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Some Characterization of the Function Space Type of Lizorkin−Triebel−Morrey

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Abstract: This paper will introduce you to some properties of normed function spaces with many groups variables field of Analysis and it helps me appreciate how normed Lebesgue–Morrey space with many groups of variables that build and studied new normed spaces nowadays. Many of the topics here are important to an Analysis class. By reading this paper, you will discover the "embedding theory" of normed spaces type of Lebesgue–Morrey by introducing few of its "new functions with groups with variables" and along the way you will see to some interesting and article elements of the branch called Analysis. A lot of problems belonging to the characterization of various spaces of differentiability function spaces and relationships between them have been solved using the theory embedding theorems. The purpose of this paper is to review several embedding inequalities of normed spaces that will arise properties of these spaces and again throughout this material. We also give "working definition, notations" of a functions and function spaces. We must note that, the analysis is based on such function spaces to build new space type of Lizorkin–Triebel–Morrey. In addition, throughout this paper we will introduce a working normed function spaces type of Lizorkin–Triebel–Morrey with standard mathematical definitions and terminology. One aspect of this paper involves normed Lebesgue–Morey type spaces that can convert space from one to another.

Keywords: The Space Type of Lesgue–Triebel−Morrey, Function Space of Differentiability Function, Many Groups of Variablesç

1. Introduction and Main Results

Let $G \subset R^n$ and $1 \le s \le n;\ s, n$ be naturals, where $n_1 + \cdots + n_s = n$. We consider the sufficient smooth function f(x), where the point $x = (x_1, \dots, x_s) \in R^n$ has coordinates $x_k = \left(x_{k.1}; \dots; x_{k,n_k}\right) \in R^{n_k}$ (k $\in e_s = \{$ 1,...,s$\}$). More precisely, $R^n = R^{n_1} \times R^{n_2} \times \cdots \times R^{n_s}$. Thus we consider the fixed, non−negative, integral vector l=$(l_1, \dots, l_s)$ such that, $l_k = \left(l_{k.1}; \dots; l_{k,n_k}\right)$, (k $\subset e_s$) that is, $l_{k,j} > 0$, $(j = 1, \dots, n_k)$ for all $k \in e_s$. Here we consider by Q the set of vectors i= $(i_1, \dots, i_s)$ where i_k= 1,2,…, n_k for every $k \in e_s$. The number of set Q is equal to: $|Q| = \prod_{k=1}^{s}(1 + n_k)$. Therefore, to the vector i $= (i_1, \dots, i_s) \in Q$, we shall correspond the vector $l^i = \left(l_1^{i_1}; \dots; l_s^{i_s}\right)$ of the set of non−negative, integral vectors l= $(l_1, \dots, l_s)$, where $l^0 = (0,0, \dots, 0), l_k^1 = \left(l_{k,1}, 0, \dots, 0\right), \dots, l_k^{i_k} = \left(0,0, \dots, l_{k,n_k}\right)$ for all $k \in e_s$. Then to the vector e^i, we let correspond the vector $\bar{l}^i = (\bar{l}_1^{i_1}, \bar{l}_1^{i_2}, \dots, \bar{l}_1^{i_s})$, where $\bar{l}_k^{i_k} = \left(\bar{l}_{k,1}^{i_1}, \bar{l}_{k,2}^{i_2}, \dots, \bar{l}_{k,n_k}^{i_k}\right)$ $(k \in e_s)$. Here the largest number $\bar{l}_{k,j}^{i_k}$ is less than $l_{k,j}^{i_k}$ for all $l_{k,j}^{i_k}$>0, when $l_{k,j}^{i_k}$=0 then we assume that $\bar{l}_{k,j}^{i_k} = 0$ for all $k \in e_s$. Theremore, we consider $D^{\bar{l}^i} f = D_1^{\bar{l}_1^{i_1}} \cdots D_s^{\bar{l}_s^{i_s}} f, D_k^{\bar{l}_k^{i_k}} f = D_{k,1}^{\bar{l}_k^{i_k}} \cdots D_{k,n_k}^{\bar{l}_k^{i_k}} f, G_{t^{\varkappa}} = G \cap I_{t^{\varkappa}}(x), I_{t^{\varkappa}}(x) = I_{t_1^{\varkappa_1}}(x_1) \times I_{t_2^{\varkappa_2}}(x_2) \times \cdots \times I_{t_s^{\varkappa_s}}(x_s), I_{t_k^{\varkappa_k}}(x_k) = \left\{y_k: |y_k - x_k| < \frac{1}{2} t_k^{|\varkappa_k|}, k \in e_s\right\}$ and $|\beta_k| = \sum_{j=1}^{n_k} \beta_{k,j}^{i_k};\ \left|\beta_k^{i_k}\right| = \sum_{j=1}^{n_k} \beta_{k,j}^{i_k}\ \frac{dt_k}{t_k} = \prod_{j \in e_k^i} \frac{dt_{k,j}}{t_{k,j}}$, we take 0< $\beta_{k,j}^{i_k} = l_{k,j}^{i_k} - \bar{l}_{k,j}^{i_k} \le 1$, when $l_k^{i_k} > 0$, but when $l_{k,j}^{i_k} = 0$, then $\beta_{k,j}^{i_k} = 0$; $t = (t_1, \dots, t_s)$, $t_k = \left(t_{k,1}, \dots, t_{k,n_k}\right), \omega = (\omega_1, \dots, \omega_s), \omega_k = \left(\omega_{k,1}, \dots, \omega_{k,n_k}\right)$ and we take $\omega_{k,j} = 1$, when k$\in e^i$, or we give $\omega_{k,j}$ =0, when k $\in e_s \,/\, e^i$, $e^i = suppl\bar{l}^i = suppl^i = supp\omega$, $1 \le \theta \le \infty$; $1 \le p < \infty$. Here $t_0 = \left(t_{0,1}, \dots, t_{0,s}\right), t_{0,k} = \left(t_{0,k,1}, \dots, t_{0,k,n_k}\right)$ −is fixed vector and $\varkappa \in (0, \infty)^n$, a$\in [0, 1], \tau \in [1, \infty], [t_k]_1 = \min\{1, t_k\}, k \in$

e_s. Here $\Delta^{\omega}(t)f = \Delta_1^{\omega_1}(t_1)\cdots\Delta_s^{\omega_s}(t_k)f$, when $2\omega = (2,2,\dots,2)$, and $\Delta_k^{\omega_k}(t_k)f = \Delta_{k,1}^{\omega_{k,1}}(t_{k,1})\cdots\Delta_{k,n_k}^{\omega_{k,n_k}}(t_{k,n_k})f, (k \in e_s)$, following $\Delta_{k,j_k}^{\omega_{k,j_k}}(t_{k,j_k})f$ are finite difference function, which has the direction with variables t_{k,j_k} and with order ω_{k,j_k}, by step t_{k,j_k} for $j = 1,\dots,n_k$ and for all and $k \in e_s$, following $\Delta_{k,j_k}^1(t_{k,j_k})f(\cdots,x_{k,j_k},\cdots) = f(\cdots,x_{k,j_k}+t_{k,j_k},\cdots) - f(\cdots,x_{k,j_k},\cdots)$, and $\Delta_{k,j_k}^{\omega_{k,j_k}}(t_{k,j_k})f(\cdots,x_{k,j_k},\cdots) = \Delta_{k,j_k}^1(t_{k,j_k})\left\{\Delta_{k,j_k}^{\omega_{k,j_k}-1}(t_{k,j_k})f(\cdots,x_{k,j_k},\cdots)\right\}$, but when $\omega_{k,j_k} = 0$, then $\Delta_{k,j_k}^0(t_{k,j_k})f(\cdots,x_{k,j_k},\cdots) = f(\cdots,x_{k,j_k},\cdots)$. Here

$$\|f\|_{p,G_{t^{\varkappa}}(x)} = \left\{\int\limits_{G_{t^{\varkappa_n}}(x_n)}\left[\cdots\left\{\int\limits_{G_{t^{\varkappa_2}}(x_2)}\left(\int\limits_{G_{t^{\varkappa_1}}(x_1)}|f(y)|^{p_1}\,dy_1\right)^{p_2/p_1}dy_2\right\}^{p_3/p_2}\cdots\right]^{p_n/p_{n-1}}dy_n\right\}^{1/p_n}$$

and dy=$\prod_{i=1}^n dy_i$.

It is said, that the subdomain $U \subset G \subset R^n$ calls domain satisfying the condition "$\sigma -$ semi–horn", if the vector $\sigma = (\sigma_1,\dots,\sigma_s)$ is such that $x + V(\sigma) \subset G$ for all $x \subset U$. It is said, that the domain $G \subset R^n$ calls domain satisfying the condition "$\sigma -$semi–horn", that is, $G \subset A(T^{\sigma})$, if we have finite subdomains $G_1,\dots,G_N \subset G$, satisfying the condition "$\sigma -$semi–horn" and the surfacing the domain G, that is,

$$G \subset \cup_{j=1}^N G_j. \tag{1}$$

But we suppose $G \in A_{\epsilon}(T^{\sigma})$, if we substitute the condition $G \subset \cup_{j=1}^N G_{j,\epsilon}$ into the condition "(1)" [1, 2].

2. Definitions and Preliminaries

Definition 1. We denote by

$$F_{p,\theta,a,\varkappa,\tau}^{<l>}(\mathrm{G, s})\ (1 < \theta < \infty)$$

normed Lizorkin–Triebel–Morrey space of function f on G, with many groups variables, with finite norm

$$\|f\|_{\mathrm{F}_{\mathrm{p,\theta,a,\varkappa,\tau}}^{<l>}(\mathrm{G,s})} = \sum_{i\in Q}\|f\|_{L_{p,\theta,a,\varkappa,\tau}^{<l^i>}(G;s)}, \tag{2}$$

where

$$\|f\|_{L_{p,\theta,a,\varkappa,\tau}^{<l^i>}(G;s)} = \left\|\left\{\int_0^{t_{0,1}^i}\cdots\int_0^{t_{0,s}^i}\left[\frac{\delta^{2\omega}(t,G)D^{\bar{l}^i}f}{\prod_{k\in e^i}t_k^{|\beta_k|}}\right]^{\theta}\prod_{k\in e^i}\frac{dt_k}{t_k}\right\}^{1/\theta}\right\|_{p,a,\varkappa,\tau:G}, \tag{3}$$

and

$$\delta^{2\omega}(t)f(x) = \int\limits_{-1}^{1}\cdots\int\limits_{-1}^{1}|\Delta^{2\omega}(t,G_t)f(x)|\,dt,$$

$$\|f\|_{p,a,\varkappa,\tau:\,G} = sup_{x\in G}\left\{\int_0^{\infty}\cdots\int_0^{\infty}\left[\prod_{k\in e_s}[t_k]_1^{\frac{-|\varkappa_k|a}{p}}\times\|f\|_{p,\,G_{t^{\varkappa}}(x)}\right]^{\tau}\prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau}. \tag{4}$$

Here $G \subset R^n$, $l^i, \bar{l}^i, p, a, \varkappa, \tau, e^i, e_s, \beta_k$ and $[t_k]_1$ are defined in chapter 1.

We must note that, when s=1, then the space $F_{p,\theta,a,\varkappa,\tau}^{<l>}(s, G)$ is equivalent to the space Lizorkin–Triebel–Morrey $\mathrm{F}_{\mathrm{p,\theta,a,\varkappa,\tau}}^{<l>}$(G), when s=n, then this space is equivalent to the space Lizorkin–Triebel–Morrey type with dominant mixed derivatives $S_{\mathrm{p,\theta,a,\varkappa,\tau}}^{<l>}$F(G), when a=0, $\tau = \infty$, then this space is equivalent to the space $F_{\mathrm{p,\theta}}^{<l>}$(s,G) Lizorkin–Triebel–Morrey type with many groups variables, which were studied in [3, 10, 17].

Definition 2. We denote by

$$\mathrm{B}_{\mathrm{p,\theta,a,\varkappa,\tau}}^{<l>}(\mathrm{s,G}) \tag{5}$$

Besov–Morrey space type of loccaly summable f, on G, with finite norm ($1 \leq \theta \leq \infty$)

$$\|f\|_{\mathrm{B}_{\mathrm{p,\theta,a,\varkappa,\tau}}^{<l>}(\mathrm{s,G})} = \sum_{i\in Q}\|f\|_{L_{p,\theta,a,\varkappa,\tau}^{<l^i>}(G;s)}, \tag{6}$$

$$\|f\|_{L_{p,\theta,a,\varkappa,\tau}^{<l^i>}(G;s)} = \left\{\int_0^{t_0}\cdots\int_0^{t_0}\left\|\frac{\Delta^{2\omega}(t;G)D^{\bar{l}^i}f}{\prod_{k\in e^i}t_k^{|\beta_k|}}\right\|_{p,a,\varkappa,\tau:\,G}^{\theta}\times\prod_{k\in e^i}\frac{dt_k}{t_k}\right\}^{1/\theta}, \tag{7}$$

and

$$\|f\|_{p,a,\varkappa,\tau:\,G} = sup_{x\in G}\left\{\int_0^\infty \cdots \int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f\|_{p,\,G_{t^\varkappa}(x)}\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau},$$

where $1 \le p < \infty, a \in [0,1], \varkappa \in (0,\infty)^n, 1 \le \tau \le \infty, [t_k]_1 = min\{1, t_k\}, t_0$–fixed, positive number.

When $\theta = \infty$, then the space "(5)" is equivalent to the space type of Nickolski − Morrey:

$$\|f\|_{H^{<l>}_{p,a,\varkappa,\tau}(s,G)} = \sum_{i=(i_1,\dots,i_s)\in Q} vrai\ sup_{t\in[0;t_0^i]} \left\|\frac{\Delta^{2\omega}(t;G)D^{\bar{l}^i}f}{\prod_{k\in e^i} t_k^{|\beta_k|}}\right\|_{p,a,\varkappa,\tau:\,G}. \tag{8}$$

When s=1 then the space "(5)" is equivalent to the space type of $B^{<l>}_{p,\theta,a,\varkappa,\tau}(G)$, when $\tau = \infty$ then the space "(5)" is equivalent to the space type $B^l_{p,\theta,a,\varkappa}(G)$, when s=n, then the space "(5)" is equivalent to the space type $S^{<l>}_{p,\theta,a,\varkappa,\tau}B(G)$, when a=0, $\tau = \infty$, then the space "(5)" is equivalent to the space type $B^{<l>}_{p,\theta}$ (s,G), which was studied by A. Dj. Djabrailov and in [4, 11, 13, 15, 17].

Definition 3. We denote by

$$W^{<l>}_{p,\theta,a,\varkappa,\tau}(s,G) \tag{9}$$

Sobolev–Morrey space type of loccaly summable f, on G, with finite norm ($1 \le \theta < \infty$)

$$\|f\|_{W^{<l>}_{p,\theta,a,\varkappa,\tau}(s,G)} = \sum_{i\in Q}\|f\|_{L^{<l^i>}_{p,\theta,a,\varkappa,\tau}(G;s)},$$

where

$$\|f\|_{L^{<l^i>}_{p,\theta,a,\varkappa,\tau}(G;s)} = sup_{x\epsilon G}$$

$$\left\{\int_0^\infty \cdots \int_0^\infty \prod_{k\epsilon e_s}\left([t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f\|_{p,\,G_{t^\varkappa}(x)}\right)^\tau \prod_{k\epsilon e_s}\frac{dt_k}{t_k}\right\}^{1/\tau}. \tag{10}$$

Here if $\tau = \infty$, then we get

$$\|f\|_{L^{<l^i>}_{p,\theta,a,\varkappa,\tau}(G;s)} = \|f\|_{L^{<l^i>}_{p,\theta,a,\varkappa,\infty}(G;s)} = sup_{\substack{x\epsilon G\\ t_k\\ k\epsilon e_s}}\left([t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f\|_{p,\,G_{t^\varkappa}(x)}\right)$$

which was studied in [6, 12, 19].

3. Proofs of Some Properties

Let us note some properties of the normed space $F^{<l>}_{p,\theta,a,\varkappa,\tau}$(s, G):

1) For any $\varkappa$>0, $1 \le \tau \le \infty, c > 0$, we obtain

$$\|f\|_{F^{<l>}_{p,\theta}(G,s)} \le \|f\|_{F^{<l>}_{p,\theta,a,\varkappa}(G,s)} \le C_2\,\|f\|_{F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s)}.$$

2) The function space $F^{<l>}_{p,\theta,a,\varkappa,\tau}(s,G)$ is complete. The spaces $L_{p,a,\varkappa,\tau}(G)$ and $F^{l}_{p,\theta,}$(s, G) are complete, that is why the space $F^{<l>}_{p,\theta,a,\varkappa,\tau}$(s, G) is complete.

3) For any real number c>0

$$\|f\|_{F^{<l>}_{p,\theta,a,c\varkappa,\tau}(G,s)} = \frac{1}{c^{\frac{1}{\tau}}}\|f\|_{F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s)}.$$

4) $\|f\|_{F^{<l>}_{p,\theta,0,\varkappa,\infty}(G,s)} = \|f\|_{F^{<l>}_{p,\theta}(G,s)}$.

5) If G limited domain and

$$p \le q, \frac{1-b}{q} \le \frac{1-a}{p}, 1 \le \tau_1 \le \tau_2 \le \infty$$

then we have

$$F^{<l>}_{q,\theta,b,\varkappa,\tau_1}(G;s) \subset_{>} F^{<l>}_{p,\theta,a,\varkappa,\tau_2}(G;s)$$

and

$$\|f\|_{p,a,\varkappa,\tau_2:\,G} \le \|f\|_{q,b,\varkappa,\tau_1:\,G}.$$

6) Observe that, $1 < \theta \leq r \leq s \leq \sigma < \infty$ and $\theta \leq p \leq \sigma$, then we get

$$B^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s).$$

If we shall be using some well–known facts in [2] about the about of normed spaces then we shall get following embedding:

$$B^{l}_{p,\theta}(G) \subset_{>} F^{l}_{p,r}(G) \subset_{>} B^{l}_{p,s}(G) \subset_{>} B^{l}_{p,\sigma}(G).$$

If $l \in N^n, r = s = 2$, then

$$B^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,2,a,\varkappa,\tau}(G,s) = W^{<l>}_{p,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s).$$

In this case when p= θ, then $F^{<l>}_{p,p,a,\varkappa,\tau}(G,s) = B^{<l>}_{p,p,a,\varkappa,\tau}(G,s)$.

Here the space $W^{<l>}_{p,a,\varkappa,\tau}(G, s)$ was defined and studied in [2, 8, 12, 13].

We proof first that properties 1, 2, 3, 4, 5 is sufficient for any for any normed $\|f\|_{p,a,\varkappa,\tau:\,G}$ [7]. Because of

$$\|f\|_{p,a,\varkappa,\tau:\,G} = sup_{x\in G}\left\{\int_0^\infty \cdots \int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f\|_{p,\,G_{t^\varkappa}(x)}\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau}$$

is given same.

Proof 6. Now we must assume that, $1 < \theta \leq r \leq s \leq \sigma < \infty$ and $\theta \leq p \leq \sigma$, then we have to proof

$$B^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s).$$

We know that, if m>n and a>0, then $a^{1/n} < a^{1/m}$. Then taking $1 < \theta \leq r \leq s \leq \sigma < \infty$ and using fifth property of the space $F^{<l>}_{p,\theta,a,\varkappa,\tau}(G, s)$ then we have

$$\|f\|_{p,\theta,a,\varkappa,\tau} \leq \|f\|_{p,r,a,\varkappa,\tau} \leq \|f\|_{p,s,a,\varkappa,\tau} \leq \|f\|_{p,\sigma,a,\varkappa,\tau}.$$

Then it follows easy from it

$$F^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s)$$

and

$$B^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s).$$

Taking Minkowski's inequality we hold

$$\left\|\left\{\int_0^{t^i_{0,1}} \cdots \int_0^{t^i_{0,s}} \left[\frac{\delta^{2\omega}(t,G)D^{\bar{l}^i}f}{\prod_{k\in e^i} t_k^{|\beta_k|}}\right]^\theta \prod_{k\in e^i}\frac{dt_k}{t_k}\right\}^{1/\theta}\right\|_{p,a,\varkappa,\tau:G} \leq \left\{\int_0^{t_0} \cdots \int_0^{t_0} \left\|\frac{\Delta^{2\omega}(t;G)D^{\bar{l}^i}f}{\prod_{k\in e^i} t_k^{|\beta_k|}}\right\|^\theta_{p,a,\varkappa,\tau:\,G} \times \prod_{k\in e^i}\frac{dt_k}{t_k}\right\}^{1/\theta}.$$

It means that,

$$B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s),$$

$$B^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s).$$

It is clear that,

$$B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s).$$

In addition,

$$B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,\sigma,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,s,a,\varkappa,\tau}(G,s) \subset_{>} B^{<l>}_{p,r,a,\varkappa,\tau}(G,s) \subset_{>} F^{<l>}_{p,\theta,a,\varkappa,\tau}(G,s).$$

We have proved that

$$\|f\|_{p,\theta,a,\varkappa,\tau} \leq \|f\|_{p,r,a,\varkappa,\tau}.$$

We must note that,

$$W_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} W_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{W}_{\mathrm{p}}^{<l>}(\mathrm{G,s}),$$

$$B_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} B_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{B}_{\mathrm{p}}^{<l>}(\mathrm{G,s}),$$

$$F_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} F_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{F}_{\mathrm{p}}^{<l>}(\mathrm{G,s}),$$

and

$$\|f\|_{W_{\mathrm{p}}^{<l>}(\mathrm{G,s})} \le \|f\|_{W_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s})} \le c\|f\|_{W_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})},$$

$$\|f\|_{B_{\mathrm{p}}^{<l>}(\mathrm{G,s})} \le \|f\|_{B_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s})} \le c\|f\|_{B_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})},$$

$$\|f\|_{F_{\mathrm{p}}^{<l>}(\mathrm{G,s})} \le \|f\|_{F_{\mathrm{p,a,ϰ}}^{<l>}(\mathrm{G,s})} \le c\|f\|_{F_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})}$$

were proved. Having this property for the space $F_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})$ using fact $F_{\mathrm{p,\theta,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} F_{\mathrm{p,\theta,a,ϰ}}^{<l>}(\mathrm{G,s})$, one can easy proof it [9].

To prove $\mathrm{B}_{\mathrm{p,\theta}}^{l}(\mathrm{G}) \subset_{>} \mathrm{F}_{\mathrm{p,r}}^{l}(\mathrm{G}) \subset_{>} \mathrm{B}_{\mathrm{p,s}}^{l}(\mathrm{G}) \subset_{>} \mathrm{B}_{\mathrm{p},\sigma}^{l}(\mathrm{G})$ we shall use first part of the property 6.

Now let us proof

$$\mathrm{B}_{\mathrm{p,\theta,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) = \mathrm{W}_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{B}_{\mathrm{p,\sigma,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}).$$

Taking first part of property 6, we get

$$\mathrm{B}_{\mathrm{p,\theta,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) = \mathrm{F}_{\mathrm{p,2,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{B}_{\mathrm{p,\sigma,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}).$$

Then we may replace $\theta = \infty$, then the space "(5)" is equivalent to the space type of Nickolski –Morrey "(8)":

$$\mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{W}_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})$$

and

$$\|f\|_{L_{p,\theta,a,ϰ,\tau}^{<l^i>}(G;s)} = sup_{\substack{x\epsilon G\\ t_k\\ k\epsilon e_s}}\left([t_k]_1^{-\frac{|ϰ_k|a}{p}} \times \|f\|_{p,\,G_{t^ϰ}(x)}\right).$$

Then we see that a Lebesgue–Triebel–Morrey type space is continuously embedded into a Sobolev–Morrey and there exists a constant c such as

$$\|f\|_{\mathrm{W}_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})} \le c\|f\|_{\mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s})}$$

for all f. In addition we get

$$\mathrm{W}_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) \subset_{>} \mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}).$$

Then we hold

$$\mathrm{F}_{\mathrm{p,2,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}) = \mathrm{W}_{\mathrm{p,a,ϰ,\tau}}^{<l>}(\mathrm{G,s}).$$

Taking

$$\|f\|_\infty \le \|f\|_1 \le n\|f\|_\infty,$$

$$\|f\|_\infty \le \|f\|_2 \le \sqrt{n}\|f\|_\infty,$$

$$\|f\|_2 \le \|f\|_1 \le \sqrt{n}\|f\|_2$$

Which was studied in [18, 19] we can proof property 6 for any θ.

4. Conclusion

In this paper I have given and studied normed Lizorkin–Triebel–Morrey type space with many groups of variables. In addition, we can give some properties of these type spaces and some of them were proved. Five of these properties are similar to the already known vanishing properties, but related to the behavior at Lebesgue–Triebel–Morrey type spaces instead of that the origin. Another is connected with the Lebesgue–Morrey type to the exterior with many groups of variables. These five additional properties, together with the vanishing property at the origin, allow us that all elements in this new spaces denoted in the sequel by $L_{p,\theta,a,ϰ,\tau}^{<l^i>}(G;s)$, may be approximated by $B_{p,\theta,a,ϰ,\tau}^{<l^i>}(G;s), F_{p,\theta,a,ϰ,\tau}^{<l^i>}(G;s)$ function spaces in Morrey norm. In this paper, we have done a quick review of some topic that ate absolutely essential to being successful in an Analysis class. We know, that generalized derivatives of the space are dependent of the function, but using integral representation one can see the function is dependent of its generalized derivatives:

$$f(x) = f_{t^\sigma}(x) + \int_0^T \sum_{i=(i_1,\dots,i_s)\in Q} v^{-1-|\sigma|-t_i\sigma_i}\, dv \times$$

$$\int_{R^n} \frac{\delta^{\sigma_i} f(x+y)}{\delta_i^{\sigma_i}} \varphi_i(v^{-\sigma} y) dy,$$

$$f_{t^\sigma}(x) = t^{-|\sigma|} \int_{R^n} f(x+y)\varphi_0(v^{-\sigma} y)\, dy.$$

Present work by the author is devoted to generalizing to the concepts of several Lebesgue–Morrey type spaces with many group variables to introduce if more refined results on the well-known inequalities and method integral representation of these type spaces can be obtained and we one obtain relationship between and convert space from one from to another. All spaces are in what we generally consider Lebesgue–Morrey type spaces.

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