving Assignment Problem

We will provide a novel way for addressing the Assignment problem by changing the ACO algorithm in this part. This new method is an easy procedure for resolving Assignment problems. Consider a colony of ants that is searching for food. An ant colony will demonstrate that ants frequently walk in a straight line (vertical and horizontal) and between their anthill and the food source. The concept of an "army" of ants marching in the file has permeated our new culture, and build up a new approach. This method is used to solve an example, and the results are compared to other methods

Step 0. Define the starting AP table and probability table as given subsequently.

Step 1: Using the following technique, identify the cell (probability table) for:

a) Select each row that has highest probability cell or cells.
b) Mark (√) row or rows that has a maximum of one probability cell in a.
c) Remove the column/s and row/s in b.
d) Repeat a., b., and c. until one probability cell occurs in each row and column.

Step 2: In step 1, select the smallest number of units (AP cost table) corresponding value of (d).

Step 3: Calculate assignment Minimum Cost

5. Experimental Results and Discussion

This section introduced the experimental results used to assess the validity and performance of the proposed method. The quality of the solutions obtained by the suggested technique, like that of previous metaheuristic algorithms, was considerably influenced by the different parameter values. As a result, a variety of alternative values were investigated to fine-tune the parameters of the suggested method. Table 1 shows a probability table with the highest probability in each row.

Now we consider 4x4 AP and we assume

1st row highest probability	p_{12}
2nd row highest probability	$p_{21} = p_{24}$
3rd row highest probability	p_{32}
4th row highest probability	$p_{43} = p_{44}$

Table 3. *A_0 and P_0 give the initial representation of the PT and AP.*

	P_0		
p_{11}	p_{12}	p_{12}	p_{14}
p_{21}	p_{22}	p_{23}	p_{24}
p_{31}	p_{32}	p_{33}	p_{34}
p_{41}	p_{42}	p_{43}	p_{44}
	A_0		
k_{11}	k_{12}	k_{12}	k_{14}
k_{21}	k_{22}	k_{23}	k_{24}
k_{31}	k_{32}	k_{33}	k_{34}
k_{41}	k_{42}	k_{43}	k_{44}

Table 4. *The pivot row (rows) is the pivot column given by the (shade d), as shown in the P_1 table and select the corresponding unit cost in A_1.*

	P_1		
p_{11}	p_{12}	p_{12}	p_{14}
p_{21}	p_{22}	p_{23}	p_{24}
p_{31}	p_{32}	p_{33}	p_{34}
p_{41}	p_{42}	p_{43}	p_{44}
	A_1		
k_{11}	k_{12}	k_{12}	k_{14}
k_{21}	k_{22}	k_{23}	k_{24}
k_{31}	k_{32}	k_{33}	k_{34}
k_{41}	k_{42}	k_{43}	k_{44}

Table 5. *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_2table and select the corresponding unit cost in A_2.*

	P_2		
p_{11}	p_{12}	p_{12}	p_{14}
p_{21}	p_{22}	p_{23}	p_{24}
p_{31}	p_{32}	p_{33}	p_{34}
p_{41}	p_{42}	p_{43}	p_{44}
	A_2		
k_{11}	k_{12}	k_{12}	k_{14}
k_{21}	k_{22}	k_{23}	k_{24}
k_{31}	k_{32}	k_{33}	k_{34}
k_{41}	k_{42}	k_{43}	k_{44}

Table 6. *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_3 table and select the corresponding unit cost in A_3. No further improvement are possible in this iteration.*

	P_3		
p_{11}	p_{12}	p_{12}	p_{14}
p_{21}	p_{22}	p_{23}	p_{24}
p_{31}	p_{32}	p_{33}	p_{34}
p_{41}	p_{42}	p_{43}	p_{44}
	A_3		
k_{11}	k_{12}	k_{12}	k_{14}
k_{21}	k_{22}	k_{23}	k_{24}
k_{31}	k_{32}	k_{33}	k_{34}
k_{41}	k_{42}	k_{43}	k_{44}

After the assignment, Minimum Cost=$k_{13} + k_{21} + k_{32} + k_{44}$

Example 1. Consider the following assignment problem. Assign the five jobs to five machines so as to minimize the cost [7].

Table 7. *Assignment matrix.*

	A	B	C	D	E
P	12	8	7	15	4
Q	7	9	1	14	10
R	9	6	12	6	7
S	7	6	14	6	10
T	9	6	12	10	6

Table 8. *The table P_0 and A_0 give the initial representation of the PT and AP.*

	A	B	C	D	E
	P_0				
P	.141	.169	.103	.116	.329
Q	.241	.150	.724	.125	.131
R	.188	.226	.060	.291	.188
S	.241	.226	.051	.291	.131
T	.188	.226	.060	.175	.219
	A_0				
P	12	8	7	15	4
Q	7	9	1	14	10
R	9	6	12	6	7
S	7	6	14	6	10
T	9	6	12	10	6

Table 9. *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_1 table and select the corresponding unit cost in A_1.*

	A	B	C	D	E
	P_1				
P	.141	.169	.103	.116	.329
Q	.241	.150	.724	.125	.131
R	.188	.226	.060	.291	.188
S	.241	.226	.051	.291	.131
T	.188	.226	.060	.175	.219
	A_1				
P	12	8	7	15	4
Q	7	9	1	14	10
R	9	6	12	6	7
S	7	6	14	6	10
T	9	6	12	10	6

Table 10. *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_2 table and select the corresponding unit cost in A_2. No further improvement are possible in this iteration.*

	A	B	C	D	E
	P_2				
P	.141	.169	.103	.116	.329
Q	.241	.150	.724	.125	.131
R	.188	.226	.060	.291	.188
S	.241	.226	.051	.291	.131
T	.188	.226	.060	.175	.219
	A_2				
P	12	8	7	15	4
Q	7	9	1	14	10
R	9	6	12	6	7
S	7	6	14	6	10
T	9	6	12	10	6

After the assignment, Minimum Cost=4+1+6+7+6=24

Example 2. The six projects are faculty of education building (ED), Engineering block (EN), Pharmacy building (PH), Management building (MA), Central lecture hall (CE) and Health science building (HS). The seven companies are as follows: Sam & Vic Ltd (SA), Jipdant Association Ltd (JA), Minno Engineering Ltd (ME), Victenik Ltd (VL), Hytect Global Ltd (HG), Kafenal Ltd (KA) and Fine Job Ltd (FJ). The bids (in millions of Naira) of tenders to the various project is given in Table 11**.** (Problem Choose from Akpan N. P. [1]). The problem now is how the projects should be assign to the companies to minimize the total cost of undertaking the projects [29].

Table 11. *Assignment costs.*

	ED	EN	PH	MA	CE	HS
SA	126	207	254	245	214	243
JA	229	238	242	228	213	285
ME	118	253	306	218	245	216
VL	172	247	218	248	217	243
HG	309	207	105	136	194	139
KA	99	168	220	140	215	116
FG	95	174	168	145	249	98

Now the cost matrix is not balanced, hence we add one dummy column (DC) with zero cost entries in the column. The cost matrix so obtained is given in Table 12.

Table 12. *Assignment costs with dummy column*

	ED	EN	PH	MA	CE	HS	DC
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FG	95	174	168	145	249	98	0

Table 13. *The table A_0 and P_0 give the initial representation of the PT and AP.*

	ED	EN	PH	MA	CE	HS	OL
	P_0						
SA	.273	.166	.135	.140	.161	.141	0
JA	.174	.168	.165	.175	.187	.140	0
ME	.302	.141	.116	.163	.145	.165	0
VL	.215	.149	.169	.149	.170	.152	0
HG	.087	.130	.257	.198	.139	.194	0
KA	.246	.145	.110	.174	.113	.210	0
FG	.244	.133	.138	.160	.093	.237	0
	A_0						
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FG	95	174	168	145	249	98	0

Table 14. *The pivot row (rows) the pivot column given by the (shaded), as shown in theP_1table and select the corresponding unit cost in A_1.*

	ED	EN	PH	MA	CE	HS	OL
	P_1						
SA	.273	.166	.135	.140	.161	.141	0
JA	.174	.168	.165	.175	.187	.140	0
ME	.302	.141	.116	.163	.145	.165	0
VL	.215	.149	.169	.149	.170	.152	0
HG	.087	.130	.257	.198	.139	.194	0
KA	.246	.145	.110	.174	.113	.210	0
FG	.244	.133	.138	.160	.093	.237	0
	A_1						
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FG	95	174	168	145	249	98	0

Table 15. *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_2 table and select the corresponding unit cost in A_2.*

	ED	EN	PH	MA	CE	HS	OL
	P_2						
SA	.273	.166	.135	.140	.161	.141	0
JA	.174	.168	.165	.175	.187	.140	0
ME	.302	.141	.116	.163	.145	.165	0
VL	.215	.149	.169	.149	.170	.152	0
HG	.087	.130	.257	.198	.139	.194	0
KA	.246	.145	.110	.174	.113	.210	0
FG	.244	.133	.138	.160	.093	.237	0
	A_2						
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FG	95	174	168	145	249	98	0

T*able 16.* *The pivot row (rows) the pivot column given by the (shaded), as shown in the P_3 table and select the corresponding unit cost in A_3.*

	ED	EN	PH	MA	CE	HS	OL
	P_3						
SA	.273	.166	.135	.140	.161	.141	0
JA	.174	.168	.165	.175	.187	.140	0
ME	.302	.141	.116	.163	.145	.165	0
VL	.215	.149	.169	.149	.170	.152	0
HG	.087	.130	.257	.198	.139	.194	0
KA	.246	.145	.110	.174	.113	.210	0
FG	.244	.133	.138	.160	.093	.237	0
	A_3						
SA	126	207	254	245	214	243	0
JA	229	238	242	228	213	285	0
ME	118	253	306	218	245	216	0
VL	172	247	218	248	217	243	0
HG	309	207	105	136	194	139	0
KA	99	168	220	140	215	116	0
FG	95	174	168	145	249	98	0

After the assignment, Minimum Cost$=207+213+118+105+140+98=881$

6. Conclusion

In this approach, new attention has been paid to the study of a new optimality solution to the AP. Many researchers have focused their efforts on resolving this problem using various ways. Hungarian method is the most prominent and renowned method in finding the best solution to an AP. However, in this research paper, we discuss a new alternative method, a modified ant colony optimization algorithm which gives often an optimal solution to the AP. We look at numerous first solutions in this paper that propose ways for achieving initial plausible solutions to balanced and unbalanced APs. This research paper includes an outline of the notion of an Ant colony algorithm as well as a discussion of its applications in the solution of APs. The proposed method can be applied to various decision-making problems. It is more efficient and simple when compared to the earlier methods. This method requires a minimum number of steps to reach optimality as compared to the existing methods. So this technique can be applied for solving intuitionistic APs occurring in real-life situations.

Conflict of Interest

The authors declare that they have no conflict of interests.

Acknowledgements

The authors thank Dr. Uthpala Dhanayaka, Head of the Department of Physical Sciences at Rajarata University in Sri Lanka. He has helpful suggestions and resources to help me finish this paper. Finally, I'd like to thank my wife, daughters, and son for their comfort and invaluable assistance during this difficult time.

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