International Journal on Data Science and Technology
2024, Vol. 10, No. 2, pp. 26-37
https://doi.org/10.11648/j.ijdst.20241002.12

Research Article

Identification of MIMO Nonlinear Gaussian Time-Varying System Based on Multi-Dimensional Taylor Network Multi-Level Approximation

Jiefei Li[1], Shaolin Hu[2, *]

[1]School of Information and Control Engineering, Jilin Institute of Chemical Technology, Jilin, China
[2]School of Automation, Guangdong University of Petrochemical Technology, Maoming, China

Abstract

Aiming at the problems of identification difficulties and low identification accuracy in modelling and identification of multiple-input multiple-output (MIMO) nonlinear Gaussian time-varying systems, this paper proposes an identification scheme based on the step-by-step approximation of multidimensional Taylor network (MTN). The aim of this paper is to improve the modelling of complex nonlinear systems so as to improve the prediction performance and control effect of the system. Different from the traditional multidimensional Taylor network identification method, this method adopts an order-by-order approximation strategy, which seeks its parameters sequentially from the lower order to the higher order, and continuously optimises the parameter weights during the parameter seeking process. Firstly, the nonlinear function model is approximated as a polynomial form by the order-by-order Taylor expansion, and then the weight parameters of each order of the Taylor expansion are calculated and updated step by step by using the algorithm based on the Variable Forgetting Factor Recursive Least Squares (VFF-RLS) method. Through iterative optimized of these parameters, dynamic weight assignment to each order of the Taylor expansion is achieved. A parameter-identified nonlinear function model is finally obtained, which can more accurately describe the dynamic behaviour and characteristics of the system. Finally, an arithmetic simulation is carried out through an example to verify the effectiveness of the proposed method.

Keywords

Gaussian Nonlinear Time-Varying System, Multi-Dimensional Taylor Network, Gradual Approximation, Variable Forgetting Factor Recursive Least Squares Algorithm

1. Introduction

For the past few years, there has been extensive and in-depth research on identification methods for time-invariant systems. However, compared to linear time-invariant systems, nonlinear time-varying systems are the most general and widely applicable system in engineering applications [1]. Recently, a number of scholars have started to delve into the problem of mode identification for nonlinear time-varying system. However, there is still limited domestic and international research on the structure of such system. Nonlinear system identification is one of the

*Corresponding author: hfkth@126.com (Shaolin Hu)

Received: 25 June 2024; **Accepted:** 11 July 2024; **Published:** 29 July 2024

main methods to address complex system modelling, and it serves as the foundation for system analysis and control. Therefore, research on the identification of nonlinear time-varying systems is of significant importance and application value [2].

Currently, with the emergence of neural networks, thise method has become an effective approach to solving nonlinear identification problems due to their powerful nonlinear approximation capabilities. Recently, a fully connected recurrent neural network (FCRNN) structure for nonlinear system recognition has been proposed by Shobana et al [3]. The fully connected recurrent neural network structure consists of feedback layers with internally adjustable weights, which not only endows the structure with the necessary memory properties but also improves its ability to handle dynamic systems. The Back Propagation (BP) algorithm is used to derive the equations used to update the weights of the proposed model. Experimental results show that the FCRNN model has better discriminative accuracy and robustness compared to feedforward neural networks (FFNN) and local recurrent neural networks (LRNN). Literature [4] proposes a model based on the classical Elman Neural Network (ENN), which connects the output neurons to an additional number of weighted self-feedback loops and adjustable weighted input signals, and updates the model parameters using the Lyapunov stability criterion, which ensures the stability of the model and the validity of the simulation. XuJun et al [5] proposed an efficient hinge hyperplane (EHH) neural network, EHH is different from the commonly used single hidden layer neural network, the hidden layer in EHH can be viewed as a directed acyclic graph (DAG), and the output is influenced by all nodes in the DAG. Each EHH neural network contains an equivalent Adaptive Hinge Hyperplane (AHH) tree, which is proposed on the basis of the Hinge Hyperplane (HH) model, and is well used in the direction of system identification. Applying the EHH neural networks to nonlinear system identification can achieve satisfactory accuracy with relatively low computational cost. However, nonlinear identification based on neural networks has a series of problems due to high data demand, overfitting, difficulty in choosing the network structure, often falling into local optimums, and the need for high computational resource requirements, which cannot satisfy the demand for constructing predictive models in real time. In addition, compared with single-input single-output systems, multi-input multi-output systems are more difficult to model due to the more complex relationship between input and output variables [6, 7].

With the development of mathematics, statistics and computational methods, many new model identification methods and techniques are emerged. For online identification of continuous-time nonlinear systems by sliding window type Gaussian process (GP) models, a scheme combining the linear recursive least squares (RLS) method with the firefly algorithm (FA) in a bootstrap approach for tracking the time-varying system parameters and nonlinear functions has been proposed in the literature [8]. The Firefly algorithm is mainly responsible for searching the hyperparameters of the covariance function, while the least squares (RLS) method is used to update the system parameters of the linear terms and the weighting parameters of the mean function. Literature [9] presents a model reference adaptive control (MRAC) method for identifying unstable nonlinear systems in nonlinear parameters. The method first ensures that the system is stabilized at the equilibrium point by Liapunov's method and then is used to estimate the unknown system parameters by nonlinear least squares. The method is advantageous in identifying open-loop unstable nonlinear systems. Literature [10] proposed a flexible coefficient autoregressive model (BFM-FCAR) based on the basis function matrix with a time series and nonlinear system modeling framework. Using this framework, many well-known nonlinear time series models can be derived by choosing appropriate basis function matrices, and the effectiveness of the method is demonstrated by experimental results. However, many model-based nonlinear system model identification still suffer from high algorithmic complexity, poor real-time performance, and some identification schemes are not generalizable.

In 2010, based on the idea of Taylor expansion, Prof. Hongsen Yan proposed a modelling scheme of multidimensional Taylor net, which is a new dynamics modelling tool that can be used to solve the problem of dynamics modelling and identification of general nonlinear systems with unknown mechanisms. Professor Hongsen Yan proposed the idea of multidimensional Taylor network optimal control [11, 12], which was applied to the field of control of nonlinear systems, and successively carried out extensive research on the control problems of nonlinear time-varying systems [13], nonlinear time-lag systems [14] and nonlinear stochastic systems [15]. However, since the method performs a uniform one-time approximation for the Taylor-expanded mathematical polynomials (hereinafter abbreviated as the MTN-DA identification model), the approximation accuracy still suffers from a large error, and furthermore, there are some limitations of these methods in the implementation of the application and in high-noise environments. Based on this, this paper proposes a multidimensional Taylor network step-by-step approximation identification scheme (hereinafter abbreviated as MTN-PA identification model) on the basis of multidimensional Taylor network, which expands the complex nonlinear function model in the form of first-order terms, second-order terms, third-order terms and other higher-order terms, and performs step-by-step approximation of each power, respectively, so as to improve the accuracy of the identification of the nonlinear system model.

The commonly used MTN prediction algorithms include DRLS algorithm, L-M algorithm and BP algorithm with momentum factor [16]. The recursive least squares (RLS) [17, 18] algorithm is also a commonly used algorithm for solving the problem of recognizing nonlinear systems containing noise factors and time-varying characteristics [19]. However, in the identification of Gaussian nonlinear time-varying systems, the traditional RLS estimation algorithm suffers from the problems of slow adaptive tracking and low accuracy, in this paper, we will use the least squares with forgetting factor (VFF-RLS) algorithm [20] to track the dynamics of nonlinear time-varying systems. The algorithm is able to adaptively change the data weights so as to better reflect the current characteristics of the system, so it is able to perform effective tracking approximation of Gaussian nonlinear time-varying systems.

The main contributions of this paper are as follows: a) based on the multidimensional Taylor network, a step-by-step approximation of the recognition model is proposed, and the proposed scheme is easy to implement; b) based on the least squares with forgetting factor as the learning algorithm of the recognition model, it can estimate the weights in real time, and it can dynamically allocate the weights during the task, so as to retain the more valuable information, and ultimately improve the accuracy of the model recognition; c) through the experimental simulation, it is proved that the recognition scheme based on multidimensional Taylor network step-by-step approximation can utilize the multilevel information of the data to dynamically assign the weights and further improve the model recognition accuracy, which provides technical support for the modeling and recognition of multi-input multi-output nonlinear Gaussian time-varying systems. d) The proposed recognition scheme is of general and practical value for the recognition of MIMO nonlinear Gaussian time-varying systems.

2. Nonlinear Systems with Multidimensional Taylor Network Approximation

MTN is a dynamics modelling tool that can be used to solve the problem of dynamics modelling and identification of general nonlinear systems with unknown mechanisms.

2.1. System Description

Considering a MIMO nonlinear time-varying system with Gaussian noise:

$$y_j(k) = f_j(x(k), k) + v_j(k) \tag{1}$$

where $y_j(k) \in R^{m\times 1}; x(k) \in R^{n\times 1}$, $y_j(k)$ is the j -th component of y, $f_j(\bullet)$ is an unknown nonlinear function, $v_j(k)$ is a Gaussian white noise with a mean of 0 and satisfies:

$$v_i(k)v_j^{\tau}(k) = \delta_{ij}R(k) \tag{2}$$

where δ(x) is the Dirac delta function and satisfies:

$$\delta_{ij} = \begin{cases} 1, & k = j \\ 0, & k \neq j \end{cases} \tag{3}$$

2.2. Multi-dimensional Taylor Network Approaching

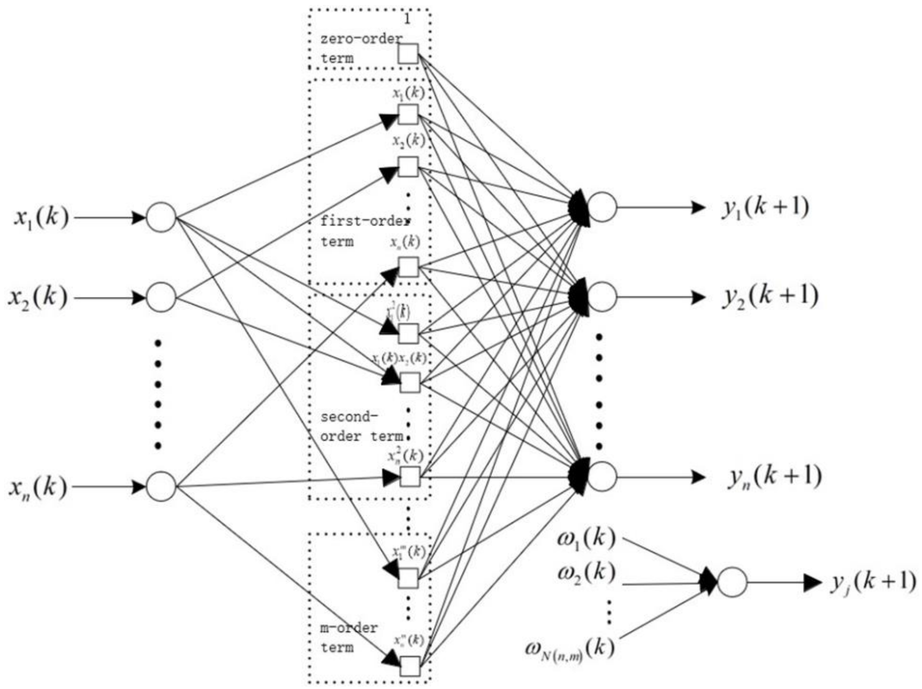

Figure 1. *Multi-dimensional Taylor Network Model Structure.*

The multi-dimensional Taylor network (MTN) adopts a forward single hidden layer structure consisting of an input layer, a hidden layer, and an output layer, as shown in Figure 1. For the MIMO nonlinear system in equation (1), the j -th output component $y_j(k+1)$ can be expressed based on the multi-dimensional Taylor network as follows:

$$\begin{aligned} y_j(k+1) &= f_j(x(k),k)+v_j(k) \\ &= \sum_{t=1}^{N(n,m)} \omega_t(k)\prod_{i=1}^{n} x_i^{\lambda_{t,i}}(k)+v_j(k) \end{aligned} \tag{4}$$

Where $\sum_{t=1}^{n}\lambda_{t,i} \le m$, ω_t represents the weight before the t -th variable product term, $N(n,m)$ represents the total number of terms in the expansion, $\lambda_{t,i}$ represents the power of variable x_i in the t -th variable product term.

Therefore, f is composed of the weighted sum of variable product terms where the sum of the powers of variables is less than or equal to m.

Literature [21] demonstrated that for a continuous function $f(x_1, x_2, \dots, x_n)$, which is defined on a closed interval, it can be approximated by a $\sum_{t=1}^{N(n,m)} \omega_t \prod_{i=1}^{n} x_i^{\lambda_{t,i}}$ expansion. $N(n,m)$ is the total number of terms in the product term of the approximation expansion, ω_t is the weight value before the j -th product term in the approximation expansion, and $\lambda_{t,i}$ is the degree of the variable x_i in the t -th product term of the expansion.

The MTN utilizes an architecture consisting of an input layer, hidden layers, and an output layer. Moreover, based on Lemma 21, as long as the number of terms is sufficiently large, the multivariate Taylor network model can approximate any function and effectively replace traditional neural networks for dynamic system modeling and control.

3. Progressive Approximation of the Multivariate Taylor Network

The multivariate Taylor network model given by equation (4) can approximate any nonlinear function. The specific approximation methods can be divided into two categories: direct approximation and progressive approximation. Direct approximation refers to the original model, while progressive approximation involves expanding the nonlinear function model in terms of first-order, second-order, third-order, and higher-order terms using a Taylor series expansion. The VFF-RLS (Variable Forgetting Factor Recursive Least Squares) algorithm with a forgetting factor is then used to progressively approximate each order of power terms.

If we perform a first-order MTN expansion on equation (1), based on equation (4), we obtain:

$$\begin{aligned} y_j(k) &= f_j(x(k),k)+v_j(k) \\ &= \sum_{t=1}^{n}\omega_t(k)x_t(k) + \sum_{t=1}^{N(n,m)}[\omega_t(k)\prod_{i=1}^{n} x_i^{\lambda_{t,i}}(k)]+v_j(k) \end{aligned} \tag{5}$$

where $2 \le \sum_{t=1}^{n}\lambda_{t,i} \le m$.

The first-order term of the multivariate Taylor expansion is included in equation (5), represented by the function $\sum_{t=1}^{n}\omega_t(k)x_t(k)$. The higher-order remainder term is $\sum_{t=1}^{N(n,m)}[\omega_t(k)\prod_{i=1}^{n} x_i^{\lambda_{t,i}}(k)]$, $2 \le \sum_{t=1}^{n}\lambda_{t,i} \le m$, Moreover, equation (5) also includes the noise $v_j(k)$. If we neglect the higher-order remainder term in equation (5), the input-output relationship can be approximately expressed as:

$$y_j(k) \approx \sum_{t=1}^{n}\omega_t(k)x_t(k)+v_j(k) \tag{6}$$

By utilizing equation (6), the VFF-RLS algorithm can be employed to estimate the value of the first-order weight parameters $\omega_t(k)$ as $\hat{\omega}_t(k)$. The VFF-RLS algorithm will be introduced in section 4.1. After obtaining $\hat{\omega}_t(k)$, we substitute $\hat{\omega}_t(k)$ into equation (6) to obtain the first-order predicted output value of the system, denoted as $\hat{y}_j(k)$.

$$\hat{y}_j(k) = \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k) \tag{7}$$

If we perform a second-order MTN expansion on equation (1), based on equation (4), we obtain:

$$y_j(k) = f_j(x(k),k)+v_j(k) = \sum_{t=1}^{n}\omega_t(k)x_t(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\omega_{t,i}^{(2)}(k)x_t(k)x_i(k) + \sum_{t=1}^{N(n,m)}[\omega_t(k)\prod_{i=1}^{n} x_i^{\lambda_{t,i}}(k)]+v_j(k) \tag{8}$$

where $3 \le \sum_{t=1}^{n} \lambda_{t,i} \le m$.

The first-order term of the multivariate Taylor expansion is included in equation (8), represented by the function, and the second-order term $\sum_{t=1}^{n}\sum_{i=1}^{n}\omega_{t,i}^{(2)}(k)x_t(k)x_i(k)$ is included. $\sum_{t=1}^{N(n,m)}[\omega_t(k)\prod_{i=1}^{n}x_i^{\lambda_{t,i}}(k)]$ is the higher-order remainder term, and the noise term $v_j(k)$ is included. If we neglect the higher-order remainder term in equation (8), it can be approximated as:

$$y_j(k) \approx \sum_{t=1}^{n}\omega_t(k)x_t(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\omega_{t,i}^{(2)}(k)x_t(k)x_i(k) + v_j(k) \quad (9)$$

Differencing equation (9) with equation (7):

$$y_j(k) - \hat{y}_j(k) = \tilde{y}_j(k) = [\sum_{t=1}^{n}\omega_t(k)x_t(k) - \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k)] + \sum_{t=1}^{n}\sum_{i=1}^{n}\omega_{t,i}^{(2)}(k)x_t(k)x_i(k) + v_j(k) \quad (10)$$

At this point, $\sum_{t=1}^{n}\omega_t(k)x_t(k) - \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k) = 0$. By employing the VFF-RLS algorithm, we can estimate the value of the second-order weight parameters $\omega_{t,i}^{(2)}(k)$ as their estimate $\hat{\omega}_{t,i}^{(2)}(k)$. After obtaining $\hat{\omega}_{t,i}^{(2)}(k)$, we substitute $\hat{\omega}_t(k)$ and $\hat{\omega}_{t,i}^{(2)}(k)$ into equation (9) to acquire the second-order predicted output value of the system, denoted as $\hat{y}_j^{(2)}(k)$.

$$\hat{y}_j^{(2)}(k) = \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\hat{\omega}_{t,i}^{(2)}(k)x_t(k)x_i(k) \quad (11)$$

Equation (11) is the approximation formula obtained through second-order approximation method based on the multidimensional Taylor network model.

Similarly, the approximation formula based on the MTN model obtained by the third-order approximation method is as follows:

$$\hat{y}_j^{(3)}(k) = \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\hat{\omega}_{t,i}^{(2)}(k)x_t(k)x_i(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\sum_{h=1}^{n}\hat{\omega}_{t,i,h}^{(3)}(k)x_t(k)x_i(k)x_h(k) \quad (12)$$

The approximation formula based on the MTN model obtained through m order approximation method is as follows:

$$\hat{y}_j^{(m)}(k) = \sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\hat{\omega}_{t,i}^{(2)}(k)x_t(k)x_i(k) + \sum_{t=1}^{n}\sum_{i=1}^{n}\sum_{h=1}^{n}\hat{\omega}_{t,i,h}^{(3)}(k)x_t(k)x_i(k)x_h(k) + \cdots + \sum_{\lambda=1}^{C_{n+m}^{m}}(\hat{\omega}_{\lambda}^{(m)}(k)\prod_{i=1}^{n}x_i^{\lambda_m(C_{n+m-1}^{m},i)}) \quad (13)$$

where $\hat{\omega}_{\lambda}^{(m)}$ represents the m th-order weighted prediction value, and $\lambda_m(C_{n+m-1}^{m},i)$ represents the power of each input component at n dimensions when expanding the MTN to the m-order.

4. Multi-dimensional Taylor Network Stepwise Approximation Learning Algorithm

Based on the structure of the MTN model, a suitable parameter training algorithm can approximate any nonlinear system. In this paper, the Variable Forgetting Factor Recursive Least Squares (VFF-RLS) algorithm with a forgetting factor is employed as the learning algorithm for the MTN-PA model.

The learning process of MTN refers to the learning of parameter weights for each term of the expansion in MTN. In this paper, the parameters of MTN are obtained through the Least Squares Learning Algorithm with Forgetting Factor (VFF-RLS) algorithm.

Denote:

$$
\begin{aligned}
&Y_j^{(1)}(k)=\sum_{t=1}^{n}\hat{\omega}_t(k)x_t(k)\\
&Y_j^{(2)}(k)=\sum_{t=1}^{n}\sum_{i=1}^{n}\hat{\omega}_{t,i}^{(2)}(k)x_t(k)x_i(k)\\
&Y_j^{(3)}(k)=\sum_{t=1}^{n}\sum_{i=1}^{n}\sum_{h=1}^{n}\hat{\omega}_{t,i,h}^{(3)}(k)x_t(k)x_i(k)x_h(k)\\
&\quad\vdots\qquad\quad\vdots\qquad\qquad\vdots\\
&Y_j^{(m)}(k)=\sum_{\lambda=1}^{C_{n+m}^{m}}(\hat{\omega}_\lambda^{(m)}(k)\prod_{i=1}^{n}x_i^{\lambda_m(C_{n+m-1}^{m},i)})
\end{aligned}
\tag{14}
$$

Based on the equation (13), the following expression can be defined:

$$
\begin{aligned}
y_j(k)&=Y_j^{(1)}(k)+Y_j^{(2)}(k)+\ldots+Y_j^{(m)}(k)+v_j(k)\\
&=\Omega_1(k)X^{(1)}(k)+\Omega_2(k)X^{(2)}(k)+\ldots+\Omega_i(k)X^{(m)}(k)+v_j(k)\\
&=\sum_{t=1}^{i}\Omega_t(k)\sum_{n=1}^{m}X^{(n)}(k)+v_j(k)
\end{aligned}
\tag{15}
$$

where $\Omega_t(k)$ represents the weight matrix of the i-th power, $Y_j^{(m)}(k)$ denotes the m-order term of $y_j(k)$, $X^{(n)}(k)$ represents the variable of the polynomial with degree n.

According to Eq. (15) one can get:

$$
\begin{aligned}
&X^{(1)}(k)=\sum_{i=1}^{n}x_i(k)\\
&X^{(2)}(k)=\sum_{1\le i_1\le i_2\le n}^{n}x_{i_1}(k)x_{i_2}(k)\\
&\quad\vdots\qquad\qquad\vdots\\
&X^{(n)}(k)=\sum_{1\le i_1\le i_2\le\cdots\le i_n\le n}^{n}x_{i_1}(k)x_{i_2}(k)\cdots x_{i_n}(k)
\end{aligned}
\tag{16}
$$

The specific steps of the VFF-RLS algorithm based on the MTN multi-order approximation are as follows:

1. Initialization of the relevant parameters for the i-th order approximation. The initial weight value of the weight parameter $\Omega_i(\tau)$ for the i-th order approximation is set to $\hat{\Omega}_i(0)=0$, and the initial value of $P_i(\tau)$ is set to $P_i(0)=I$, where I denotes the identity matrix.

2. Compute the estimation error for the i-th order approximation and denote it as $\tilde{Y}_j^{(i)}(\tau)$.

$$\tilde{Y}_j^{(i)}(\tau)=y_j(\tau)-\left(Y_j^{(1)}(\tau)+Y_j^{(1)}(\tau)+\cdots+Y_j^{(i)}(\tau)\right) \tag{17}$$

3. Compute the weight iteration parameters.

$$K_i(\tau)=P_i(\tau)X^{(i)T}(\tau)/[\beta_i+X^{(i)}(\tau)P_i(\tau)X^{(i)T}(\tau)] \tag{18}$$

$$P_i(\tau)=\beta_i(\tau)^{-1}[I-K_i(\tau)X^{(i)}(\tau)]P_i(\tau) \tag{19}$$

$$\beta_i(\tau)=\beta_{i(\min)}+(1-\beta_{i(\min)})2^{L_i(\tau)} \tag{20}$$

$$L_i(\tau)=-INT(\rho[\tilde{Y}^{(i)}(\tau)]^2) \tag{21}$$

where $INT(\cdot)$ represents the smallest integer close to $\rho e^2(\tau)$, $\beta_{i(\min)}$ is a constant, ρ is the sensitive gain that can control the rate at which β approaches 1, and it happens when the estimation error $\tilde{Y}_j^{(i)}(\tau)$ approaches 0, $\beta=1$.

β represents the forgetting factor that reflects the influence of input-output data on the Recursive Least Squares (RLS) algorithm. The basic principle is to adjust the weights of parameter estimation based on historical data, assigning a larger weight to data close to time step" k " and a smaller weight to data farther away from time step" k ".

When the parameters of a nonlinear Gaussian MIMO system change over time, traditional gradient descent methods and ordinary least squares methods may struggle to identify the system parameters and adapt to the effects caused by parameter changes. However, the Variable Forgetting Factor Recursive Least Squares (VFF-RLS) method, which incorporates a forgetting factor, can effectively reduce the weight of past data and increase the weight of current data for parameter identification.

In the VFF-RLS algorithm, $0<\beta<1$ is usually chosen such that the larger the value of β, the greater the impact of new data on system parameter identification and the smaller the impact of old data. When $0.95<\beta<1$ equals a certain value, the VFF-RLS algorithm can effectively track time-varying parameters. Based on the MTN's step-by-step approximation algorithm's convergence speed, both the parameter identification capability and the learning algorithm performance are effectively improved.

4. Weight update:

$$\hat{\Omega}_i(\tau+1)=\hat{\Omega}_i(\tau)+K_i(\tau)\tilde{Y}_j^{(i)}(\tau+1) \tag{22}$$

5. Repeat steps B to D until $\tau=k$, in order to obtain the weight parameters $\hat{\Omega}_i(k)$ for the i-th order approximation.

6. Return to step A to calculate the weight parameters for the next order approximation, until the weight parameters $\hat{\Omega}_i(k)$ are estimated for each order approximation.

7. Repeat steps A to F to calculate the $j+1$-th output component $\hat{y}_{j+1}(k)$.

5. Simulation Results and Analysis

Consider the following MIMO (Multiple-Input Multiple-Output) nonlinear time-varying system:

$$y_1(k)=\sin(k)x_1(k)+\frac{0.25x_2(k)\cos(k)}{k+\sin(x_2(k))}+e^{x_3(k)}x_2(k)x_4(k)+\cos(x_1(k))x_3(k)+v_1(k) \tag{23}$$

$$y_2(k)=\cos(k)x_1(k)+\frac{0.2x_2(k)x_3(k)}{12.56+k\sin(k+x_1(k))}+v_2(k)0.03\cos(x_3(k))x_3(k)+0.3e^{x_4(k)\cos(x_2(k)x_1(k))} \tag{24}$$

In this case, we focus on a MIMO nonlinear Gaussian time-varying system. We compare the MTN-DA model and the MTN-PA model with the introduction of the VFF-RLS algorithm. Both models adopt a 5-56-2 structure, which means they have 5 input nodes expanded up to the 3rd order. The inputs, denoted as $\{x_1(k),x_2(k),x_3(k),x_4(k)\}^T$, are randomly sampled from a uniform distribution within the interval $[-0.5,0.5]$. The iteration is performed for a total of 400 iterations, with 300 samples used for learning and 100 samples used for testing. y_1 and y_2 represent the system outputs. "MTN-PA+VFF-RLS" refers to the proposed identification scheme, and "MTN-DA+VFF-RLS" represents the comparative scheme.

Figure 2, figure 3, and figure 4 as well as Figure 8, figure 9, and figure 10 show the identification results of subsystems y_1 and y_2 based on the MTN multistage approximation. Figure 2, figure 8 represents the first-order approximation, Figure 3, figure 9 represents the second-order approximation, and Figure 4, figure 10 represents the third-order approximation. As the order of approximation increases, the approximation accuracy improves. When reaching the third-order approximation, the errors are reduced to 0.0131 and 0.0090, respectively.

Figure 5 and figure 12 display the identification error curves of systems y_1 and y_2 using the MTN-DA+VFF-RL scheme. Compared to the MTN-PA+VFF-RLS identification scheme, the accuracy of the MTN-DA+VFF-RL scheme falls between the first-order and second-order approximations, indicating a poorer approximation performance.

Figure 6 and figure 13 provide a comparison among the desired output, MTN-PA+VFF-RLS, and MTN-DA+VFF-RLS for subsystems y_1 and y_2 Experimental results indicate that the MTN-PA+VFF-RLS identification scheme exhibits significant improvement in identification accuracy compared to the MTN-DA+VFF-RLS scheme.

Figure 7 and figure 13 depict the identification error curves of subsystems y_1 and y_2 for both the MTN-PA+VFF-RLS and MTN-DA+VFF-RL schemes. The tracking performance of MTN-PA+VFF-RL is significantly better than that of the MTN-DA+VFF-RLS scheme during the initial stages of identification under the influence of the forgetting factor, resulting in smaller identification errors.

Table 1 and table 2 compares the MSE calculation results and the test Mean Square Error (MSE) results of the two identification schemes. For MIMO nonlinear Gaussian systems, the MTN-PA identification scheme outperforms the MTN-DA identification scheme in terms of identification accuracy.

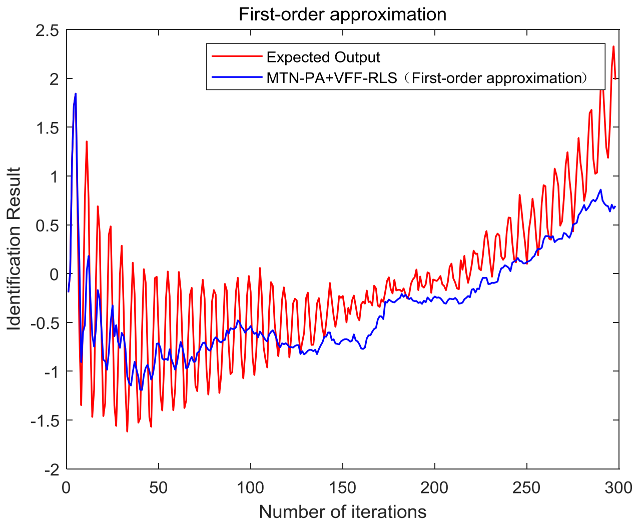

***Figure 2**. First-order approximation (y_1).*

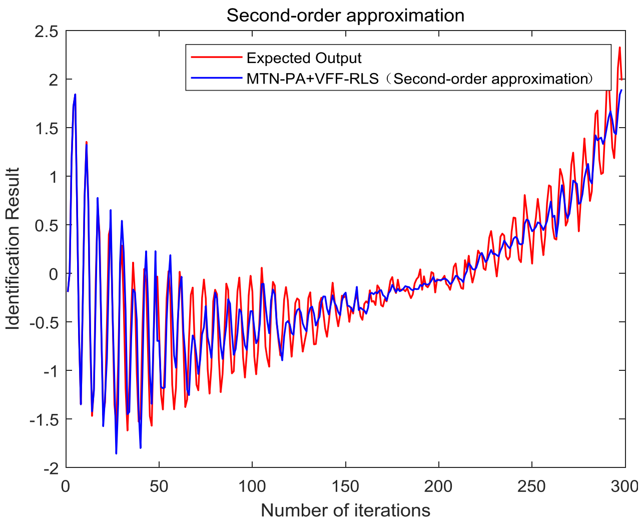

***Figure 3**. Second-order approximation (y_1).*

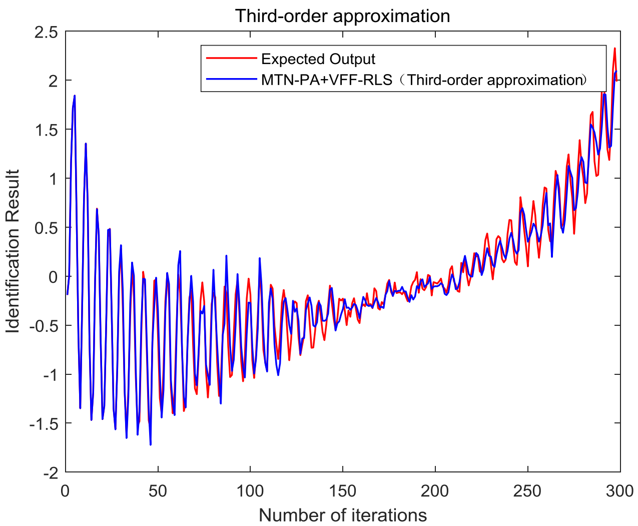

***Figure 4**. Third-order approximation (y_1).*

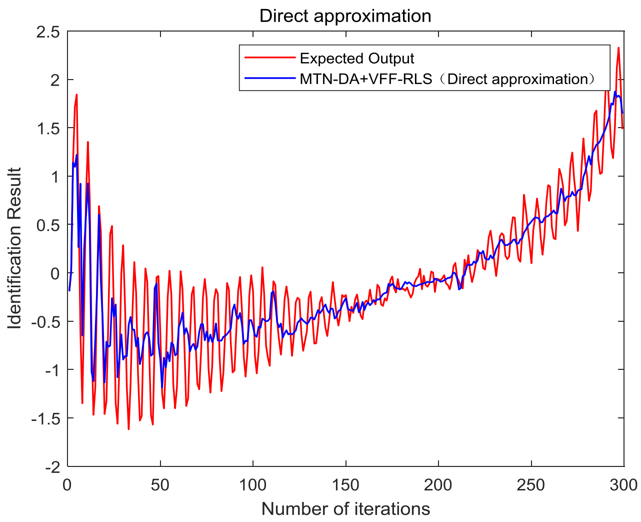

***Figure 5**. Direct approximation (y_1).*

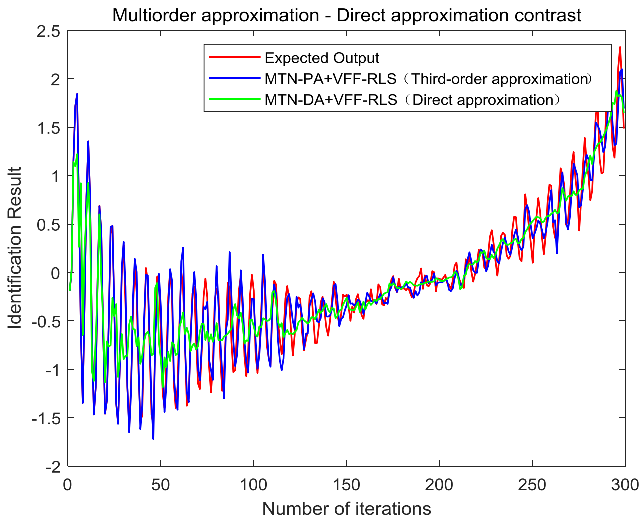

***Figure 6**. Comparison of multilevel and direct approximation (y_1).*

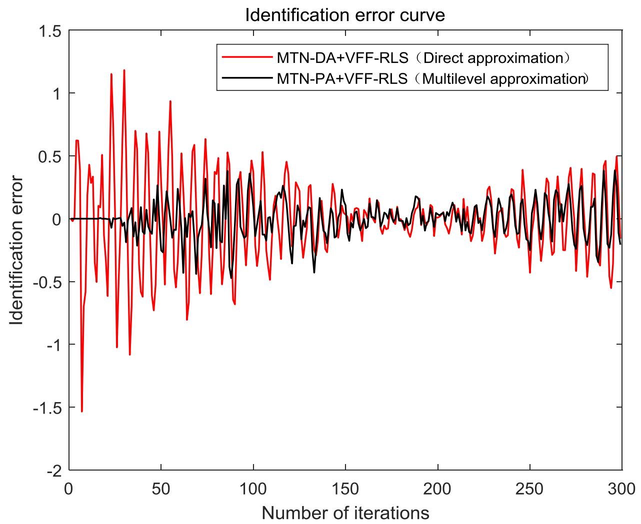

***Figure 7**. Identification error curve (y_1).*

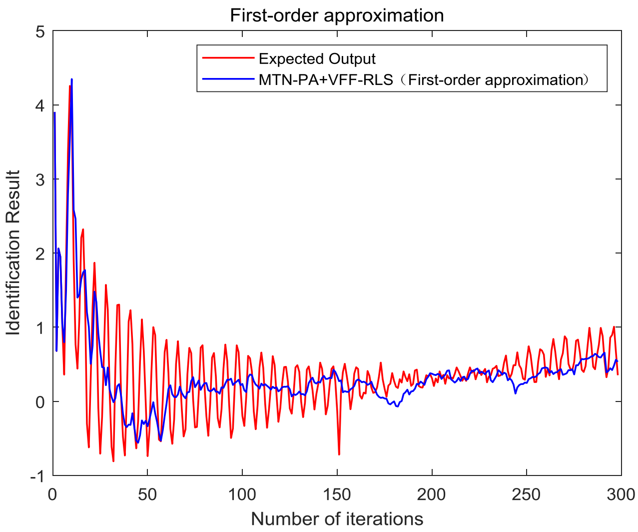

Figure 8. *First-order approximation (y_2).*

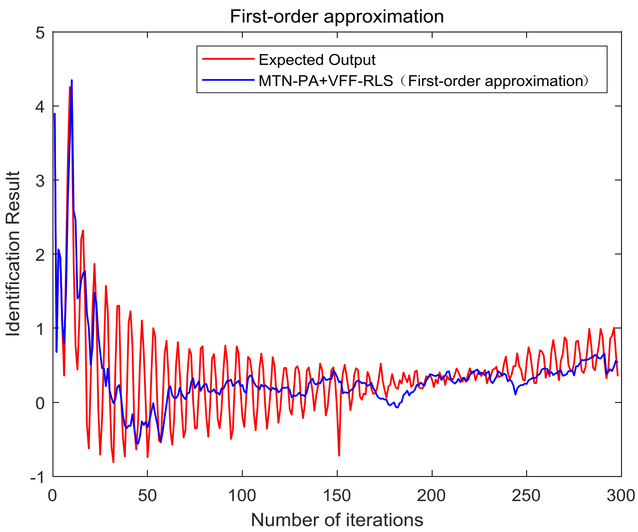

Figure 9. *Second-order approximation (y_1).*

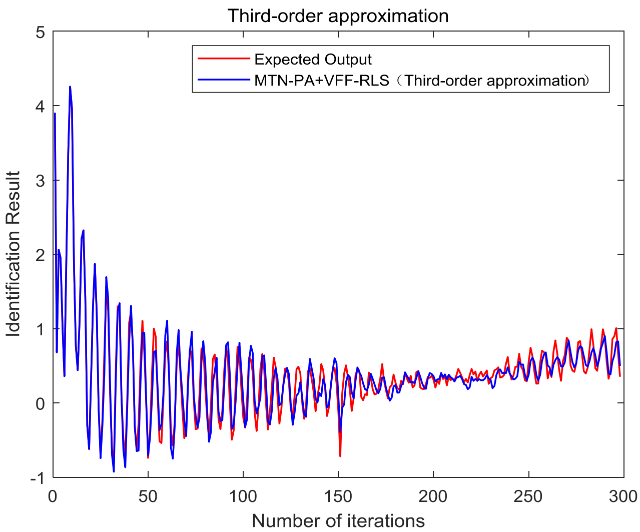

Figure 10. *Third-order approximation (y_1).*

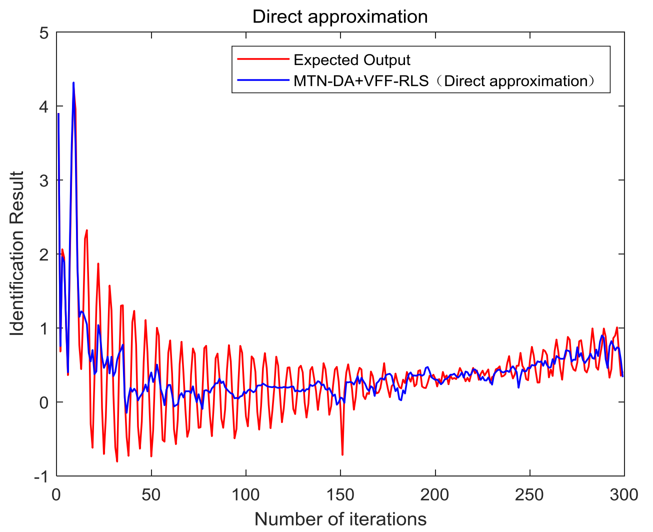

Figure 11. *Direct approximation (y_1).*

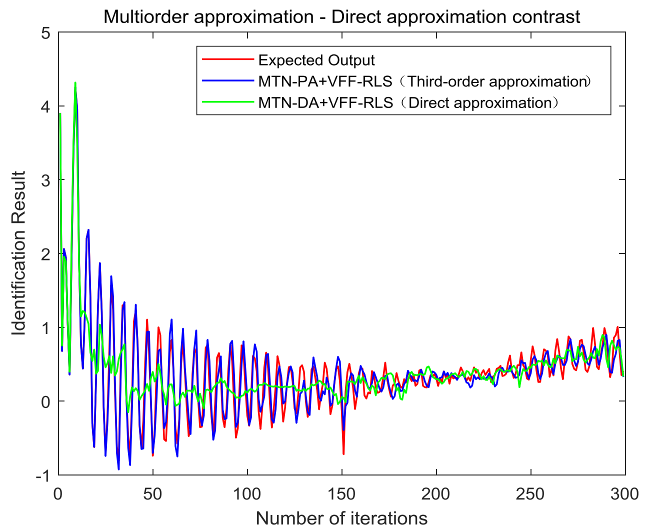

Figure 12. *Comparison of multilevel and direct approximation (y_1).*

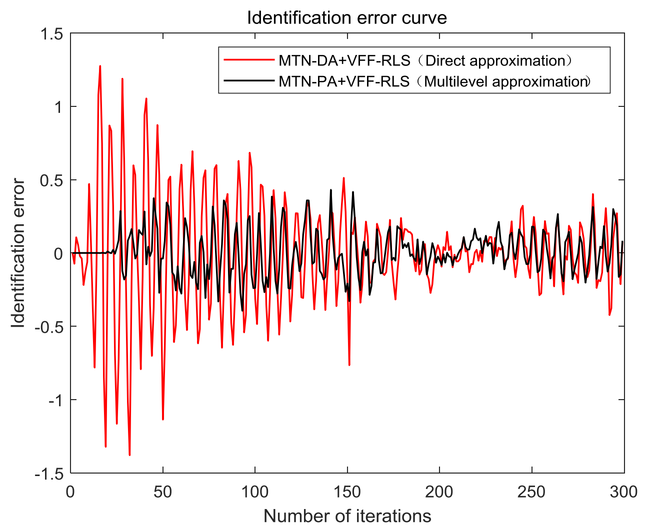

Figure 13. *Identification error curve (y_1).*

Table 1. *Comparison of MSE Calculation Results for Different Identification Methods (y_1).*

Identification Method	MSE	Performance Enhancement
Direct approximation	0.0764	-
first-order approximation	0.1321	-
Second-order approximation	0.0412	46.07%
Third-order approximation	0.0131	82.85%

Table 2. *Comparison of MSE Calculation Results for Different Identification Methods (y_2).*

Identification Method	MSE	Performance Enhancement
Direct approximation	0.0802	-
first-order approximation	0.1196	-
Second-order approximation	0.0337	57.98%
Third-order approximation	0.0090	88.78%

6. Conclusions

The study of multi-input multi-output nonlinear Gaussian time-varying systems is of great significance for system modelling and control design. In this paper, a recognition scheme based on multi-dimensional Taylor network with step-by-step approximation is proposed, and in order to improve the recognition accuracy, the least squares method based on with forgetting factor is used as the learning algorithm of the recognition model, which is able to dynamically allocate the weights in the task to improve the recognition accuracy. The experimental results show that this scheme is feasible and effective, which not only can effectively simplify the system model and improve the recognition accuracy, but also can cope with the challenges of real-time applications and high-noise environments, and provide technical support for the wide application and development of nonlinear system recognition technology.

Abbreviations

MTN	Multi-Dimensional Taylor Network
RLS	Recursive Least Squares
BP	Back Propagation
L-M	Levenberg-Marquardt
VFF-RLS	Variable Forgetting Factor Recursive Least Squares
FCRNN	Fully Connected Recurrent Neural Network
FFNN	Feedforward Neural Networks
EHH	Elman Neural Network
DAG	Directed Acyclic Graph
AHH	Adaptive Hinge Hyperplane
MTN-DA	Direct Approach of Multidimensional Taylor Network
MTN-PA	Multilevel Approximation of Multidimensional Taylor Networks
MIMO	Multiple Inputs and Multiple Outputs
MSE	Mean Square Error

Acknowledgments

This section serves to recognize contributions that do not meet authorship criteria, including technical assistance, donations, or organizational aid. Individuals or organizations should be acknowledged with their full names. The acknowledgments should be placed after the conclusion and before the references section in the manuscript.

Author Contributions

Jiefei Li: Conceptualization, Data curation, Formal Analysis, Investigation, Methodology, Software, Validation, Visualization, Writing – original draft, Writing – review & editing

Shaolin Hu: Funding acquisition, Project administration, Resources, Supervision

Funding

This work is supported by Nature Science Foundation of

Guangdong Province (2023A1515012341).

Data Availability Statement

No data was used.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] G. Xie, D. Zhang and F. Qian, "The identification of stochastic time-varying parameters for nonlinear system," 2016 35th Chinese Control Conference (CCC), Chengdu, China, 2016, pp. 2137-2141, https://doi.org/10.1109/ChiCC.2016.7553683

[2] Zhang, Chao, and Hong-Sen Yan. "Nonlinear stochastic time-varying system identification based on multi-dimensional Taylor network with optimal structure." *2016 16th International Conference on Control, Automation and Systems (ICCAS)*. IEEE, 2016, https://doi.org/10.1109/ICCAS.2016.7832296

[3] Shobana, R., Bhavnesh Jaint, and Rajesh Kumar. "Design of a novel robust recurrent neural network for the identification of complex nonlinear dynamical systems." *Soft Computing* 28.3 (2024): 2737-2751. https://doi.org/10.1007/s00500-023-09187-5

[4] Kumar, Rajesh. "Memory recurrent Elman neural network-based identification of time-delayed nonlinear dynamical system." *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 53.2(2022): 753-762. https://doi.org/10.1109/TSMC.2022.3186610

[5] Xu, Jun, et al. "Efficient hinging hyperplanes neural network and its application in nonlinear system identification." *Automatica* 116(2020): 108906. https://doi.org/10.1016/j.automatica.2020.108906

[6] Batselier, Kim, Zhongming Chen, and Ngai Wong. "A Tensor Network Kalman filter with an application in recursive MIMO Volterra system identification." Automatica 84 (2017): 17-25. https://doi.org/10.1016/j.automatica.2017.06.019

[7] Gray, W. Steven, G. S. Venkatesh, and Luis A. Duffaut Espinosa. "Nonlinear system identification for multivariable control via discrete-time Chen–Fliess series." *Automatica* 119 (2020): 109085. https://doi.org/10.1016/j.automatica.2020.109085

[8] Sugiyama, Koichi, and Tomohiro Hachino. "On-line identification of continuous-time nonlinear systems using Gaussian process models." *2018 Joint 10th International Conference on Soft Computing and Intelligent Systems (SCIS) and 19th International Symposium on Advanced Intelligent Systems (ISIS)*. IEEE, 2018. https://doi.org/10.1109/SCIS-ISIS.2018.00191

[9] Farzanegan, Behzad, et al. "An identification approach for unstable nonlinear systems with nonlinear parameterization using MRAC." *2020 28th Iranian Conference on Electrical Engineering (ICEE)*. IEEE, 2020. https://doi.org/10.1109/ICEE50131.2020.9260758

[10] Chen, Guang-Yong, et al. "Basis function matrix-based flexible coefficient autoregressive models: A framework for time series and nonlinear system modeling." *IEEE transactions on cybernetics* 51.2(2019): 614-623. https://doi.org/10.1109/TCYB.2019.2900469

[11] Yan, Hong-Sen, and Zheng-Yi Duan. "Tube-based model predictive control using multidimensional Taylor network for nonlinear time-delay systems." *IEEE Transactions on Automatic Control* 66.5(2020): 2099-2114. https://doi.org/10.1109/TAC.2020.3005674

[12] Sun, Qi-Ming, et al. "Data-driven nonlinear near-optimal regulation based on multi-dimensional Taylor network dynamic programming." *IEEE Access* 8 (2020): 36476-36484. https://doi.org/10.1109/ACCESS.2020.2975391

[13] Zhang, Jiao-Jun, and Hong-Sen Yan. "MTN optimal control of MIMO non-affine nonlinear time-varying discrete systems for tracking only by output feedback." *Journal of the Franklin Institute* 356.8 (2019): 4304-4334. https://doi.org/10.1016/j.jfranklin.2019.03.008

[14] Zhu, Shan-Liang, Ming-n Wang, and Yu-Qun Han. "Adaptive multi-dimensional Taylor network control for nonlinear stochastic systems with time-delay." *Proceedings of the Institution of Mechanical Engineers, Part I: Journal of Systems and Control Engineering* 236.3 (2022): 579-591. https://doi.org/10.1177/09596518211040285

[15] Yan, Hong-Sen, and Guo-Biao Wang. "Adaptive tracking control for stochastic nonlinear systems with time-varying delays using multi-dimensional Taylor network." *ISA transactions* 132 (2023): 246-257. https://doi.org/10.1016/j.isatra.2022.06.004

[16] Yu, Chien-Cheng, and Bin-Da Liu. "A backpropagation algorithm with adaptive learning rate and momentum coefficient." *Proceedings of the 2002 International Joint Conference on Neural Networks. IJCNN'02 (Cat. No. 02CH37290)*. Vol. 2. IEEE, 2002. https://doi.org/10.1109/IJCNN.2002.1007668

[17] Li, Jianshu, Yuan Zheng, and Zhi Lin. "Recursive identification of time-varying systems: Self-tuning and matrix RLS algorithms." *Systems & Control Letters* 66 (2014): 104-110. https://doi.org/10.1016/j.sysconle.2014.01.004

[18] Qin, Boya, et al. "Low-complexity variable forgetting factor constant modulus RLS-based algorithm for blind adaptive beamforming." *2013 Asilomar Conference on Signals, Systems and Computers*. IEEE, 2013. https://doi.org/10.1109/ACSSC.2013.6810253

[19] Zhang, Chao, and Hong-Sen Yan. "Identification and adaptive multi-dimensional Taylor network control of single-input single-output non-linear uncertain time-varying systems with noise disturbances." *IET Control Theory & Applications* 13.6 (2019): 841-853. https://doi.org/10.1049/iet-cta.2018.5542

[20] Shrivastava, Prashant, et al. "Lithium-ion battery model parameter identification using modified adaptive forgetting factor-based recursive least square algorithm." *2021 IEEE 12th Energy Conversion Congress & Exposition-Asia (ECCE-Asia)*. IEEE, 2021. https://doi.org/10.1109/ECCE-Asia49820.2021.9479079

[21] Lin, Yi, H. S. Yan, and Bo Zhou. "Nonlinear time series prediction method based on multi-dimensional Taylor network and its applications." *Control and Decision* 29.5 (2014): 795-801.

Biography

Jiefei Li is a graduate student at Jilin College of Chemical Engineering. He is a student of Prof Hu Shaolin and his main research interests are control science and engineering.

Shaolin Hu is a Master's supervisor of Guangdong University of Petrochemical Technology. He received his Bachelor's degree from Anhui Normal University in 1986, Master's degree from Xi'an Jiaotong University in 1989, PhD degree from Xi'an Jiaotong University in 2000, post-doctoral fellowships from University of Science and Technology of China and Nanjing University of Science and Technology, and a visiting professor of the Royal Institute of Technology of Sweden under the State Sending Programmes. He is a recipient of the Swedish Government Scholarship for International Senior Visiting Scholars, the China Aerospace Foundation Award and the title of Outstanding Returned Scholar of the Shaanxi Provincial Government, and his main research interests are in satellite-based navigation, process monitoring and control, fault diagnosis and safety of complex systems.

Research Field

Jiefei Li: Control Science and Engineering, process control.

Shaolin Hu: Artificial Intelligence Technology Applications, Control Science and Engineering, Intelligent Sensing and System Security, Fault diagnosis and safety of complex system.