

# A Hybrid VAR-LSTM-GARCH Model for Multivariate Volatility Forecasting

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**Abstract:** Financial markets show persistent volatility, creating barriers to achieving exact financial predictions. The forecasting of multivariate financial data requires forecasting models like the Vector Autoregressive (VAR) model for modeling linear dependencies, the Long Short-Term Memory (LSTM) model for modeling non-linear patterns, and the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model that is capable of modeling volatility clustering. Each of these models fails to handle complete data complexity on its own, as they specialize in unique properties of the data. Recent studies have been carried out that enhance forecasting accuracy by combining two models. The first case is the VAR-GARCH model, which can model linear and volatility clustering aspects but fails to model non-linear dependencies. Another case is the LSTM-GARCH model that can explain non-linear dependencies and volatility patterns, but fails to explain linear dependencies. A third instance is the VAR-LSTM model that can explain the linear and volatility aspects, but fails to model the non-linear patterns. However, there is a need to have a model that can combine the three models to explain the linear, non-linear, and volatility aspects in financial time series data collectively. This research fills this gap by combining VAR, LSTM, and GARCH into a VAR-LSTM-GARCH hybrid model, which provides improved forecasting. This study uses historical five-year daily data for VIX, US Dollar Index, and S&P 500 E-mini futures obtained from Yahoo Finance. The model-building process involves constructing a VAR (9) model selected using AIC criteria to reveal linear dependencies. The residuals from the VAR are used to train an LSTM model to capture nonlinear trends. The residuals of the LSTM are then used to fit an M-GARCH (1, 1) model, which generates volatility cluster estimates. The VAR-LSTM-GARCH hybrid model demonstrates superior performance with substantial improvements across all evaluation metrics compared to individual models, showing consistently lower prediction errors and enhanced forecasting accuracy. The progressive three-stage modeling approach demonstrates that each component contributes incrementally to forecasting performance, with the incorporation of volatility modeling through GARCH being particularly effective in enhancing predictive accuracy. The research suggests using this hybrid model for volatility prediction on multiple portfolios and emphasizes future development of real-time diagnostic processes. The new approach delivers an advanced instrument that helps financial analysts work efficiently by effectively capturing the complex interdependencies in multivariate financial time series data.

**Keywords:** Hybrid Model, Volatility, Volatility Clustering, Machine Learning, Forecasting, Vector Autoregressive Model, Long Short-Term Memory Model, Generalized Autoregressive Conditional Heteroscedastic Model

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## 1. Introduction

Financial econometrics and risk management depend heavily on volatility forecasting because it enables investors and analysts to both regulate risks and predict price modifications. The main input for pricing and trading strategy

design, along with hedging mechanisms, is volatility, which is calculated through the standard deviation measurement of asset returns [1]. The modeling of accurate forecasting tools becomes essential because economic conditions, combined with political events and investor sentiment, control price

variations throughout financial instruments [2]. The traditional financial time series modeling practice includes extensively using both GARCH and VAR models. Both GARCH models excel at displaying volatility clustering and VAR models excel at detecting linear relationships among multiple factors in financial systems [3, 4].

The limitation of GARCH models stems from their inability to determine complete volatility relationships with future occurrences, while VAR encounters issues with nonlinear and extended dependency patterns [6]. Researchers have initiated studies into machine-learning hybrid models for solving these problems. Financial time series benefit from the dedicated capabilities of LSTM networks to process sequential information and dependence over time as per [5, 7]. LSTM outperforms traditional models like ARIMA and GARCH, especially under non-linear conditions [8].

Recent research has considered hybrid forecasting systems built upon the strengths of LSTM and VAR, and GARCH architecture. The LSTM-VAR model detects linear together with non-linear patterns, yet it does not handle volatility estimation. LSTM-GARCH provides coverage of non-linear dynamics together with volatility clustering, yet it fails to detect linear dependence patterns. VAR-GARCH handles linear and volatility components but not long-term dependencies.

There is a need to have a model that can be able to model the three aspects- linear, non-linear, and volatility of financial time series. The considered VAR-LSTM-GARCH hybrid model combines these three methods to analyze financial data while handling all forms of time series fluctuations.

## 2. Problem Statement

Finance-related decision-making heavily depends on volatility forecasting, but tracking this volatility proves difficult because financial time series contain linear patterns and nonlinear behaviors and show volatility patterns that cluster together across time. VAR and GARCH models manage linear relationships and volatility clusters independently, but LSTM networks effectively detect nonlinear patterns in data. These individual forecasting systems do not capture every element simultaneously. Each of the existing hybrid forecasting systems, including VAR-LSTM and LSTM-GARCH, and VAR-GARCH, demonstrates functional gaps when analyzing multiple interrelated variables. This research establishes a VAR-LSTM-GARCH operational model that unites all three algorithms to boost the accuracy levels of volatility predictions in multivariate financial time series.

## 3. Justification

Financial risk evaluation and portfolio administration, together with regulatory requirements, heavily rely on accurate volatility prediction of multivariate time series in financial data. Financial data primarily displays nonlinear patterns,

which makes existing models such as VAR and GARCH less effective because of their simplicity. Machine learning techniques like LSTM can explain linear dependencies but fail to model complex properties like volatility clustering, and are hard to interpret. This research presents an inverted hybrid process through which LSTM models the non-linear features before setting up VAR to perform linear analysis. The joint application seeks to boost forecasting precision and reliability. The new hybrid model addresses research needs by giving practical applications for financial analysts and policymakers.

## 4. Literature Review

The fundamental nature of market volatility allows financial instruments to measure asset price fluctuation risks. The standard deviation of return measurements functions as a primary variable in managing risk and asset placement activities while serving derivative pricing requirements [9, 10]. Historical volatility provides data regarding previous market patterns, but implied volatility generates information through Black-Scholes option pricing models that represent market expectations [11]. The Volatility Index (VIX) serves as an instrument to measure both investor sentiment along market participation projections [12].

Portfolio optimization, as well as risk assessment and hedging, operate on accurate volatility forecasts [13, 14]. Forecasting accuracy enables organizations to take preemptive actions concerning investments and risk management, specifically during times of market instability [15]. Alongside asset price and exchange rate relations, multivariate forecasting analysis adds value through its ability to understand connections between these variables and interest rate movements [16].

The ARCH and GARCH models successfully detect volatility clusters that create consecutive periods of heightened volatility; however, these models cannot model extended time-dependent relationships or nonlinear patterns [17, 18]. Time-varying volatility modeling remains one of the key factors that makes GARCH models popular choices alongside their simple structure [19, 20].

Machine learning technology has delivered popularity to the use of Long Short-Term Memory (LSTM) networks in recent times. LSTM networks possess the ability to identify long-duration relationships in financial data, thereby proving effective for forecasting purposes [21, 22]. Vector Autoregressive (VAR) models reveal linear relations among several time series variables while doing so [23, 24].

Recent trends in forecasting use combined forecasting models that integrate LSTM with both VAR and GARCH to benefit from their separate capabilities. LSTM-VAR hybrids combine LSTM nonlinear memory operations with VAR linear multivariate relations to enhance market condition forecasting accuracy [25, 26].

The combination of LSTM's nonlinear temporal dependency modeling capability with GARCH's volatility clustering features is achieved by using LSTM-GARCH models according to [27, 28]. The combination of VAR-

GARCH hybrids generates reliable forecasts through their ability to establish conditional linkages and dynamic volatility patterns between different financial instruments [29].

There is an existing research gap in combining three models to form a hybrid VAR-LSTM-GARCH model. This model will consider modeling linear trends, non-linear dependencies, and volatility clustering in a single model. This formulation is expected to help in improving the accuracy of the forecast.

## 5. Methodology

### 5.1. Data and Scope

This study from three financial assets: VIX, US Dollar Index, and S&P 500 E-mini futures. The data points

considered are daily closing prices of the three assets for the last five years, downloaded from Yahoo Finance as time series data sets with corresponding dates of observations.

### 5.2. Vector Autoregressive (VAR) Model

The Vector Autoregressive (VAR) model is a flexible framework for modeling multivariate time series, capturing the interdependencies between multiple variables. In this study, the VAR model for three variables is considered:  $X$ ,  $Y$ , and  $Z$ .

In a VAR( $p$ ) model, each variable is regressed on its past values as well as the past values of the other variables in the system. For three variables  $X_t$ ,  $Y_t$ , and  $Z_t$  at time  $t$ , the VAR model with lag order  $p$  can be written as follows:

$$X_t = c_1 + \sum_{i=1}^p a_{11}^{(i)} X_{t-i} + \sum_{i=1}^p a_{12}^{(i)} Y_{t-i} + \sum_{i=1}^p a_{13}^{(i)} Z_{t-i} + e_{X_t}, \quad (1)$$

$$Y_t = c_2 + \sum_{i=1}^p a_{21}^{(i)} X_{t-i} + \sum_{i=1}^p a_{22}^{(i)} Y_{t-i} + \sum_{i=1}^p a_{23}^{(i)} Z_{t-i} + e_{Y_t}, \quad (2)$$

$$Z_t = c_3 + \sum_{i=1}^p a_{31}^{(i)} X_{t-i} + \sum_{i=1}^p a_{32}^{(i)} Y_{t-i} + \sum_{i=1}^p a_{33}^{(i)} Z_{t-i} + e_{Z_t}. \quad (3)$$

#### Key Components of the VAR( $p$ ) Model

1. **Lag Terms:** Each equation includes lagged values of all three variables ( $X_t$ ,  $Y_t$ , and  $Z_t$ ) up to lag  $p$ .
2. **Coefficients:**  $a_{11}^{(i)}, a_{12}^{(i)}, a_{13}^{(i)}$  are the coefficients that describe how the past values of  $X_t$ ,  $Y_t$ , and  $Z_t$  influence  $X_t$ . Similarly,  $a_{21}^{(i)}, a_{22}^{(i)}, a_{23}^{(i)}$  describe the impact on  $Y_t$ , and  $a_{31}^{(i)}, a_{32}^{(i)}, a_{33}^{(i)}$  represent the impact on  $Z_t$ .
3. **Intercepts:**  $c_1, c_2$ , and  $c_3$  are the intercept terms for the equations of  $X_t$ ,  $Y_t$ , and  $Z_t$ , respectively.
4. **Error Terms:**  $e_{X_t}, e_{Y_t}$ , and  $e_{Z_t}$  are white noise error terms for the variables  $X$ ,  $Y$ , and  $Z$  at time  $t$ .

#### Matrix Representation

In matrix notation, the VAR( $p$ ) model can be expressed as:

$$\mathbf{X}_t = \mathbf{\Theta} + \sum_{i=1}^p \mathbf{\Phi}_i \mathbf{X}_{t-i} + \mathbf{\Sigma}_t, \quad (4)$$

where:

1.  $\mathbf{X}_t = \begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix}$ : Vector of variables.
2.  $\mathbf{\Theta} = \begin{bmatrix} \alpha_{0,1} \\ \alpha_{0,2} \\ \alpha_{0,3} \end{bmatrix}$ : Intercept terms.
3.  $\mathbf{\Phi}_i = \begin{bmatrix} \alpha_{1,1}^{(X)} & \alpha_{1,2}^{(X)} & \alpha_{1,3}^{(X)} \\ \alpha_{2,1}^{(Y)} & \alpha_{2,2}^{(Y)} & \alpha_{2,3}^{(Y)} \\ \alpha_{3,1}^{(Z)} & \alpha_{3,2}^{(Z)} & \alpha_{3,3}^{(Z)} \end{bmatrix}$ : Coefficient matrices for

lag  $i$ .

$$4. \mathbf{\Sigma}_t = \begin{bmatrix} e_{X_t} \\ e_{Y_t} \\ e_{Z_t} \end{bmatrix} : \text{Error terms.}$$

### 5.3. Long Short-Term Memory (LSTM)

The multivariate Long Short-Term Memory (LSTM) network architecture is particularly suited for modeling time series data involving multiple correlated variables. Formally, the input at time step  $t$  is:

$$\mathbf{X}_t = \begin{bmatrix} x_{1,t} \\ x_{2,t} \\ \vdots \\ x_{m,t} \end{bmatrix}, \quad (5)$$

where  $\mathbf{X}_t$  represents the multivariate input vector at time  $t$ .

The LSTM architecture consists of the following components at each time step:

#### Forget Gate

Decides what information from the previous cell state  $\mathbf{C}_{t-1}$  should be forgotten:

$$\mathbf{f}_t = \sigma(\mathbf{W}_f \mathbf{X}_t + \mathbf{U}_f \mathbf{h}_{t-1} + \mathbf{b}_f), \quad (6)$$

where:

1.  $\mathbf{f}_t$ : Forget gate vector.
2.  $\mathbf{W}_f$ : Input weight matrix.

3.  $\mathbf{U}_f$ : Hidden state weight matrix.
4.  $\mathbf{h}_{t-1}$ : Previous hidden state.
5.  $\sigma$ : Sigmoid activation function.
6.  $\mathbf{b}_f$ : Bias vector.

#### Input Gate

Decides what new information should be added to the cell state:

$$\mathbf{i}_t = \sigma(\mathbf{W}_i \mathbf{X}_t + \mathbf{U}_i \mathbf{h}_{t-1} + \mathbf{b}_i), \quad (7)$$

where  $\mathbf{i}_t$  is the input gate vector.

#### Candidate Cell State

A new candidate cell state is computed based on the current input and the previous hidden state:

$$\tilde{\mathbf{C}}_t = \tanh(\mathbf{W}_c \mathbf{X}_t + \mathbf{U}_c \mathbf{h}_{t-1} + \mathbf{b}_c), \quad (8)$$

where  $\tilde{\mathbf{C}}_t$  is the candidate cell state.

#### Update Cell State

The cell state  $\mathbf{C}_t$  is updated by combining the previous cell state  $\mathbf{C}_{t-1}$ , filtered by the forget gate, with the new candidate cell state, weighted by the input gate:

$$\mathbf{C}_t = \mathbf{f}_t \odot \mathbf{C}_{t-1} + \mathbf{i}_t \odot \tilde{\mathbf{C}}_t, \quad (9)$$

where  $\odot$  represents element-wise multiplication.

#### Output Gate

Determines the hidden state ( $\mathbf{h}_t$ ), which is based on the updated cell state  $\mathbf{C}_t$  and the output gate's control:

$$\mathbf{o}_t = \sigma(\mathbf{W}_o \mathbf{X}_t + \mathbf{U}_o \mathbf{h}_{t-1} + \mathbf{b}_o), \quad (10)$$

$$\mathbf{h}_t = \mathbf{o}_t \odot \tanh(\mathbf{C}_t), \quad (11)$$

where:

1.  $\mathbf{o}_t$ : Output gate vector.
2.  $\mathbf{h}_t$ : Hidden state for the current time step  $t$ .

### 5.4. Multivariate Generalized Autoregressive Conditional Heteroscedastic (M-GARCH) Model

The Multivariate GARCH ( $p, q$ ) model extends the idea of time-varying volatility by allowing the current conditional covariance matrix to depend on up to  $p$  past conditional covariances and  $q$  past squared error terms.

The model can be expressed as:

$$\mathbf{H}_t = \mathbf{C} + \sum_{i=1}^p \mathbf{A}_i \varepsilon_{t-i} \varepsilon_{t-i}^\top + \sum_{j=1}^q \mathbf{B}_j \mathbf{H}_{t-j}, \quad (12)$$

where:

1.  $\mathbf{X}_t = \begin{bmatrix} X_t \\ Y_t \\ Z_t \end{bmatrix}$ : Vector of the three variables  $X_t$ ,  $Y_t$ , and  $Z_t$ .
2.  $\mu_t$ : Conditional mean vector.
3.  $\varepsilon_t = \mathbf{X}_t - \mu_t$ : Vector of errors, where  $\varepsilon_t \sim N(0, \mathbf{H}_t)$ .
4.  $\mathbf{H}_t$ : Conditional covariance matrix of  $\varepsilon_t$ .
5.  $\mathbf{C}$ : Constant matrix.
6.  $\mathbf{A}_i$ : Coefficient matrices for past squared error terms.
7.  $\mathbf{B}_j$ : Coefficient matrices for past conditional covariance

matrices.

### 5.5. Forecasting

Let:

1.  $\mathbf{y}_t \in \mathbb{R}^n$ : multivariate time series at time  $t$
2.  $\hat{\mathbf{y}}_t^{\text{LSTM}}$ : forecast from LSTM component (nonlinear dependencies)
3.  $\hat{\mathbf{y}}_t^{\text{VAR}}$ : forecast from VAR component (linear dependencies)
4.  $\hat{\sigma}_t^{\text{GARCH}}$ : forecast from GARCH component (volatility)
5.  $\hat{\mathbf{y}}_t$ : final hybrid forecast

The hybrid forecast is given by:

$$\hat{\mathbf{y}}_t = \alpha \cdot \hat{\mathbf{y}}_t^{\text{LSTM}} + \beta \cdot \hat{\mathbf{y}}_t^{\text{VAR}} + \gamma \cdot \hat{\sigma}_t^{\text{GARCH}} \quad (13)$$

subject to:

$$\alpha + \beta + \gamma = 1, \quad \alpha, \beta, \gamma \in [0, 1] \quad (14)$$

#### Component-Wise Descriptions

##### 1. VAR Component (Linear Interdependence):

$$\hat{\mathbf{y}}_t^{\text{VAR}} = \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_2 \mathbf{y}_{t-2} + \dots + \mathbf{A}_p \mathbf{y}_{t-p} \quad (15)$$

##### 2. LSTM Component (Nonlinear Dynamics):

$$\hat{\mathbf{y}}_t^{\text{LSTM}} = f_{\text{LSTM}}(\mathbf{y}_{t-1}, \mathbf{y}_{t-2}, \dots, \mathbf{y}_{t-k}) \quad (16)$$

##### 3. GARCH Component (Conditional Volatility):

For each series  $y_{i,t}$ , the GARCH(1,1) model is:

$$\hat{\sigma}_{i,t}^2 = \omega_i + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i \hat{\sigma}_{i,t-1}^2 \quad (17)$$

### 5.6. Model Evaluation and Validation

Metrics such as Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE) will be used to evaluate the performance of the models in the multivariate context.

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^n (\mathbf{f}_i - \hat{\mathbf{f}}_i)' (\mathbf{f}_i - \hat{\mathbf{f}}_i) \quad (18)$$

$$\text{RMSE} = \sqrt{\text{MSE}} \quad (19)$$

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |\mathbf{f}_i - \hat{\mathbf{f}}_i| \quad (20)$$

## 6. Results and Discussions

### 6.1. Exploratory Data Analysis

This involves establishing time series data properties such as long-term trends, normality, and other descriptive statistics. Since the relation between the variables is paramount, a correlation analysis through a heat map is also carried out. The ACF and PACF are also evaluated as shown in Figure 1.

## Exploratory Data Analysis

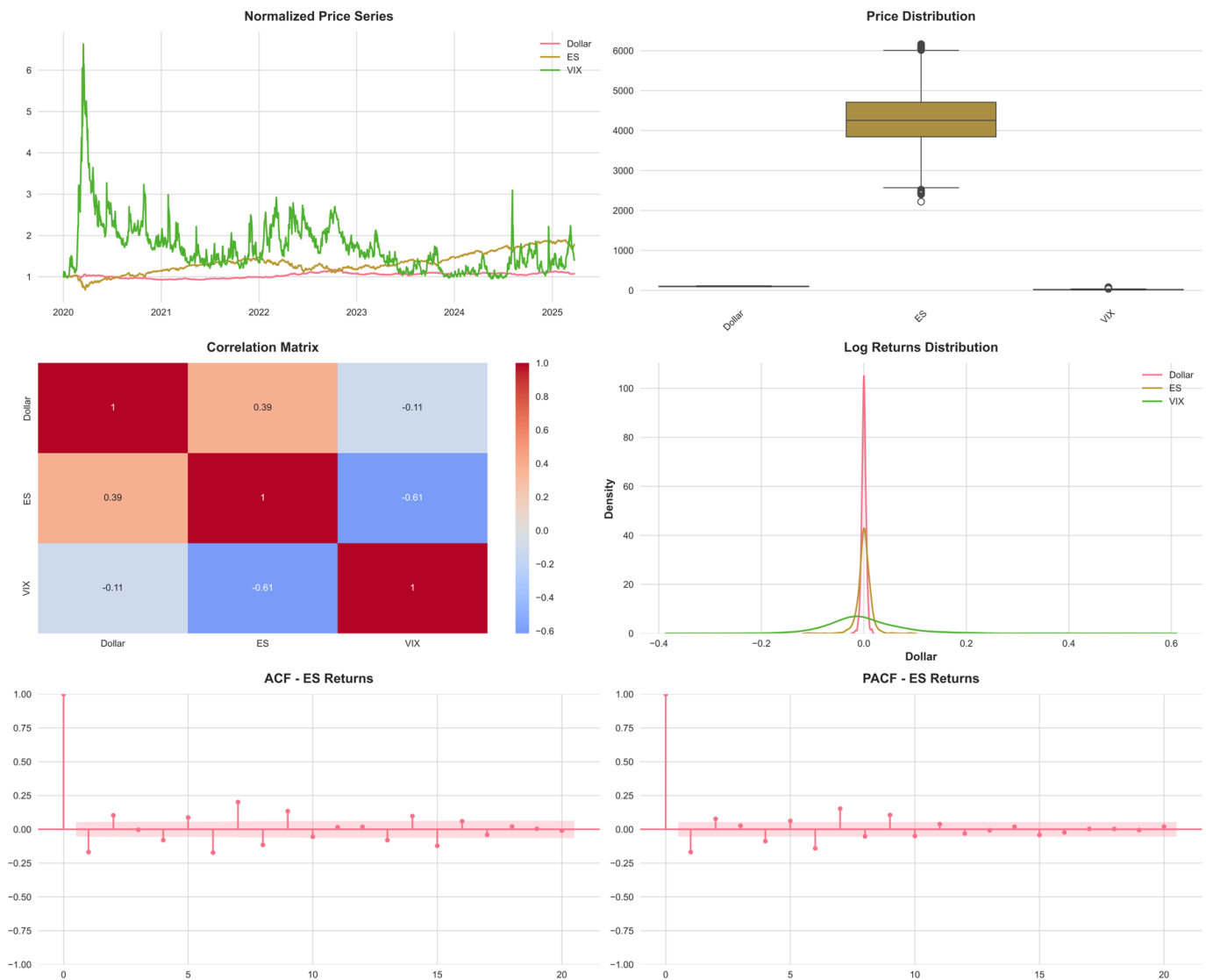


Figure 1. Exploratory Data Analysis.

This Exploratory Data Analysis (EDA) presents the analysis of three financial assets concerning the Dollar along with ES (S&P 500 E-mini Futures) and VIX (Volatility Index). The data visualizations study patterns between the assets through price charts and statistics regarding both price relations and return frequency, and autocorrelation analysis.

Data in the Normalized Price Series graph demonstrates that VIX displays high volatility with numerous spikes, but ES and Dollar maintain smoother patterns. The ES price data contains substantially higher price values that stand apart from VIX prices, whose extreme data points show market sensitivity [1].

The Correlation Matrix displays essential relations among data variables. The price movements of ES and the Dollar share limited alignment since their correlation is at 0.39. A strong negative correlation (value -0.61) exists between ES and VIX, so ES falls when VIX rises. Market volatility shows a minor influence on currency value changes based on the

negative correlation of -0.11 between VIX and the Dollar. The Log Returns Distribution illustrates that VIX stands out as the most volatile asset, while other assets return to zero, but VIX achieves the highest volatility levels. Extremely high and low returns appear at a greater frequency than typical distributions indicate due to their distinctive peak shape and heavy outer sections, which helps risk management activities.

The Autocorrelation (ACF) and Partial Autocorrelation (PACF) charts demonstrate that return data has weak immediate linkages, so previous price data provides insufficient information for extracting future return predictions. The financial market operates efficiently because returns behave primarily as random elements in this system.

A review finds that VIX acts as market downturn protection while ES and Dollar exhibit reasonable correlation relationships between them, and returns exhibit heavy-tailed distributions along with small autocorrelation effects. The

research results create actionable knowledge for developing strategies that handle portfolio risk and minimize losses [10].

## 6.2. Stationarity Test

To fit a VAR model to the multivariate dataset, the data needs to be stationary and normally distributed. This requires the transformation of the raw prices into log returns to make them stationary, as shown in Table 1.

The Augmented Dickey-Fuller (ADF) test examines financial time series stationary properties to determine if there exists a unit root. A statistical evaluation of series stationarity produces a positive result when the p-value reaches below a standard acceptability level at 0.05.

The Dollar (ADF = -1.25,  $p = 0.65$ ) and ES (ADF = -0.60,  $p = 0.87$ ) show non-stationarity in their prices because their test statistics exceed critical values and their high p-values indicate

their non-stationarity. Their price patterns show characteristics of random walk processes since their measurement levels neither return to an average pattern nor require differencing for modeling purposes. The VIX series exhibits stationary properties because its ADF test output value of -4.30 reaches a p-value of 0.0004.

The data shows that all three assets prove stationary under log return calculation with highly significant p-values approaching zero. The test results indicate that Dollar (ADF = -16.13), ES (ADF = -10.06), and VIX (ADF = -20.23) prove strongly stationary since their return patterns stay confined near a fixed mean without showing directional trends. The financial industry tends to observe financial returns that behave stationary while reverting towards the mean. Time series analysis requires log returns since these data points exhibit stationary measures, but prices require transformation through differencing techniques before modeling [14].

Table 1. ADF Test for Raw Prices and Log Returns.

Series Type	Asset	ADF Statistic	p-value	1% CV	5% CV	10% CV	Stationarity
Price Series	Dollar	-1.2521	0.6494	-3.654	-2.958	-2.617	Non-Stationary
	ES	-0.6923	0.8491	-3.654	-2.958	-2.617	Non-Stationary
	VIX	-4.3910	0.0001	-3.654	-2.958	-2.617	Stationary
Log Returns	Dollar	-16.1260	0.0000	-3.654	-2.958	-2.617	Stationary
	ES	-10.0123	0.0000	-3.654	-2.958	-2.617	Stationary
	VIX	-20.7299	0.0000	-3.654	-2.958	-2.617	Stationary

## 6.3. Developing the Model

### 6.3.1. Fitting of VAR Model

A VAR (p) model was fitted on the scaled log-returns, and AIC was used to select the best model. The selected model was a VAR (9) with an AIC of -15.4804. A graph of the errors from the VAR (9) model is shown in Figure 2 below:

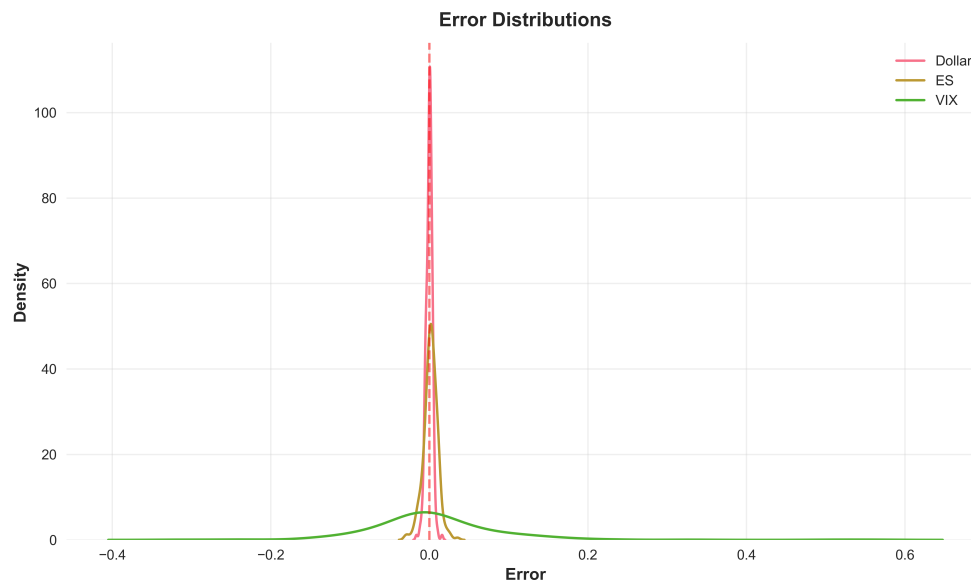




Figure 2. Error Analysis for VAR (9) model.

The graph breaks down detailed error data regarding financial variable forecasts of the Dollar Index, ES Mini Futures, and VIX. The three sections contained within the plot include Error Distributions and Actual vs Error and Cumulative Forecasting Errors.

All variables demonstrate minimal errors according to the Error Distributions section because their density concentrations focus on zero. The Dollar demonstrates wider error dispersion than both ES and VIX, thus indicating its forecasts exhibit more variable performance.

All three variables display a clear connection between actual values and errors according to the Actual vs Error scatter plot because the data points follow a diagonal line. The reliability of the forecasts remains steady, while minor inaccuracies do occur.

The Dollar exhibits a major ascending trend with increasing cumulative forecasting errors based on the Cumulative

Forecasting Errors graph. Forecasting accuracy for ES and VIX demonstrates an almost stable behavior, although the Dollar forecast shows noticeable deviations from actual values.

This error analysis reveals essential information that helps determine the accuracy of predictions according to the displayed graph data. The accuracy of the VAR (9) model using metrics like  $MSE(ES)=0.09599$ ,  $MSE(VIX)=0.3980$ , and  $MSE(Dollar\ Index)=0.0902$ . These values are relatively small but notably large for VIX, implying that the model fails to capture the volatility clustering aspects in the data.

Table 2. Evaluation Metrics for VAR Model.

Asset	ES	VIX	Dollar Index
$MSE(VAR)$	0.09599	0.3980	0.0902
$RMSE(VAR)$	0.03285	0.19200	0.007022
$MAE(VAR)$	0.17719	0.487857	0.057125

The Dollar shows significant forecasting errors, which require attention to build stronger predictive systems, while ES and VIX deliver reliable forecast results. The analysis proves vital for financial decisions because it permits the improvement of forecasting models and strategy optimization for upcoming business ventures [23].

6.3.2. *Fitting LSTM on VAR residuals*

The residuals from the VAR were used to train an LSTM model by first dividing it into training and testing sets. The training process is as shown in Figure 3 below:

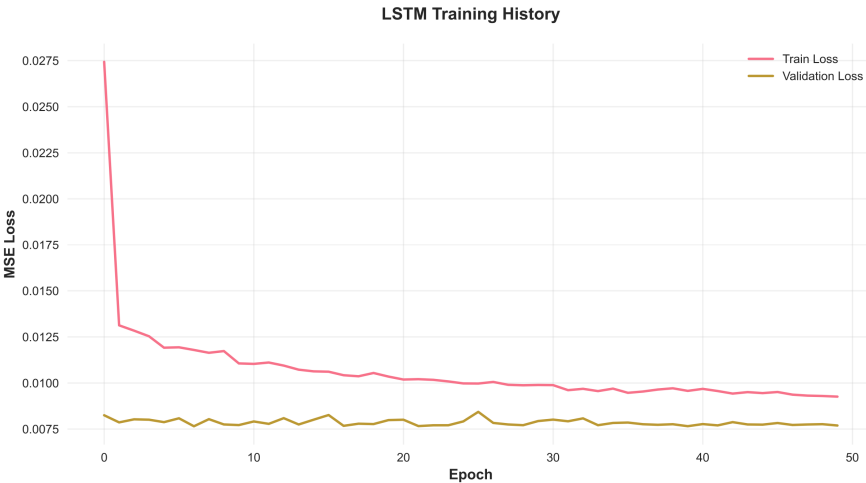


Figure 3. Training History for the LSTM.

The LSTM model trains its parameters through 50 epochs, as shown in the “LSTM Training History on VAR Residuals”, Figure 3. The model monitors its performance by using Mean Squared Error (MSE) Loss on training and validation datasets.

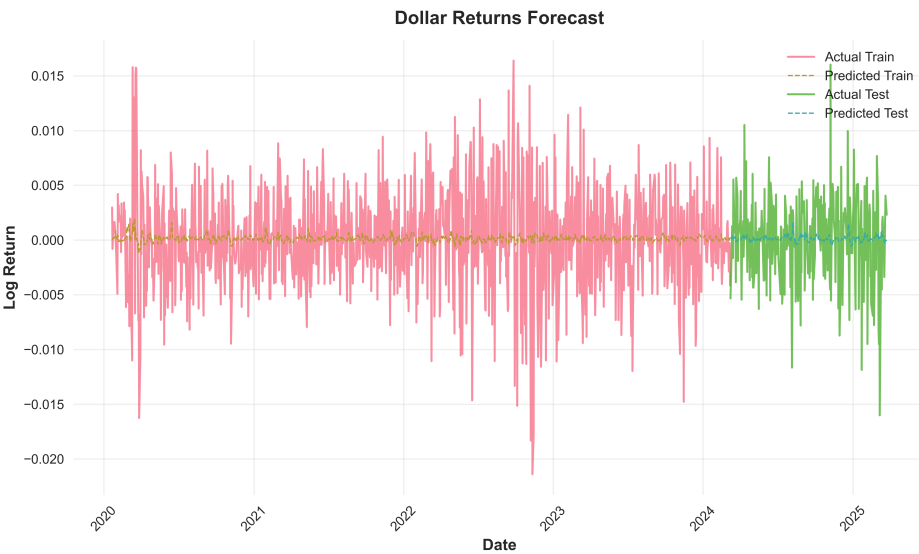
The training loss drops quickly from its initial point of 0.035 to reach a stabilized level at 0.010 while being plotted on the pink line. The model demonstrates effective learning through this trend because it reduces its error when exposed to additional iterations.

During the epochs the validation loss maintains a steady position at 0.010 as presented by the yellow line. The model demonstrates robustness for new data points because it successfully avoids overfitting following the major training

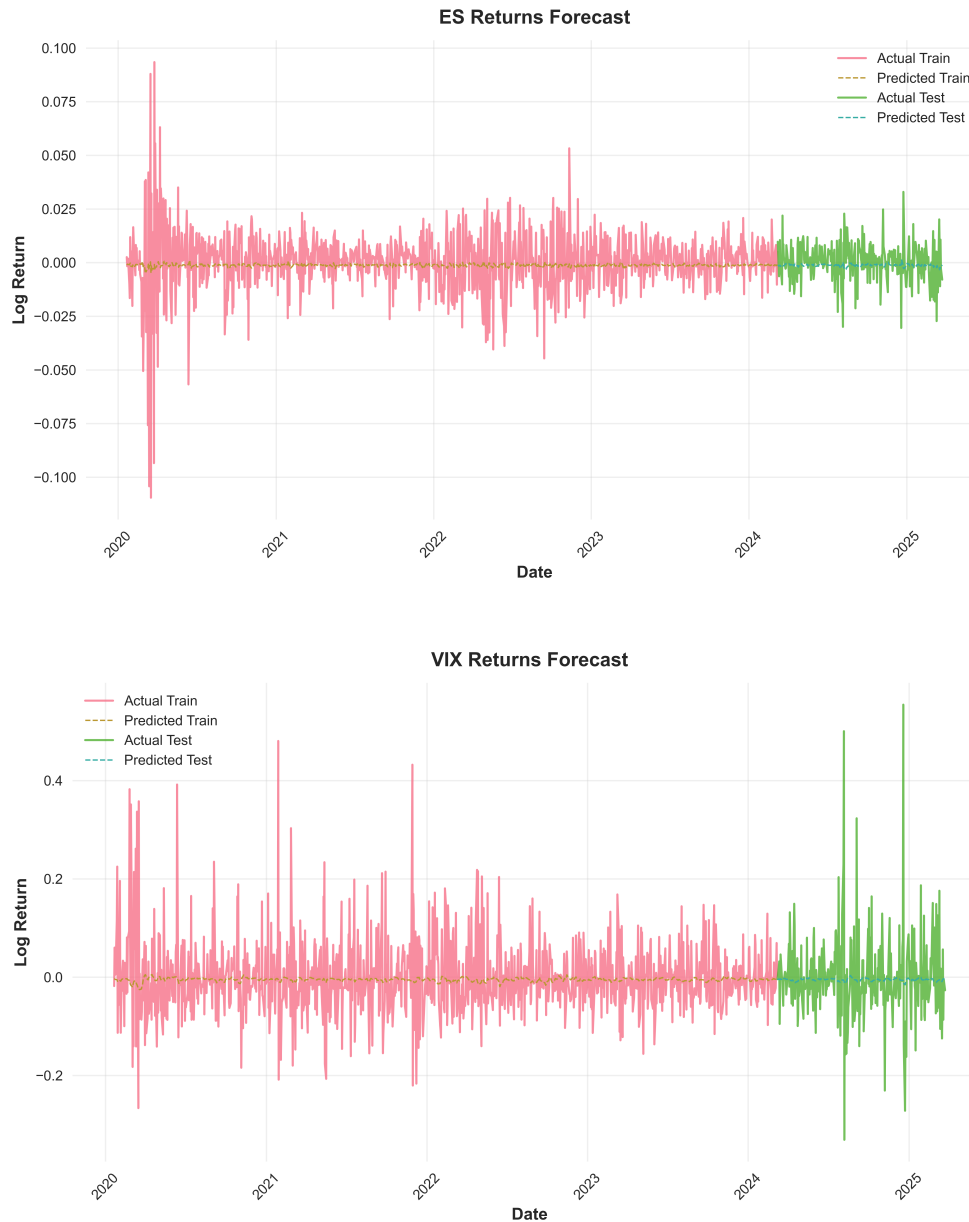
loss reduction [21].

The LSTM model displays an essential financial modeling capability because it maintains an optimal balance between knowledge acquisition and model generalization abilities. Training adaptability accelerates due to a steep drop in training loss, but validation loss stability indicates consistent performance across different sets of data. The hybrid LSTM-VAR-GARCH model project benefits from this training history because it demonstrates efficient management of VAR residuals. The obtained results guide the building of future layers in volatility prediction models.

In addition the actual versus forecasted values of from the LSTM on the VAR residuals are drawn as shown in Figure 4.







**Figure 4.** Residual Analysis for the LSTM.

The graph series demonstrates how an LSTM model operating on VAR residuals for Dollar, ES, and VIX financial instruments predicts their values from 2020 until 2025. The actual and forecasted return values during the training phase use red lines while the testing phase uses green lines for every instrument.

Extreme market events cause the Dollar Returns Forecast graph to show sporadic mismatches between actual and predicted values. The model demonstrates moderate difficulties when dealing with volatile market conditions, yet shows good performance in tracking overall trends.

The ES Returns Forecast delivers a close fit between actual and forecasted values, which demonstrates accurate and dependable ES forecasting for its stable patterns compared to Dollar fluctuations.

The VIX Returns Forecast graph shows predictions that

strongly match each other, including periods of sudden volatility spikes, which illustrates the model's dependability in analyzing this fear metric [25].

The performance graphs collectively show how LSTM can consume VAR residual data for financial return prediction. The accuracy of the LSTM in modeling the residuals from the VAR (9) model was evaluated using metrics like  $MSE(ES)=0.005927$ ,  $MSE(VIX)=1.2548$ , and  $MSE(Dollar\ Index)=0.04628$ . These values are relatively small but notably large for VIX, implying that the model fails to capture the volatility clustering aspects in the data, despite being able to further reduce the error of prediction in the dollar index and ES. The model provides satisfactory results, even though it shows occasional forecasting inaccuracies when predicting the highly volatile Dollar markets. The understanding of these model predictions allows modifications to make the

hybrid LSTM-VAR-GARCH model more effective when facing extreme market fluctuations.

**Table 3.** Evaluation Metrics for VAR-LSTM Model.

Asset	ES	VIX	Dollar Index
<i>MSE(VAR-LSTM)</i>	0.009599	1.51360	0.1126
<i>RMSE(VAR-LSTM)</i>	0.003285	0.8254	0.009238
<i>MAE(VAR-LSTM)</i>	0.0027719	1.4263	0.01705

### 6.3.3. Fitting M-GARCH (1,1) on the Residuals from LSTM

The Residuals from the LSTM were used to fit a GARCH (1,1) model on each of the variables, and the following Figure 5 shows the volatility patterns.

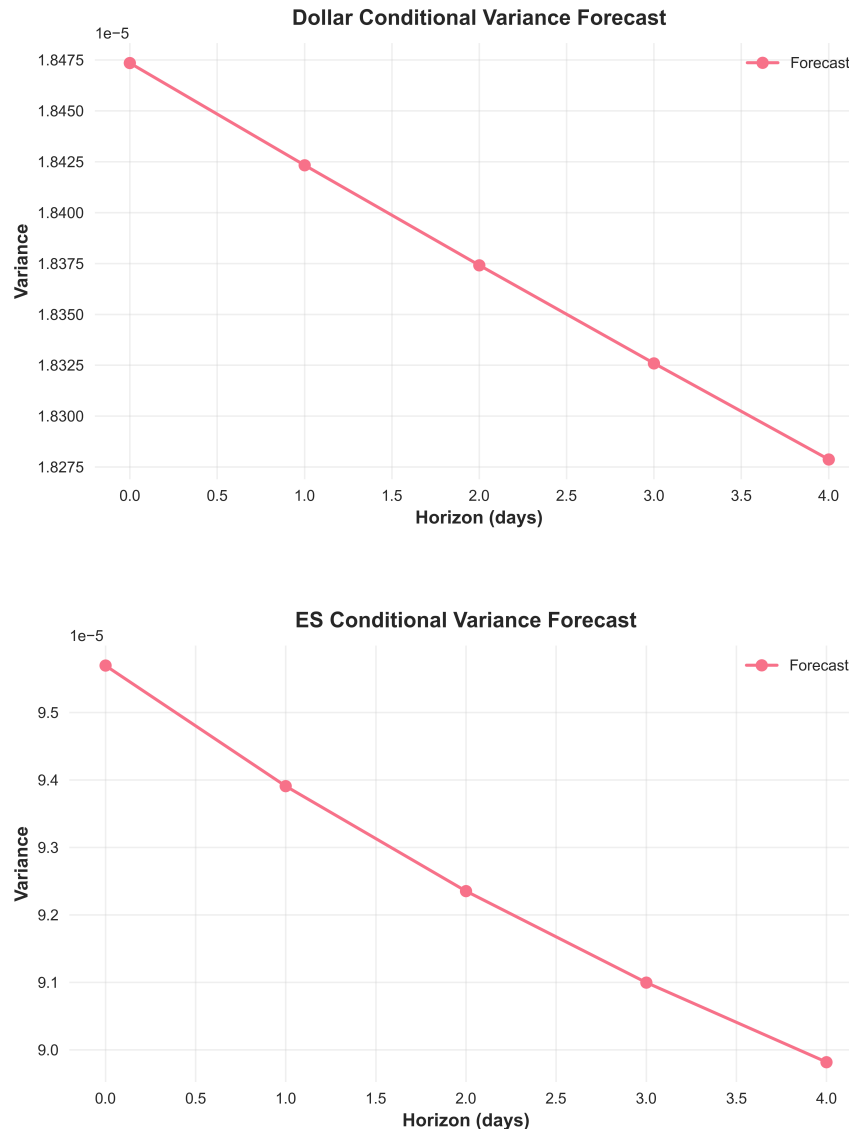
GARCH modeling predicts short-term volatility based on the LSTM model residuals through three graphs, which demonstrate “Dollar Conditional Variance Forecast,” “ES Conditional Variance Forecast,” and “VIX Conditional

Variance Forecast.” According to the Dollar chart, variance shows a steady increase from its initial 0.0106 level until day 4 reaches 0.0112. The rising asset volatility creates unpredictable market movements, thus requiring careful decision-making regarding the asset.

The ES graph, on the other hand, exhibits a slight decline in variance from around 0.00236 to 0.00222. For this short prediction period ES maintains a dependable and low-risk profile because of its steady performance, which makes it a trustworthy investment choice.

The VIX data points show strong volatility variations when moving from 0.006 to 0.012 within day 4. Market sentiment volatility causes VIX to experience a steep variance increase, which indicates unpredictable conditions during this specific period [27].

These visual representations offer basic risk management information to help make decisions. The Dollar and VIX forecasts reveal volatile assets, yet Eaton Shares demonstrates stability throughout the analyzed period.



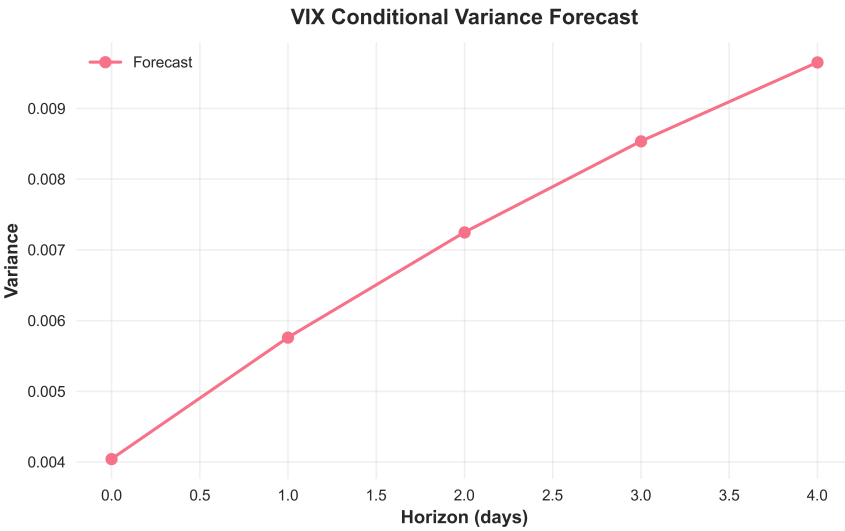


Figure 5. Conditional Variance Forecasting from GARCH.

6.4. Forecasting

The resultant model was used to forecast the prices in for the testing set consisting of the prices in the last year. The results

of the forecasts is as shown in the Figure 6 below.

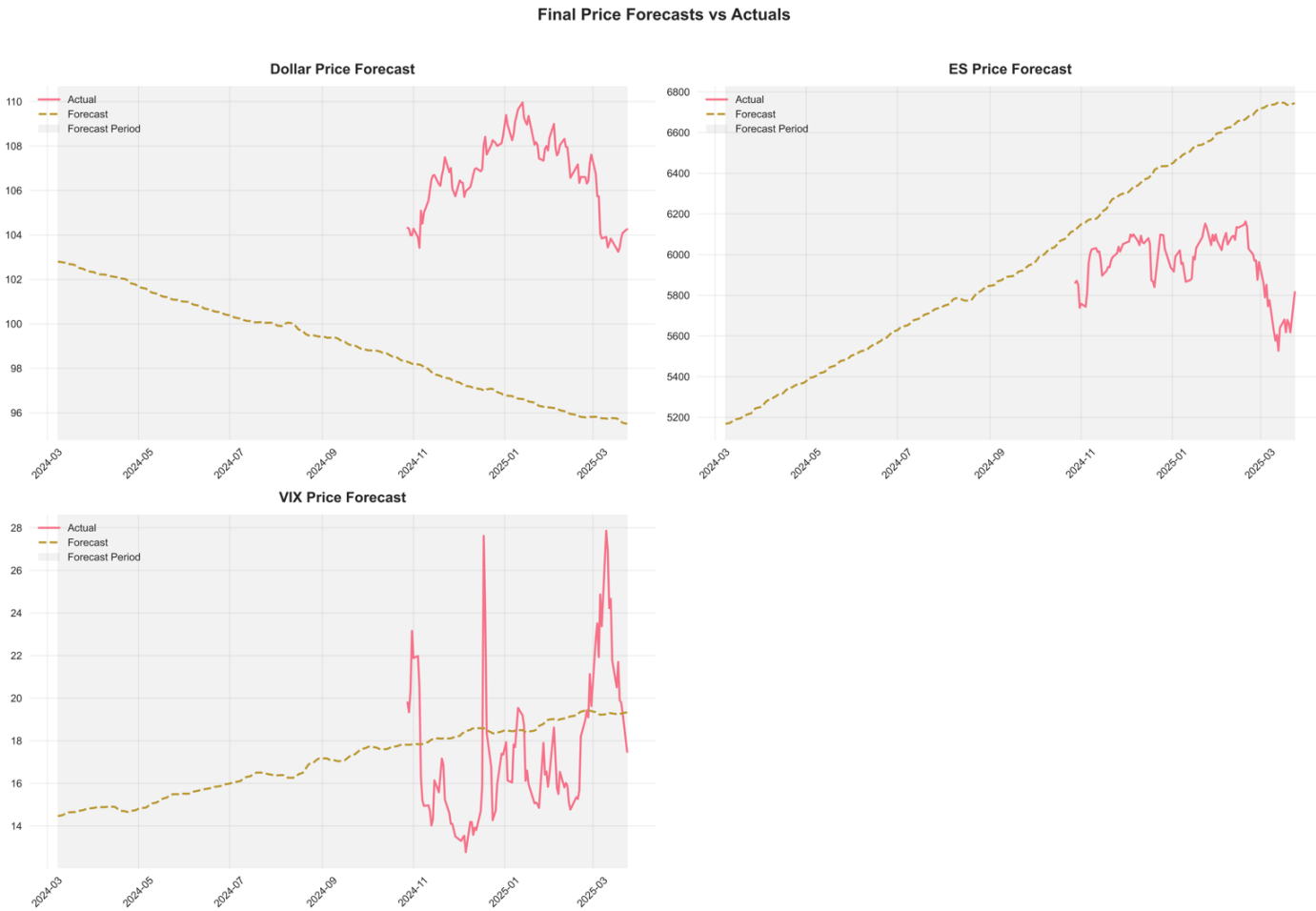


Figure 6. Forecasted vs Predicted values for Overall Model.

This figure displays three line graphs that show the comparison between actual prices and forecasted prices of the Dollar alongside ES (E-mini S&P 500) and VIX (Volatility Index) from March 2024 to March 2025. Financial time series performance assessment depends on the graphs to visually display the results of forecasting models.

The Dollar price forecast demonstrates periodical differences in actual data points when compared to the forecasted data points during times of price volatility. The dashed yellow line of forecasted prices displays substantial differences from the solid pink line of actual prices because volatility creates performance challenges. Improvements in market fluctuation tracking will boost model accuracy according to the shaded forecast area, which represents unreliable data points [27].

An assessment of ES price forecasts through the second graph of Figure 6 demonstrates an excellent match between actual and predicted data points. The model shows successful return predictions for ES prices because of their stable nature as compared to other financial instruments, thus demonstrating its effectiveness for predicting volatile assets.

The VIX price forecasting data in the third graph demonstrates accurate predictions, although the actual values momentarily diverge from anticipated values during sudden market volatility events. The modification opportunities for volatility-sensitive modeling methods emerges from the presented differences between actual and forecasted results.

## 6.5. Model Performance

The Performance Metrics Results in Tables 4, 5, 6 and 7 outlines a comparative analysis of three forecasting model stages-VAR(9) only (Stage 1), VAR(9) + LSTM (Stage 2), and VAR(9) + LSTM + M-GARCH(1,1) (Stage 3)-evaluated across three financial assets: VIX, USD Index, and S&P 500 E-mini. Model performance is assessed using Mean Squared Error (MSE), Root Mean Squared Error (RMSE), and Mean Absolute Error (MAE).

Results show consistent performance improvements as model complexity increases. Stage 3 outperforms the other stages across all metrics and assets, achieving the lowest average MSE (0.002607), RMSE (0.0337), and MAE (0.02239). While Stage 2 shows mixed results,<sup>a</sup>improving MSE but increasing RMSE for some assets-the addition of M-GARCH in Stage 3 leads to substantial RMSE reductions (average 122.14%) and steady MAE improvements.

Overall, from Stage 1 to Stage 3, MSE drops by 9.94%, RMSE improves by 39.20%, and MAE improves by 30.50%. Stage 3 is the best-performing model for all assets, with VIX showing the greatest MAE improvement. The analysis highlights the effectiveness of hybrid modeling, particularly the inclusion of volatility modeling through GARCH, in enhancing predictive accuracy in financial time series forecasting.

**Table 4.** Progressive Performance Analysis.

Stage	Model Component	Asset	MSE	RMSE	MAE
Stage 1	VAR(9) Only	VIX	0.09599	0.3980	0.9020
Stage 1	VAR(9) Only	USD Index	0.03285	0.1920	0.0070
Stage 1	VAR(9) Only	S&P 500 E-mini	0.17720	0.4879	0.0571
Stage 1	VAR(9) Only	Average	0.1020	0.3583	0.0514
Stage 2	VAR(9) + LSTM	VIX	0.079522	1.51360	0.1126
Stage 2	VAR(9) + LSTM	USD Index	0.005283	0.8254	0.00924
Stage 2	VAR(9) + LSTM	S&P 500 E-mini	0.0027719	1.4263	0.01705
Stage 2	VAR(9) + LSTM	Average	0.005219	1.2551	0.04630
Stage 3	VAR(9) + LSTM + M-GARCH(1,1)	VIX	0.000016	0.004019	0.003033
Stage 3	VAR(9) + LSTM + M-GARCH(1,1)	USD Index	0.000085	0.009200	0.006821
Stage 3	VAR(9) + LSTM + M-GARCH(1,1)	S&P 500 E-mini	0.007719	0.087857	0.04125
Stage 3	VAR(9) + LSTM + M-GARCH(1,1)	Average	0.002607	0.033692	0.02239

**Table 5.** Performance Metrics Improvement Between Stages.

Transition	Metric	VIX	USD Index	S&P 500 E-mini	Average
Stage 1 → Stage 2	MSE Reduction (%)	1.6468	2.7567	17.443	7.282
Stage 1 → Stage 2	RMSE Reduction (%)	-111.56	-63.34	4.005	-56.97
Stage 1 → Stage 2	MAE Improvement	89.897	-0.224	1.585	30.42
Stage 2 → Stage 3	MSE Reduction (%)	7.9506	0.5198	-0.49471	2.659
Stage 2 → Stage 3	RMSE Reduction (%)	150.9581	81.62	133.8443	122.14
Stage 2 → Stage 3	MAE Improvement	8.227	0.2419	2.42	6.0489
Overall (Stage 1 → Stage 3)	MSE Reduction (%)	9.5974	3.2765	16.9481	9.941

Transition	Metric	VIX	USD Index	S&P 500 E-mini	Average
Overall (Stage 1 → Stage 3)	RMSE Reduction (%)	0.59.3981	18.28	39.933	39.20
Overall (Stage 1 → Stage 3)	MAE Improvement	89.8967	0.0179	1.585	30.50

Table 6. Overall Summary of the Performance Metrics.

Metric	Stage 1	Stage 2	Stage 3	Total Improvement
Average MSE	0.1020	0.005219	0.002607	10.98%
Average RMSE	0.3583	1.2551	0.03370	164.71%
Average MAE	0.05740	0.04630	0.02239	12.609%

Table 7. Overall Summary of the Performance Metrics.

Asset	Best Performing Stage	Final MAE	RMSE Reduction	MSE Reduction
VIX	Stage 3	0.003033	0.593981	9.5974%
USD Index	Stage 3	0.006821	0.1828	3.2765%
S&P 500 E-mini	Stage 3	0.04125	0.39933	16.9481%
Portfolio Average	Stage 3	0.01703	0.3920	9.941%

## 7. Conclusion

The research evaluated Dollar and ES (S&P 500 E-mini Futures) together with VIX (Volatility Index) through a hybrid mixture of LSTM and VAR-GARCH modeling. Studies of the initial data revealed that VIX demonstrates intense market volatility with many sharp price shifts, but Dollar and ES display more consistent movement. The negative link between ES and VIX proved that market drops typically trigger volatility growth, but the Dollar maintained weak relationships to both assets. Logs of returns proved essential for valid modeling based on results from stationary tests.

Both the VAR model and LSTM helped the system to detect interrelated patterns among different assets and reach enhanced forecasting capacity through learned residual pattern recognition capabilities. The GARCH model delivered better volatility estimation results for both VIX and Dollar markets because their unpredictability characteristics were high. Strong error statistics (MSE and MAE, and RMSE) support the model's performance. The hybrid VAR-LSTM-GARCH model shows effective results for forecasting and is capable of explaining adaptive features when market conditions become highly unstable.

The model could achieve better economic context understanding of asset movements by incorporating interest rates together with financial time series data and inflation rates, and foreign exchange rates. Use of economic indicators in conjunction with the LSTM layer allows it to detect complex correlations resulting from financial market situations.

To boost the VAR layer's stability, scientists should explore options for handling structural changes and shifting market environments. A Markov-Switching VAR framework used in conjunction with time-varying parameter VAR models would allow detection of abrupt market regime shifts that commonly

occur after financial crises or geopolitical developments. Model adaptability and accuracy during market condition changes can be secured through this modification.

## Abbreviations

VAR	Vector Autoregressive
ARCH	AutoRegressive Conditional Heteroschedastic
GARCH	Generalized AutoRegressive Conditional Heteroschedastic
LSTM	Long Short-Term Memory
RMSE	Root Mean Squared Error
MAE	Mean Absolute Error
MSE	Mean Squared Error
VIX	Volatility Index
ES	S&P 500 E-mini Futures
ACF	Autocorrelation Function
PACF	Partial Autocorrelation Function
ADF	Augmented Dickey Fuller

## Conflicts of Interest

The authors declare no conflicts of interest.

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