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Modified Schaffernak's Solution for Seepage Through Earth Dam

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Abstract: Seepage analysis forms an important and basic part of geotechnical engineering owing to its importance in ground water contamination control, slope stability analysis and dam design. Furthermore, It is important for determining the distribution of seepage uplift pressures and the resulting seepage forces as well as the estimation of the volume of seepage losses through the body and the foundation of earth dams. Casagrande (1940) and Schaffernak (1916) improved on Dupuit's solution of seepage through earth dam without considering tail water. In this work, modification of Schaffernak's model was done to accommodate tail water. Values obtained using the new model though similar to that of the three other models (Dupuit, Casagrande and Schaffernak) shows that existence of tail water affects the value of seepage. The new model is very consistent from 3-6 m height. Though the seepage equation from the new model is similar to that of Casagrande, they differ because the value of seepage face for the tail water 'a', used for computations are not the same. For each of the model, there is a linear relationship between the seepage and the height of water upsream. Interestingly, there is a sharp change in seepage at 6m height of dam with increase in slope between 6 m and 9 m for each model except at the slope of 1:2.5 where a decrease in slope was recorded for the new model.

Keywords: Seepage, Calculus, Earth Dam, Scaffernak's Solution, New Seepage Equation

1. Introduction

Seepage is the flow of fluid usually water through permeable soils under hydraulic gradient. Seepage analysis and computations is important in ground water contamination control, slope stability analysis and estimation of seepage forces and their distribution throughout the body and the foundation of earth dams. It is also essential for assessing the suitability of soil materials, determining the distribution of seepage uplift pressures and the resulting seepage forces as well as the estimation of the volume of seepage losses through the body and the foundation of earth dams, dykes or levees [1-8].

According to Budhu [9], many catastrophic failures in geotechnical engineering result from instability of soil masses due to seepage which results in loss of lives, infrastructural damage and in general, huge economic loss.

Several researches have been carried out on seepage through dam using different methods such as neural network [10], element free method [11], natural element method [12], boundary fitted coordinate [12], and self-potential and electrical resistivity [13]. Other method that have been employed in seepage analysis include finite element method [1, 13, 14], saturated seepage theory [15] and finite difference method [1, 16].

Prominent researchers worked on seepage analysis through dams. Prominent among them are [17-20]. Moreover, Schaffernak's and Casagrande's methods are predominantly used in seepage analysis through dam.[17] do not take cognizance of the entrance or exit conditions of the line of seepage or the development of a surface of seepage. With his assumptions, the discharge, per unit width, through a vertical section of the dam is;

$$q = ky\frac{dy}{dx} \tag{1}$$

Integrating and substituting the boundary conditions, he obtained Dupuits formula,

$$q = \frac{K(h_1^2 - h_2^2)}{2L} \quad (2)$$

Where K is the permeability, h_1 is the height of water at the entrance h_2 is the tail water, d is line BA and α is angle EOA.[19], taking exception to Dupuits assumption that the hydraulic gradient is equal to the slope $^{dy}/_{ds}$, where S is measured along the free surface. Hence for Casagrande's method

$$q = -Ky\,\frac{dy}{ds} \quad (3)$$

Integrating and substituting the boundary conditions, we have

$$q = Kasin^2\alpha \quad (4)$$

Where;

$$q = s - \sqrt{S^2 - \frac{h^2}{Sin^2\alpha}} \quad (5)$$

Casagrande also assumed that there is no tail water.

[18] is the first approximate method that accounts for the development of the surface of seepage. He also assumed the absence of tail water.

Putting his assumptions into consideration he derived that discharge per unit width;

$$q = -Ky\,\frac{dy}{dx} = Kasin\alpha tan\alpha \quad (6)$$

Where

$$a = \frac{d}{cos\alpha} - \sqrt{\frac{d2}{Cos^2\alpha} - \frac{h_2}{Sin^2\alpha}} \quad (7)$$

These models especially Casagrande and Schaffernak are still in use today. Considering the fact that tail water are present in some earthdams, this necessitates the need to put into consideration the effect of tail water on the resulting equation. This research involves the modification of Schaffernak's solution to accommodate the presence of tail water.

2. Methodology

Considering a homogenous earth dam resting on an impervious base with tail water, the cross section of such typical earth dam is as shown in figure 1.

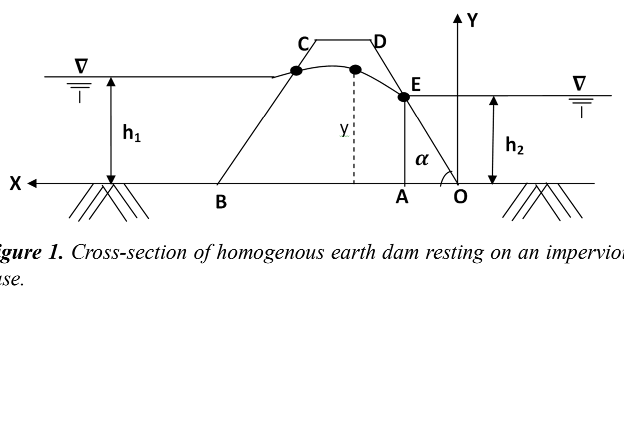


Figure 1. *Cross-section of homogenous earth dam resting on an impervious base.*

From Dupuits assumption, the hydraulic gradient is the slope of the free surface [21-22, 3].

$$i_x = \frac{dy}{dx} \quad (8)$$

Where y is the equation of the free surface and x is taken as positive to the left of the origin 0 as shown on the figure.

From Darcy's law, the velocity of flow in x-direction is;

$$V_x = K_x i_x \quad (9)$$

From equation (1)

$$V_x = K_x \frac{dy}{dx} \quad (10)$$

Where Kx is the coefficient of permeability of the material of the earth dam in the x-direction. Since the dam is homogenous, Kx will vary with space on the dam along the vertical section of the dam, the discharge per unit width of the dam is given as;

$$q = VA = KA\frac{dy}{dx} \quad (11)$$

At a vertical section located at an arbitrary distance x from the origin, the area of flow is y x 1 for a unit width of the dam.

This implies that discharge per unit width of the dam is:

$$q = Ky\,\frac{dy}{dx} \quad (12)$$

Considering ΔOEA from figure 1 as shown in figure 2,

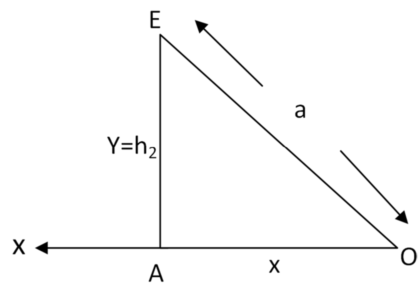


Figure 2. *Exit geometry of the model dam.*

$$Cos\alpha = \frac{x}{a}\ Sin\alpha = \frac{y}{a}\ y = xtan\alpha$$
$$x = acos\alpha, y = h_2 = asin\alpha \quad (13)$$

$$\frac{dy}{dx} = tan\alpha \quad (14)$$

Substituting equations 13 and 14 into equation 3, we have that

$$q = ky\,\frac{dy}{dx} = kh_2 tan\alpha$$
$$h_2 = asin\alpha \quad (15)$$

$$q = k.asin\alpha\ tan\alpha = kydy \quad (16)$$

Applying boundary conditions. Knowing that the free surface started with C and terminated as tail water E

$At\ E: x = acos\alpha, h_2 = y = asin\alpha$

$At\ C: x = d, y = h_1$

∴ from equation (16), we have;

$$y\frac{dy}{dx} = asin\alpha tanx$$

Separating variables

$$ydy = asin\alpha tanxdx$$

$$\int_{h_2}^{h_1} ydy = \int_{acos\alpha}^{d} (asin\alpha^2 tan\alpha) \quad (17)$$

$$\frac{y^2}{2}\Big|_{h_2}^{h_1} = asin\alpha tan\alpha [x]_{acos\alpha}^{d} \quad (18)$$

$$\frac{h_1^2}{2} - \frac{h_2^2}{2} = asin\alpha tan\alpha\,(d - acos\alpha) \quad (19)$$

$$\frac{h_1^2 - h_2^2}{2} = adasin\alpha tan\alpha - a^2 sin\alpha tan\alpha cos\alpha \quad (20)$$

$$h_1^2 - h_2^2 = 2adsin\alpha tan\alpha - 2a^2 sin\alpha tan\alpha cos\alpha \quad (21)$$

$$h_1^2 - h_2^2 = 2adsin\alpha tan\alpha - 2a^2 sin\alpha sin\alpha \quad (22)$$

$$h_1^2 - h_2^2 = 2adsin\alpha tan\alpha - 2a^2 sin^2\alpha \quad (23)$$

$$2a^2 sin^2\alpha - 2adsin\alpha tan\alpha = h_2^2 - h_1^2 \quad (24)$$

This is a quadratic equation in a, which is unknown, solving for ‘a’ by the quadratic formula, we have:

$$\Rightarrow a = \frac{-\acute{b} \pm \sqrt{\acute{b}^2 - 4\acute{a}\acute{c}}}{2\acute{a}} \quad (25)$$

$$\Rightarrow \acute{a} = 2sin^2\alpha, \acute{b} = 2dsin\alpha tan\alpha, \acute{c} = (h_2^2 - h_1^2)\ [23]$$

$$\Rightarrow a = \frac{2dsin\alpha tan\alpha \pm \sqrt{(2dsin\alpha tan\alpha)^2 - 8sin^2\alpha\,(h_2^2 - h_1^2)}}{4sin^2\alpha} \quad (26)$$

$$a = \frac{2dsin\alpha tan\alpha}{4sin^2\alpha} \pm \frac{\sqrt{4d^2 sin^2\alpha tan^2\alpha - 8sin^2\alpha(h_2^2 - h_1^2)}}{4sin^2\alpha} \quad (27)$$

$$= \frac{dsin\alpha tan\alpha}{2sin^2\alpha} \pm \frac{\sqrt{4d^2 sin^2\alpha tan^2\alpha - 8sin^2\alpha(h_2^2 - h_1^2)}}{4sin^2\alpha} \quad (28)$$

$$= \frac{dtan\alpha}{2sin\alpha} \pm \frac{\sqrt{4d^2 sin^2\alpha tan^2\alpha - 8sin^2\alpha(h_2^2 - h_1^2)}}{4sin^2\alpha} \quad (29)$$

$$= \frac{dsin\alpha}{2cos\alpha sin\alpha} \pm \frac{\sqrt{4d^2 sin^2\alpha tan^2\alpha - 8sin^2\alpha(h_2^2 - h_1^2)}}{4sin^2\alpha} \quad (30)$$

$$= \frac{d}{2cos\alpha} \pm \sqrt{\frac{4d^2 sin^2\alpha tan^2\alpha}{16sin^4\alpha} - \frac{8sin^2(h_2^2 - h_1^2)}{16sin^4\alpha}} \quad (31)$$

$$= \frac{d}{2cos\alpha} \pm \sqrt{\frac{d^2}{4cos^2\alpha} - \frac{(h_2^2 - h_1^2)}{2sin^2\alpha}} \quad (32)$$

According to Ike [21], the positive sign gives unrealistic result for a, so, we have that;

$$a = \frac{d}{2cos\alpha} - \sqrt{\frac{d^2}{4cos^2\alpha} - \frac{(h_2^2 - h_1^2)}{2sin^2\alpha}} \quad (33)$$

$$a = \frac{d}{2cos\alpha} - \sqrt{\left(\frac{d}{2cos\alpha}\right)^2 - \frac{(h_2^2 - h_1^2)}{2sin^2\alpha}} \quad (34)$$

The discharge q can be calculated by substituting equation (2.27) into equation (2.9):

$$\Rightarrow q = kh_2 tan\alpha \quad (35)$$

$$q = kasin\alpha tan\alpha \quad (36)$$

Equation (34 and 36) are the modified Schaffernak’s equation for seepage through earth dam considering tail water.

With the information obtained from [24]. The specification for the earth dam used for the analysis are as follows;

Length of dam (constant) = 60 m

Height of dam = 3 – 9 m (varies at intervals of 3 m)

Freeboard (constant) = 0.5 m

Height of tail water (constant) = 1 m

Slope considered = 1:3, 1:2.5 and 1:2

A Matlab programme written for the calculation of each of the method as shown in appendix A, was used in running the calculation. Regression analysis was also used in the analysis of result.

3. Results and Discussion

The results of the seepage discharge with varying dam heights are presented in table 1 and figure 3.

Table 1. *Height of dam and the corresponding seepage for each slope.*

a. Slope 1:3

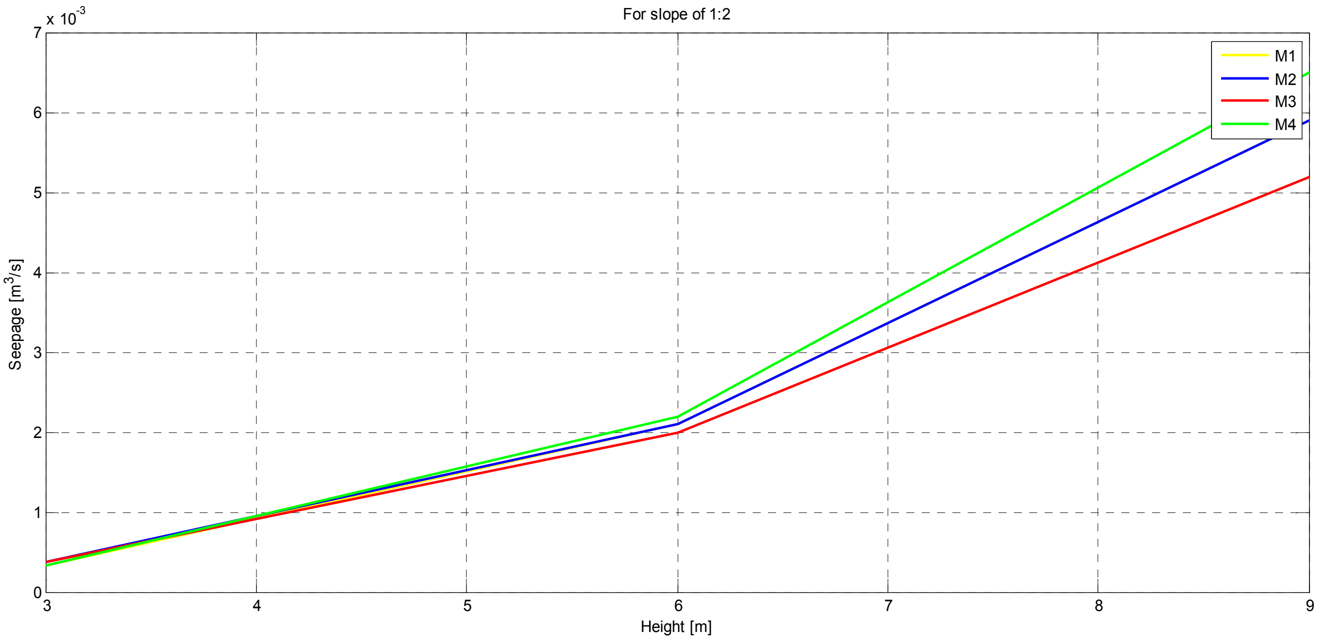
Height (m)	Dupuits Model (M_1)	Casagrande’s Model (M_2)	(Schaffernak’s Model (M_3)	New Model (M_4)
	Seepage (m^3/s)			
3	3.5732×10^{-4}	4.0314×10^{-4}	3.8621×10^{-4}	3.6083×10^{-4}
6	0.0024	0.0024	0.0022	0.0027
9	0.0076	0.0084	0.0064	0.0118-0.0064i

b. Slope 1:2.5

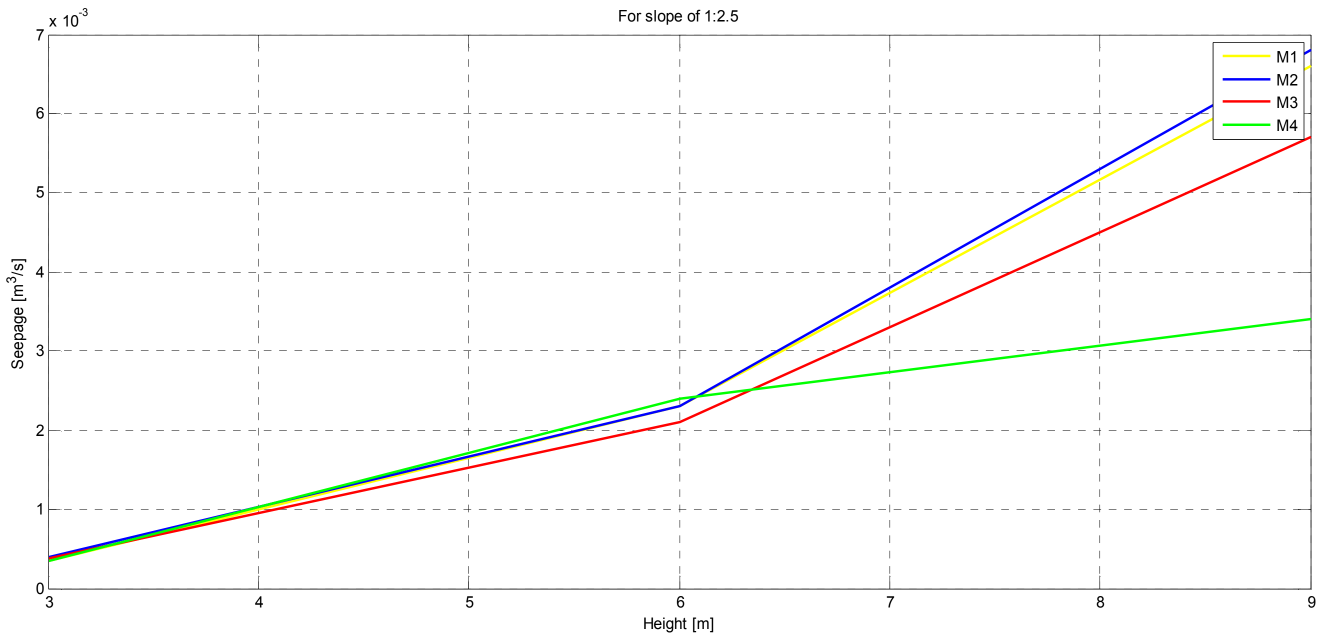
Height (m)	Dupuits Model (M_1)	Casagrande’s Model (M_2)	(Schaffernak’s Model (M_3)	New Model (M_4)
	Seepage (m^3/s)			
3	3.4511×10^{-4}	3.9307×10^{-4}	3.7955×10^{-4}	3.473×10^{-4}
6	0.0023	0.0023	0.0021	0.0024
9	0.0066	0.0068	0.0057	0.0034

c. Slope 1:2

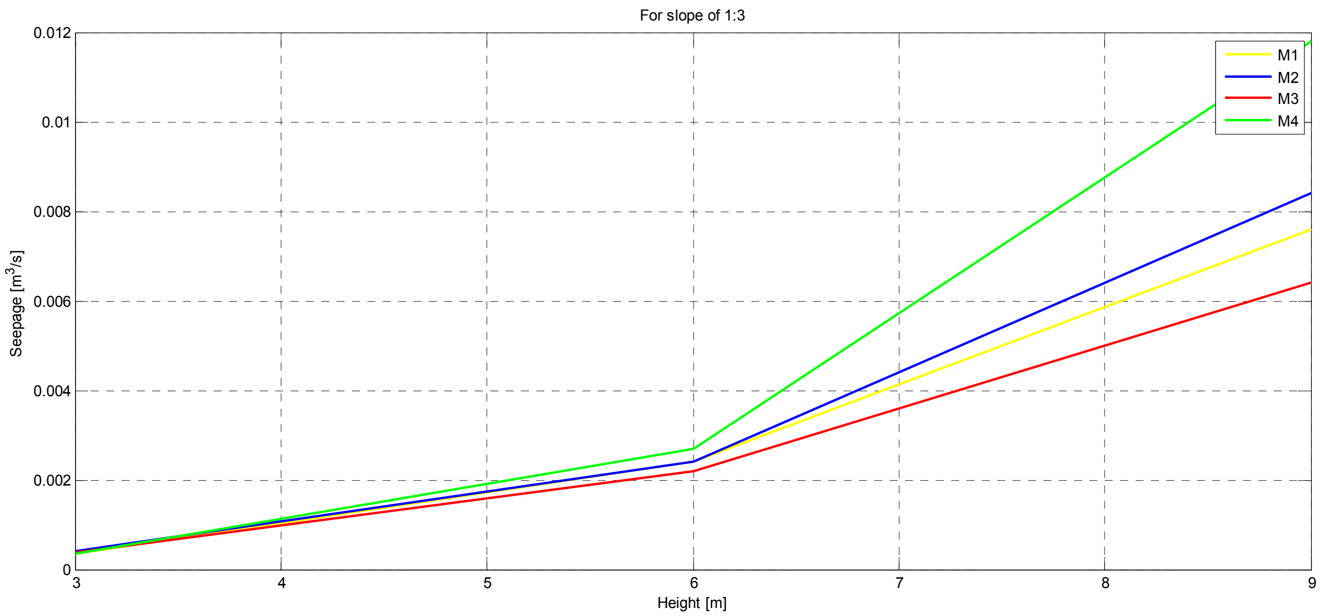
Height (m)	Dupuits Model (M_1)	Casagrande's Model (M_2)	(Schaffernak's Model (M_3)	New Model (M_4)
	Seepage (m^3/s)			
3	$3.3372 x 10^{-4}$	$3.8363 x 10^{-4}$	$3.7322 x 10^{-4}$	$3.3498 x 10^{-4}$
6	0.0021	0.0021	0.0020	0.0022
9	0.0059	0.0059	0.0052	0.0065



(a)



(b)



(c)

Figure 3. *Graph of seepage against height of dam.*

NB: M_1=Dupuit's Model M_2=Casagrande's model M_3=Schaffernak's model M_4= New model.

From tables 1 (a-c) and the graphs [figure 3 (a-c)]; for the slope of 1:3; at 3 m, the new model gave the second to the lowest value of seepage, at 6 m height of dam, it gave the highest value of seepage and at 9 m, it gave a complex value.

For the slope of 1:2.5 at 3 m, second to the lowest value of the seepage was computed, at 6 m height of dam, the highest value of seepage was obtained and at 9 m the value of seepage was lowest.

For the slope of 1:2 at 3 m, second the lowest value of the seepage was obtained, at 6 m, the value of seepage obtained using the new model was highest and at 9 m, the model recorded the highest value of seepage.

The new model is very consistent from 3-6 m. Though the seepage equation from the new model is similar to that of Casagrande, they differ because the value of seepage face for the tail water a, used for computations are not the same.

For each of the model there is a linear relationship between the seepage and the height of water upsream. Interestingly, there is a sharp change in seepage at 6 m height of dam with increase in slope between 6 m and 9 m for each model except at the slope of 1:2.5 where a decrease in slope was recorded for the new model. The increase in slope evident in all the models is likely to be a geometric effect.

The complex value obtained at 9 m height of dam and slope of 1:3 is in order because both the real ($\emptyset$) and imaginary part (φ) of an analytic function satisfy perfectly well Laplace equation in two dimension [25].

To prove this, obtained the gradient of each family of curves, obtaining;

$$\frac{dy}{dx} = \frac{-\partial\emptyset/_{\partial x}}{\partial\emptyset/_{\partial y}} \tag{37}$$

$$\frac{dy}{dx} = \frac{-\partial\varphi/_{\partial x}}{\partial\varphi/_{\partial y}} \tag{38}$$

Since $w = \emptyset + i\varphi$ is analytic $\emptyset$ and φ must satisfy the Cauchy-Newmann equations [25].

4. Conclusions

A modification of Schaffernak's model considering the effect of tailwater was achieved. Most importantly, the new model which has a regression constant (R = 0.95) shows that tail water affects the result of seepage through dam and should be considered in seepage analysis. Finally, obtaining both real and complex value, which satisfy Laplace equation shows that the new model is more encompassing and involving and should be incorporated into seepage calculation software for more accurate results. In line with recent discoveries requesting that rainstorm effect should be considered in seepage analysis [26-27], tailwater effect should also be considered.

Appendix

```
clc;
clearall
k=1.0*exp(-5);
h1=2.5;
h2=1;
l=49.5;
q=(k*(h1^2-h2^2))/(2*l)
h1=5.5; h2=1;l=40.5;
q1=(k*(h1^2-h2^2))/(2*l)
h1=8.5;h2=1;l=31.5;
q2=(k*(h1^2-h2^2))/(2*l)
```

```
h1=2.5;h2=1;l=51.25;
q3=(k*(h1^2-h2^2))/(2*l)
h1=5.5;h2=1;l=43.75;
q4=(k*(h1^2-h2^2))/(2*l)
h1=8.5;h2=1;l=36.25;
q5=(k*(h1^2-h2^2))/(2*l)
h1=2.5;h2=1;l=53;
q6=(k*(h1^2-h2^2))/(2*l)
h1=5.5;h2=1;l=47;
q7=(k*(h1^2-h2^2))/(2*l)
h1=8.5;l=41;
q8=(k*(h1^2-h2^2))/(2*l)
clc;
clearall
h=2.5;d=54.75;r=18.43494882;
w=r*pi/180;k=1.0*exp(-5);
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q=k*a*sin(w)^2
h=5.5;d=48.45;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q1=k*a*sin(w)^2
h=8.5;d=42.15;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q2=k*a*sin(w)^2
h=2.5;d=55.625;r=21.80140949;
w=r*pi/180;k=1.0*exp(-5);
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q3=k*a*sin(w)^2
h=5.5;d=50.375;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q4=k*a*sin(w)^2
h=8.5;d=45.125;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q5=k*a*sin(w)^2
h=2.5;d=56.5;r=26.56505118;
w=r*pi/180;k=1.0*exp(-5);
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q6=k*a*sin(w)^2
h=5.5;d=52.3;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q7=k*a*sin(w)^2
h=8.5;d=48.1;
s=sqrt(h^2+d^2);
a=s-sqrt(s^2-(h^2/(sin(w)^2)));
q8=k*a*sin(w)^2
clc;
clearall;
k=1.0*exp(-5);
d=52.5;h=2.5;r=18.43494882;
w=r*pi/180;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q=k*a*sin(w)*tan(w)
h=5.5;d=43.5;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q1=k*a*sin(w)*tan(w)
h=8.5;d=34.5;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q2=k*a*sin(w)*tan(w)
d=53.75;h=2.5;r=21.80140949;
w=r*pi/180;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q3=k*a*sin(w)*tan(w)
h=5.5;d=46.25;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q4=k*a*sin(w)*tan(w)
h=8.5;d=38.75;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q5=k*a*sin(w)*tan(w)
d=55;h=2.5;r=26.56505118;
w=r*pi/180;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q6=k*a*sin(w)*tan(w)
h=5.5;d=49;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q7=k*a*sin(w)*tan(w)
h=8.5;d=43;
a=(d/cos(w))-sqrt(((d^2)/(cos(w)^2))-((h^2)/(sin(w)^2)));
q8=k*a*sin(w)*tan(w)
clc;
clearall;
k=1.0*exp(-5);
h1=2.5;h2=1;d=49.5;r=18.43494882;
w=r*pi/180;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q=k*a*sin(w)*tan(w)
h1=5.5;d=40.5;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q1=k*a*sin(w)*tan(w)
h1=8.5;d=31.5;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q2=k*a*sin(w)*tan(w)
h1=2.5;d=51.25;r=21.80140949;
w=r*pi/180;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q3=k*a*sin(w)*tan(w)
h1=5.5;d=43.75;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q4=k*a*sin(w)*tan(w)
h1=8.5;d=36.25;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
```

```
q5=k*a*sin(w)*tan(w)
h1=2.5;d=53;r=26.56505118;
w=r*pi/180;
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q6=k*a*sin(w)*tan(w)
h1=5.5;d=47
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q7=k*a*sin(w)*tan(w)
h1=8.5;d=41
m=d/(2*cos(w));n=(h1^2-h2^2)/(2*sin(w)^2);
a=m-sqrt(m^2-n);
q8=k*a*sin(w)*tan(w)
```

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