



Robust Exponential Stability of Periodic Solutions for Static Recurrent Neural Networks with Delays

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Abstract: In this paper, we study the existence of periodic solutions of time-invariant static recurrent neural networks by using the fixed point theory, Poincare map and Lyapunov function combined with inequality techniques. The static recurrent neural network is a kind of neural network which studies the external states of neurons as variables. And its global robust exponential stability. This paper introduces the research status of artificial neural network, summarizes the research background and development of static recurrent neural network dynamic system, and introduces the main work of this paper. Using the fixed point theory, M. The existence of periodic solutions and the global robust exponential stability of the static recursive neural network with variable delays and the existence of almost periodic solutions of the static recursive neural network of the partitioned time are studied by combining the properties of the matrix and the Lyapunov function combined with the inequality technique. Global exponential stability, the stability conditions of the corresponding problem are obtained respectively, and the results of the related research are generalized. Using Lyapunov. The stability of the quasi - static neural recursive neural network and the stability of the periodic solution are studied. The condition of the stationary static recursive neural network is obtained and the correctness of the condition is illustrated. Considering the influence of stochastic perturbation on the dynamic behavior of static recurrent neural network, the static recursive neural network with time delay and the static recursive neural network with distributed time delay are studied by using the infinitesimal operator, Ito formula and the convergence theorem of martingales. Global critical exponential stability of quasi - static neural network with stochastic perturbation. The static recursive neural network with Markovian modulation and the time-delay static recurrent neural network model considering both random perturbation and Markovian switching are studied. The linear matrix inequality, the finite state space Markov chain property and the Lyapunov-krasovskii function, The judgment condition of the global exponential stability of the system is obtained. Firstly, the global exponential stability problem of quasi - static neural neural network with time - delay and recursive neural network is studied by using the generalized Halanay inequality. Then the stability of the Markovian response sporadic static recurrent neural network is studied by combining the properties of Markov chain.

Keywords: Robust Exponential Stability, Static Recurrent Neural Networks

1. Introduction

In the past two decades, the theory and application of artificial neural network have attracted the great interest of scientists and become one of the hotspots in the field of nonlinear science. This is mainly because the artificial neural network is a nonlinear information processing system, Has a wide range of applications. Neural network can be divided into engineering neural network and mathematical neural network. Engineering neural network is a kind of information processing function in hardware or software form. The mathematical neural network is a mathematical model proposed by engineering neural network, which is usually called recursive neural network, mainly to study its dynamic characteristics and provide theoretical support for engineering neural network and guarantee. According to the basic variables of the system, the mathematical model of recurrent neural network can be divided into static neural network and local neural network. At present, most researches on recurrent neural networks focus on the local neural network model, and the static model is relatively few. However, many important neural networks are attributed to the static model, therefore, the study of static model has important theoretical significance and practical value.

In the electronic implementation of artificial neural

networks, time delay is unavoidable due to the limited conversion speed of network neuron amplifiers. Similarly, pulsed phenomena and random perturbations are inevitable in the implementation of neural networks and are widespread, and in many cases they tend to occur in the same system. Therefore, this paper studies the time-delay static recurrent neural network, Considering the influence of pulse and random disturbance on neural network.

From the point of view of biological neural network system, the human brain is often in periodic or chaotic state, so it is very important to research the periodic oscillation and chaos phenomenon of neural network, and the periodicity includes periodicity. Problem, the study of almost periodic movement is often more realistic than the research cycle of movement. On the other hand, in practical applications, the neural network system is sometimes affected by the limitations of the amplifier conversion speed, etc., often lead to drastic changes in time lag with time.

2. Preparations

The study of the almost periodic solution of neural networks with delay is more practical.

$$\begin{cases} \frac{dx_i}{dt} = -a_i x_i(t) + f_i \left(\sum_{j=1}^n w_{ij} x_j(t) + I_i(t) \right) + g_i \left(\sum_{j=1}^n m_{ij} x_j(t - \tau_{ij}(t)) + J_i(t) \right), \\ A_+ = \{ A = \text{diag}(a_i)_{n \times n} : \underline{A} \leq A \leq \bar{A}, i.e. \underline{a}_i \leq a_i \leq \bar{a}_i \} \\ W_+ = \{ W = (w_{ij})_{n \times n} : \underline{W} \leq W \leq \bar{W}, i.e. \underline{w}_{ij} \leq w_{ij} \leq \bar{w}_{ij} \} \\ M_+ = \{ M = (m_{ij})_{n \times n} : \underline{M} \leq M \leq \bar{M}, i.e. \underline{m}_{ij} \leq m_{ij} \leq \bar{m}_{ij} \} \\ a_i > 0 \quad i, j = 1, 2, \dots, n. \end{cases} \tag{1}$$

Where $\underline{A}, \bar{A}, \underline{W}, \bar{W}, \underline{M}, \bar{M}$ are constant matrix and $\underline{A} > 0; 0 \leq \tau_{ij}(t) \leq \tau$.

The initial condition of the system (1) has the following form:

$$x_i(\theta) = \phi_i(\theta), \theta \in [-\tau, 0], i = 1, 2, \dots, n,$$

Where $\phi = (\phi_1, \phi_2, \dots, \phi_n)^T \in C([- \tau, 0], R^n)$. the solution through the point $(0, \phi)$ of the system has been credited as

$$x(t, \phi) = (x_1(t, \phi), x_2(t, \phi), \dots, x_n(t, \phi))^T,$$

We define its norm is $\|x(\bullet, \phi)\| = \sum_{i=1}^n |x_i(\bullet, \phi)|$.

Notes $x_t(\phi) = x(t + \theta, \phi), \theta \in [-\tau, 0], t \geq 0$, so $x_t(\phi) \in C([- \tau, 0], R^n), \forall t \geq 0$,

For any $\phi \in C([- \tau, 0], R^n)$, we define its norm is

$$\|\phi\|_r = \sup_{-\tau \leq \theta \leq 0} \sum_{i=1}^n |\phi_i(\theta)|.$$

Throughout the paper, we suppose that $\tau_{ij}(t), I_i(t), J_i(t)$ are w - periodic function, and $\forall i, j = 1, 2, \dots, n$, the following conditions are satisfied

$$(H_1): \tau'_{ij}(t) \leq \delta_{ij} < 1;$$

$$(H_2): \text{There exist constants } \alpha_i, \beta_i > 0, \text{ making}$$

$$|f_i(x) - f_i(y)| \leq \alpha_i |x - y|, |g_i(x) - g_i(y)| \leq \beta_i |x - y|, \forall x, y \in R;$$

$$(H_3): C = \underline{A} - (\alpha W_0^+ + \beta M_0^+) \text{ is the M matrix, in which } M_0^+ = \left(\frac{m_{ji}^0}{1 - \delta_{ji}} \right)_{n \times n},$$

$$m_{ij}^0 = \max \{ |m_{ij}|, |\bar{m}_{ij}| \}, W_0^+ = (w_{ij}^0), w_{ij}^0 = \max \{ |w_{ij}|, |\bar{w}_{ij}| \}, A = \text{diag}(\underline{a}_1, \underline{a}_2, \dots, \underline{a}_n)_{n \times n},$$

$$\alpha = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_n)_{n \times n}, \beta = \text{diag}(\beta_1, \beta_2, \dots, \beta_n)_{n \times n}$$

Definitions

w- Periodic solution $x(t, \phi^*)$ of system (1.1) (that is $x(t+w, \phi^*) = x(t, \phi^*), t > 0$) is known as robust exponential stability, if for any initial function $\phi \in C([- \tau, 0], R^n)$ and for any $A \in A^+, W \in W^+$, there exist constants $\lambda > 0, k > 0$, making the solution $x(t, \phi)$ through the point $(0, \phi)$ of the system satisfying $\|x(t, \phi) - x(t, \phi^*)\| \leq ke^{-\lambda t} \|\phi - \phi^*\|_r$

3. Main Work

Theorem 1.1 Suppose the condition (H1) - (H3) holds, then the system (1) has a unique global exponentially stable w-periodic solution.

$$-\xi_i a_i + \sum_{j=1}^n a_j w_{ji} \xi_j + \sum_{j=1}^n \beta_j \frac{m_{ji}^0}{1 - \delta_{ji}} \xi_j < 0, i = 1, 2, \dots, n.$$

Therefore, it is possible to select a sufficiently small positive constant $\varepsilon > 0$ and a sufficiently large positive integer m, making

$$(\varepsilon - \xi_i) a_i + \sum_{j=1}^n a_j w_{ji} \xi_j + e^{\varepsilon \tau} \sum_{j=1}^n \beta_j \frac{m_{ji}^0}{1 - \delta_{ji}} \xi_j < 0, i = 1, 2, \dots, n. \quad (2)$$

$$ke^{-\varepsilon(mw - \tau)} \leq \frac{1}{2} \quad (3)$$

Where

$$k = \left\{ \frac{\max_{1 \leq i \leq n} \{ \xi_i \} + \varepsilon^{-1} e^{\varepsilon \tau} \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} w_{ij}^0}{\min_{1 \leq i \leq n} \{ \xi_i \}} \right\}$$

Constructing Lyapunov functional

$$V(t) = \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} w_{ij}^0 \int_{t - \tau_{ij}(t)}^t |x_j(s, \phi) - x_j(s, \varphi)| e^{\varepsilon(s + \tau)} ds$$

We could obtain the following from the (H2)

$$\begin{aligned} & \frac{d|x_i(t, \phi) - x_i(t, \varphi)|}{dt} \\ &= -a_i |x_i(t, \phi) - x_i(t, \varphi)| + \left(f_i \left(\sum_{j=1}^n w_{ij} x_j(t, \phi) + J_i(t) \right) - f_i \left(\sum_{j=1}^n w_{ij} x_j(t, \varphi) + J_i(t) \right) \right) \\ &+ \left(g_i \left(\sum_{j=1}^n m_{ij} x_j(t - \tau_{ij}(t), \phi) + J_i(t) \right) - g_i \left(\sum_{j=1}^n m_{ij} x_j(t - \tau_{ij}(t), \varphi) + J_i(t) \right) \right) \text{sign}(x_i(t, \phi) - x_i(t, \varphi)) \\ &\leq -a_i |x_i(t, \varphi) - x_i(t, \phi)| + a_i \sum_{j=1}^n |w_{ij}| |x_j(t, \varphi) - x_j(t, \phi)| \\ &+ \beta_i \sum_{j=1}^n |m_{ij}| |x_j(t - \tau_{ij}(t), \varphi) - x_j(t - \tau_{ij}(t), \phi)| \\ &\leq -\underline{a}_i |x_i(t, \varphi) - x_i(t, \phi)| + \alpha_i \sum_{j=1}^n w_{ij}^0 |x_j(t, \varphi) - x_j(t, \phi)| \\ &+ \beta_i \sum_{j=1}^n m_{ij}^0 |x_j(t - \tau_{ij}(t), \varphi) - x_j(t - \tau_{ij}(t), \phi)| \end{aligned} \quad (4)$$

We could obtain the following from the (H1), (2) and (4)

$$\begin{aligned}
 D^+V &\leq \sum_{i=1}^n \xi_i e^{\varepsilon t} \left(-\underline{a}_i |x_i(t, \varphi) - x_i(t, \phi)| + \alpha_i \sum_{j=1}^n w_{ij}^0 |x_j(t, \varphi) - x_j(t, \phi)| \right. \\
 &\quad \left. + \beta_i \sum_{j=1}^n m_{ij}^0 |x_j(t - \tau_{ij}(t), \varphi) - x_j(t - \tau_{ij}(t), \phi)| \right) \\
 &\quad + \varepsilon \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} m_{ij}^0 \left(|x_j(t, \phi) - x_j(t, \varphi)| e^{\varepsilon(i+\tau)} \right. \\
 &\quad \left. - (1 - \tau'_{ij}(t)) |x_j(t - \tau_{ij}(t), \phi) - x_j(t - \tau_{ij}(t), \varphi)| e^{\varepsilon(t - \tau_{ij}(t) + \tau)} \right) \\
 &\leq \sum_{i=1}^n \left(\xi_i (\varepsilon - \underline{a}_i) |x_i(t, \varphi) - x_i(t, \phi)| + \alpha_i \sum_{j=1}^n w_{ij}^0 |x_j(t, \varphi) - x_j(t, \phi)| \right. \\
 &\quad \left. + \xi_i \beta_i e^{\varepsilon \tau} \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} m_{ij}^0 |x_j(t, \varphi) - x_j(t, \phi)| \right) e^{\varepsilon t} \\
 &= \sum_{i=1}^n \left((\varepsilon - \xi_i) \underline{a}_i + \sum_{j=1}^n \alpha_j w_{ij} \xi_j + e^{\varepsilon \tau} \sum_{j=1}^n \beta_j \frac{w_{ji}^0}{1 - \delta_{ji}} \xi_j \right) |x_i(t, \phi) - x_i(t, \varphi)| e^{\varepsilon t} < 0
 \end{aligned} \tag{5}$$

We could obtain the following from the (5)

$$V(t) \leq V(0), t \geq 0 \tag{6}$$

Because

$$V(t) \geq \sum_{i=1}^n \xi_i e^{\varepsilon t} |x_i(t, \phi) - x_i(t, \varphi)| \geq e^{\varepsilon t} \min_{1 \leq i \leq n} \{\xi_i\} \sum_{i=1}^n |x_i(t, \phi) - x_i(t, \varphi)|,$$

$$\begin{aligned}
 V(0) &= \sum_{i=1}^n \xi_i |x_i(0, \phi) - x_i(0, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} m_{ij}^0 \int_{0 - \tau_{ij}(0)}^0 |x_j(s, \varphi) - x_j(s, \phi)| e^{\varepsilon(s+\tau)} ds \\
 &\leq \max_{1 \leq i \leq n} \{\xi_i\} \sum_{i=1}^n |x_i(0, \phi) - x_i(0, \varphi)| \\
 &\quad + \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} m_{ij}^0 \sup_{-\tau \leq s \leq 0} \sum_{j=1}^n |\phi_j(s) - \varphi_j(s)| \int_{0 - \tau_{ij}(0)}^0 e^{\varepsilon(s+\tau)} ds \\
 &\leq \left(\max_{1 \leq i \leq n} \{\xi_i\} + \varepsilon^{-1} e^{\varepsilon \tau} \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} m_{ij}^0 \right) \|\phi - \varphi\|_r.
 \end{aligned}$$

$$\|x_t(\phi) - x_t(\varphi)\|_r = \sup_{-\tau \leq \theta \leq 0} \sum_{i=1}^n |x_i(t + \theta, \phi) - x_i(t + \theta, \varphi)| \leq k e^{-\varepsilon(t-\tau)} \|\phi - \varphi\|_r. \tag{9}$$

When $t = mw$, we could obtain the following from the (3)

$$\|x_{mw}(\phi) - x_{mw}(\varphi)\|_r \leq k e^{-\varepsilon(mw-\tau)} \|\phi - \varphi\|_r \leq \frac{1}{2} \|\phi - \varphi\|_r \tag{10}$$

Defines the Poincaré map $P : C([- \tau, 0], R^n) \rightarrow C([- \tau, 0], R^n)$ is the following:

So

$$\begin{aligned}
 \sum_{i=1}^n |x_i(t, \phi) - x_i(t, \varphi)| &\leq e^{-\varepsilon t} \frac{\max_{1 \leq i \leq n} \{\xi_i\} + \varepsilon^{-1} e^{\varepsilon \tau} \sum_{i=1}^n \xi_i \beta_i \sum_{j=1}^n \frac{1}{1 - \delta_{ij}} w_{ij}^0}{\min_{1 \leq i \leq n} \{\xi_i\}} \|\phi - \varphi\|_r \\
 &= k e^{-\varepsilon t} \|\phi - \varphi\|_r
 \end{aligned} \tag{7}$$

So

$$\sum_{i=1}^n |x_i(t + \theta, \phi) - x_i(t + \theta, \varphi)| \leq k e^{-\varepsilon(t-\theta)} \|\phi - \varphi\|_r \tag{8}$$

So

$$P\phi = x_w(\phi), \forall \phi \in C([- \tau, 0], R^n).$$

We could obtain the following from the (9), (10)

Therefore, P^m is a contractive map in Banachspace $C([- \tau, 0], R^n)$. By Banach fixed point theorem, P^m has a unique fixed point $\phi^* \in C([- \tau, 0], R^n)$ making

$P^m \phi^* = \phi^*$, Further, $P^m(P\phi^*) = P(P^m \phi^*) = P\phi^*$, So $P\phi^*$ is also the fixed point of P^m , and $P^m \phi^* = \phi^*$ is known by the uniqueness of the fixed point, ie $x_w(\phi^*) = \phi^*$

Making $x(t, \phi^*)$ is the solution through the point $(0, \phi^*)$ of the system (1.1), noticed that

$$\|x(t, \phi) - x(t, \phi^*)\| = \sum_{i=1}^n |x_i(t, \phi) - x_i(t, \phi^*)| \leq ke^{-\varepsilon t} \|\phi - \phi^*\|, \xrightarrow{t \rightarrow \infty} 0,$$

ie $x(t, \phi^*)$ is the w -period of the system (1). For any given initial function $\phi \in C([- \tau, 0], R^n)$ and arbitrarily fixed $A \in A_+, W \in W_+$, Supposing $x(t, \phi)$ is the solution of the system(1), We could obtain the following from the (7) ie, The system (1) has a unique global exponentially stable w -periodic solution.

End of proof

4. Conclusion

This paper introduces the research status of artificial neural network, summarizes the research background and development of static recurrent neural network dynamic system, and introduces the main work of this paper. Using the fixed point theory, M. The existence of periodic solutions and the global robust exponential stability of the static recursive neural network with variable delays and the existence of almost periodic solutions of the static recursive neural network of the partitioned time are studied by combining the properties of the matrix and the Lyapunov function combined with the inequality technique. Global exponential stability, the stability conditions of the corresponding problem are obtained respectively, and the results of the related research are generalized. Using Lyapunov. The stability of the quasi - static neural recursive neural network and the stability of the periodic solution are studied. The condition of the stationary static recursive neural network is obtained and the correctness of the condition is illustrated.

Considering the influence of stochastic perturbation on the dynamic behavior of static recurrent neural network, the static recursive neural network with time delay and the static recursive neural network with distributed time delay are studied by using the infinitesimal operator, Ito formula and the convergence theorem of martingales. Global critical exponential stability of quasi - static neural network with stochastic perturbation.

The static recursive neural network with Markovian modulation and the time-delay static recurrent neural network model considering both random perturbation and Markovian switching are studied. The linear matrix inequality, the finite state space Markov chain property and the Lyapunov-krasovskii function, The judgment condition of the global exponential stability of the system is obtained. Firstly, the global exponential stability problem of quasi - static neural network with time - delay and recursive neural network

is studied by using the generalized Halanay inequality. Then the stability of the Markovian response sporadic static recurrent neural network is studied by combining the properties of Markov chain.

5. Insufficient and Prospect

- In the study of the stability of the static neural network equilibrium point in the paper, if the pulse is not considered, it is stable under the given judgment condition, that is, the time delay system without considering the pulse is stable, the system considering the pulse In the corresponding conditions are stable, but if you do not consider the pulse of the system is unstable, how to use the pulse control system makes the system stable, this is not resolved, I will try to solve this problem.
- The same problem, that is, does not consider the diffusion term is stable, under the corresponding conditions, consider the diffusion of static neural network system is still stable.
- Some of the examples in the article have not been matlab or other mathematical tools to simulate the results, which is what I need to improve the place.
- Mentioned in the pulse, random disturbance, Markov chain, so you can also consider the pulse and random perturbation of these two factors, or taking into account the pulse and Markov chain these two factors, or these three factors at the same time, This time-delay static recurrent neural network is more general.

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