



Research Article

Numerical Analysis of Magnetohydrodynamic (MHD) Glycerin Copper-Silver Hybrid Nano-Coolant Using Tiwari-Das Model

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Abstract

The existence of machines, electronic devices, environmental regulations and the blooming technology demands for endless efficient heat transfer fluid in automotive radiators, electronics heat sinks and dispersants in transformers coolers. The nanofluid flow is advanced with the aim of bridging the gap in fields such as engineering, industries and medical. The potential use of hydromagnetic (Cu/Ag-glycerine) hybrid nanofluid as a coolant is examined. The Tiwari-Das nanofluid model is applied in the formulation of the partial differential equations describing the flow. The introduction of similarity transformation variables converts the equations to nonlinear systems. The deployment of the state-space method linearizes the ordinary differential equations which numerically solves using the fourth-order Runge-Kutta method with a shooting technique in MATLAB. The ode45 function is invoked. The outputs of the simulation are displayed graphically and in tabular form. The results show the fluid velocity decays with the rise of magnetic strength and Prandtl number while it escalates with radiation parameter, Grashof and Darcy numbers correspondingly. Further, the temperature of the fluid increases with magnetic and radiation parameters and Eckert number. The opposite trend is observed with soaring Prandtl and Darcy numbers respectively. The local skin friction coefficient dwindles with the growth of porosity, magnetic field and radiation parameters. The rate of energy transfer is elevated with increase in magnetic field, radiation and porosity. The decrease in fluid velocity and the hike rate of heat transfer with magnetic field are utilized in thermal transfer, cooling and in heat flow management.

Keywords

Magnetohydrodynamics, Hybrid-Nano-Coolant, Analysis

1. Introduction

The use of nanofluids as a working fluid in heat transfer applications continues to yield promising results. This is attributed to their better thermal network and rheological properties Okello [14]; Rahman [16]. Nanofluids engineered using a single type nanoparticle have been deployed as coolants in industrial processes to help dissipate heat and in high-

density heat flux devices such as computer microchips, laser diodes, transformers, miniaturized electronic devices, nuclear reactors and in car radiators El-Khouly [4]; Mutuku [12]; Nanan Kwanchai [13]. To further enhance the cooling capability of nanofluids research has shown dispersing two or more nanoparticles could further improve their cooling

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ability and overall application as heat transfer fluid in thermal processes. This has been attributed to the synergetic effect of the different nanoparticles suspended in the fluid. The hybrid nanofluids developed as a result of suspension of two or more nanoparticles in conventional heat transfer fluids have found use in specialized cooling systems. Xian [21] investigating the use of (CGnP – TiO_2) hybrid nanoparticles dispersed in a mixture of distilled water and ethylene glycol reported increments of 4.94%, 35.87%, and 20.48% in Nusselt number, heat transfer coefficient, and effectiveness of the radiator. Delving into the performance of ethylene-glycol-based hybrid nanofluid for car radiator cooling Okello [15] posted better cooling performance by $\text{Cu} - \text{Al}_2\text{O}_3/\text{ethylene} - \text{glycol}$ compared to the rest of the nanocoolants. Studying the use of Tungsten (III) oxide (WO_3) – silver/transformer oil hybrid nano-coolants, Aberoumand and Jafarimoghaddam [1] observed an increment of 41% in thermal conductivity in higher weight fraction. Mashhadzadeh [10] presented improved breakdown voltage and heat transfer by 9% and 8% respectively in the transformer while using ($\text{TiO}_2/\text{CNT} - \text{transformer oil}$) hybrid nanofluid. They also reported reduction in the levels of gases C_2H_4 and C_2H_6 dissolved in the transformer oil by over 70% as a result of the use of hybrid nanofluid. The experimental study by Hassaan [5] involving use of hybrid nanofluids ($\text{MWCNTs} - \text{Al}_2\text{O}_3/\text{water}$) at different nanoparticle concentrations and Reynolds number (2500 – 12560) in a shell and tube heat exchanger realized a (10 – 28.5%) growth in Nusselt number over distilled water. The effectiveness of the heat exchanger rose by 22.7% when using hybrid nanofluid compared to distilled water. Heat transfer analysis by Jangid [7] on ($\text{Copper} - \text{alumina}/\text{water}$) hybrid nanofluid in a vertical microtube revealed enhancement in Nusselt number by 3.102% at higher alumina nanoparticle concentration in the hybrid mixture and a 3.098% increase at a higher copper nanoparticle concentration in the mixture signifying the effectiveness of alumina nanoparticles compared to copper nanoparticles. A probe by Kanti [8] on thermal performance of ($\text{Al}_2\text{O}_3 - \text{TiO}_2(50:50)/\text{water}$) hybrid nanofluid in turbulent flow noted a maximum enhancement of 15.77% and 70.4% in viscosity and Nusselt number respectively for 1 vol% of hybrid nanofluid compared to the base fluid. Similarly, the hybrid nanofluids achieved a remarkable reduction in total entropy generation of 46% in contrast to the base fluid. Study by Mezrakchi [11] on various hybrid nanocoolants in a shell and tube thermal exchanger across eight discrete inlet velocities revealed superior heat transfer capabilities of the hybrid nanofluid compared to the base fluid across all the conditions. The ($\text{CuO} - \text{ZnO}/\text{water}$) hybrid nanocoolants emerged as a better coolant with a substantial increase of over 9% in temperature reduction as opposed to the approximately 6% attained by other hybrid nanofluids at a lower velocity of 0.5m/s. The evaluation by Kargaran [9] on the

cooling efficiency of a photovoltaic panel equipped with a three-dimensional oscillating heat pipe integrated with hybrid nanofluids ($(\text{GO} - \text{CuO}), (\text{CNT} - \text{CuO}), (\text{MWCNT} - \text{CuO})$) at 0.1 and 0.2g/L concentration of each nanofluid observed improved electrical power output by 12.07% in ($\text{CNT} - \text{CuO}$) hybrid nanofluid at 0.2g/L concentration. The ($\text{CNT} - \text{CuO}$) hybrid nanofluid outperformed other nanofluids registering a maximum exergy efficiency of 31.2%. The numerical investigation by Huminic and Huminic [6] into the use of ($\text{MWCNT} - \text{Fe}_3\text{O}_4/\text{water}$) hybrid nanofluid as an automotive coolant reported a 31% enhancement in heat transfer compared to water at low Reynolds numbers. The simulation was performed for the range of Reynold numbers (50 – 1000) and for nanoparticle volume concentrations of 0.1% and 0.3%. The study by Bouras [3] on improving cooling of high voltage transformers by incorporating nanofluids revealed improvement in the rate of heat transfer by 10% as a result of increasing solid volume nanoparticle concentration by 10% which made the fluid to become more conductive. Investigations by Boukounacha [2] on thermal performance of mineral oils adulterated with different nanoparticles ($\text{SiC}, \text{TiO}_2, \text{Al}_2\text{O}_3$) showed increased values of thermal conductivity, viscosity, and density as a result of addition of nanoparticles. The enhanced thermal conductivity contributed to better cooling of the transformer by the nanocoolant. Study on performance of a solar dish system with hybrid nanofluid as a coolant by Sheikholeslami [18] reported increase in maximum power output by 3.23% as a result of using nanocoolant. The examination by Shaalan [17] on the cooling efficacy of a ZnO/TiO_2 hybrid nanofluid in a photovoltaic (PV) system noted a 12.1% increase in electrical efficiency and a 10.4% reduction in panel temperature which were attributed to the use of nanocoolant. Investigations into the thermophysical properties of TiO_2 nanofluids by V and B [20] for prospective use in enhanced cooling applications in renewable energy systems and smart grids reported a 28% enhancement in thermal conductivity at 0.5% nanoparticle volume concentration making it ideal for use as a coolant in renewable energy systems and hybrid grids. From the aforementioned, a study on MHD glycerin Copper-Silver hybrid nanocoolant using Tiwari-Das model will be an insightful contribution to literature and offer an alternative coolant for industrial and engineering applications.

2. Mathematical Formulation

A steady, incompressible, laminar, two-dimensional (x, y) boundary layer flow of an electrically conducting ($\text{Cu}/\text{Ag} - \text{glycerine}$) hybrid nanofluid past a vertical semi-infinite flat plate under the influence of a transversely imposed magnetic field is considered (see Figure 1).

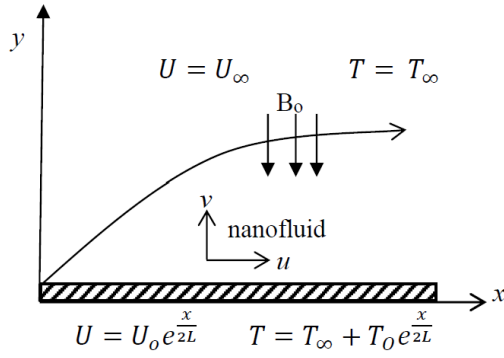


Figure 1. Flow configuration and coordinate system.

A transverse magnetic field with variable strength B_0 is applied parallel to the y -axis. There is no applied voltage and the magnetic Reynolds number is small, hence the induced magnetic field and Hall effects are negligible. The use of the model proposed by Tiwari and Das [19] is based on its inclusion of volume fraction to the thermophysical properties of the base

fluid and it quantifies the concentration of the combining particles. It is more accurate. However, the short-comes such as particles sedimentation and agglomeration associated with the model are minimized by use of surfactants, choice of size and uniformed shape of nanometals, higher viscosity of the base fluid effects and the external applied magnetic fields suspends the particles. Assuming that the base fluid and the nanoparticles are at thermal equilibrium without slip condition, the mixture is uniform, non-interactive particles, with constant thermophysical properties and the solution is dilute. Under the a forestated assumptions the governing equations for the continuity, momentum and energy in laminar incompressible boundary layer flow in a nanofluid can be expressed as;

Mass equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_\infty \frac{\partial U_\infty}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 U_\infty}{\partial y^2} + g\beta(T - T_\infty) - \frac{\sigma_{hnf}}{\rho_{hnf}} \beta_0^2 u - \frac{\mu_{hnf}}{k} u \quad (2)$$

Energy equation

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{hnf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\rho c_p \partial y} \quad (3)$$

The boundary conditions governing the flow are:

$$\left. \begin{aligned} y = 0, U_\infty = U_0 e^{\frac{x}{2L}}, v = 0, T = T_\infty + T_0 e^{\frac{x}{2L}} \\ y \rightarrow 0, U_\infty \rightarrow \infty, T \rightarrow \infty \end{aligned} \right\} \quad (4)$$

2.1. Thermophysical Properties

The choice for the base fluid and the nanometal considers the physical and chemical properties of the combines. Glycerin is non-toxic, biodegradable, stably boils at higher temperature, miscible with other liquids, higher heat capacity and non-decomposes at higher temperatures. Silver has excellent electrical and thermal conductivity. Its antimicrobial properties inhibit germination and growth of motile microorganisms enhancing efficiency and reliability of the nanofluid. Silver is less corrosive compared to other metals such as copper. 60% of water is mixed with 40% of glycerin using volume weighted average model to obtain the density, specific heat capacity, thermal conductivity and viscosity of the base fluid as follows

$$\rho_{bf} = 0.6\rho_w + 0.4\rho_g \quad (5)$$

$$C_{p,bf} = \frac{0.6\rho_w C_{p,w} + 0.4\rho_g C_{p,g}}{\rho_{bf}} \quad (6)$$

$$k_{bf} = 0.6k_w + 0.4k_g \quad (7)$$

During deformation, non-Newtonian nanofluid exhibit non-linear relation obtained as

$$\mu_{bf} = \mu_w^{0.6} + \mu_g^{0.4} \quad (8)$$

$$\mu_{bf} = e^{0.6 \ln(\mu_w)} + e^{0.4 \ln(\mu_g)} \quad (9)$$

Taking volume fractions of copper and silver as ϕ_c and ϕ_g , the total nanoparticle volume fraction is stated as

$$\phi_{np} = \phi_c + \phi_a. \quad (10)$$

Assuming the mixture is uniform and the nanometals are spherical, the effective thermophysical characteristic of density, heat capacity, viscosity and thermal expansion are obtained as

$$\rho_{hnf} = (1 - \phi_{np})\rho_{bf} + \phi_c \rho_c + \phi_a \rho_a \quad (11)$$

$$(\rho C_p)_{hnf} = (1 - \phi_{np})\rho_{bf} C_{p,bf} + \phi_c \rho_c C_{p,c} + C_{p,a} \phi_a \rho_a \quad (12)$$

Specific heat capacity of hybrid nanofluid

$$(C_p)_{hnf} = \frac{(\rho C_p)_{hnf}}{\rho_{hnf}} \quad (13)$$

We used Brinkman model to approximate the combined viscosity as

$$\mu_{hnf} = \frac{\mu_{bf}}{(1 - \phi_{hnf})^{2.5}} \quad (14)$$

The thermal conductivity of the hybrid nanofluid is determined by Maxwell-Garnette model given as

$$k_{hnf} = k_{fb} \left(\frac{k_c + 2k_{bf} - 2\phi_c(k_{bf} - k_c)}{k_c + 2k_{bf} + \phi_c(k_c - k_{bf})} \right) \chi \left(\frac{k_a + 2k_{bf} - 2\phi_a(k_{bf} - k_a)}{k_a + 2k_{bf} + \phi_a(k_a - k_{bf})} \right) \quad (15)$$

Where k_c, k_a are the effective thermal conductivity of copper and silver respectively

$$\beta_{hnf} = (1 - \phi_c - \phi_a)\beta_{bf} + \phi_c\beta_c + \phi_a\beta_a \quad (17)$$

The combined electrical conductivity of the nanometal

$$\sigma_{hnf} = \phi_c\sigma_c + \phi_a\sigma_a \quad (16)$$

With $\beta_{bf}, \beta_c, \beta_a$ as thermal expansion of the base fluid, copper and silver respectively.

The combined thermal expansion of the base fluid and nanoparticles is stated as

Table 1. Thermophysical properties.

Property	Water	Glycerin	Copper	Silver
Density (ρ)	997.05	1260	8935	10500
Specific heat capacity (C_p)	4180	2415	385	235
Thermal Conductivity (k)	0.606	0.285	400	429
Thermal expansion (β)	2.07×10^{-4}	5.1×10^{-4}	5.0×10^{-5}	1.9×10^{-5}
Electrical conductivity (σ)	-	-	5.96×10^7	6.30×10^7
Viscosity (μ)	0.00089	1.5	-	-

To enhance thermal absorption, the base fluid is dyed black and the mean absorption coefficient empirically determined as $\overline{k_{bf}} = 450 \text{ m}^{-1}$. The nanofluid flows through porous medium of porosity $K=0.85-0.98$. The power law is used to estimate the mean absorption coefficient of the nanometal as follows

$$k^* = \overline{k_{bf}}(1 + \phi_a C_a + \phi_c C_c)$$

The fitting constants for copper $C_c = 250 \text{ m}^{-1}$ and silver $C_a = 800 \text{ m}^{-1}$ are also empirically stated.

2.2. Non-Dimensionalization

The equations (1-3) and the boundary conditions 4 are expressed in dimensionless form by introducing the similarity variables defined as

$$\psi(x, y) = (U\gamma x)^{\frac{1}{2}} f, \eta = y \sqrt{\frac{U}{\gamma x}}, \theta = \frac{T - T_\infty}{T_o e^{2l}} \quad (18)$$

The free stream function can be stated as

$$\left. \begin{aligned} u &= \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \\ u &= U_o e^{\frac{x}{l}} f', v = -\left(\frac{U_o \gamma_{hnf}}{l e^{2l}}\right)^{\frac{1}{2}} \left(f + \frac{\eta}{2} f'\right) \end{aligned} \right\} \quad (19)$$

Assuming the fluid flow has small variation in temperature, we expand T^4 about T_∞ using Taylor series up to second term.

$$T^4 \approx T_\infty^4 + 4T_\infty^3(T - T_\infty)$$

Since T_∞^4 is a constant initial ambient temperature

$$\frac{\partial T^4}{\partial y} = -4T_\infty^3, \frac{\partial q_r}{(\rho C_p)_{hnf} \partial y} = \frac{-16\sigma^* T_\infty^3}{3k^*(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2}$$

Substituting equations (18)-(19) into equations (1-3), equation (1) is satisfied while equations (2)-(3) converts to dimensionless nonlinear ODEs for momentum, energy and the boundary conditions obtained as

$$f''' + \frac{1}{2} f f'' - f'^2 + \lambda \theta - M f' + \delta f' \quad (20)$$

$$\theta'' + Pr(0.5 f \theta' - f' \theta) - Rd \theta'' + Ec M f'^2 \quad (21)$$

with the boundary conditions

$$\left. \begin{aligned} f = 0, f' = 1, f'' = s_1, \theta = 1, \theta' = s_2 \text{ at } y = 0 \\ f' \rightarrow 1, \theta \rightarrow 0 \text{ at } y = \infty \end{aligned} \right\} \quad (22)$$

$$M = \frac{\sigma_{hnf} \beta_o^2 x}{\rho_{hnf} U_o}, Pr = \frac{\mu_{hnf} c_{p_{hnf}}}{k_{hnf}}, Ec = \frac{U_o^2}{c_p T_o}, \lambda = \frac{g \beta_{hnf} T_o x}{U_o^2}, \delta = \frac{\mu_{hnf}}{K U_o \rho_{hnf}}, Rd = \frac{16 \sigma^* T_\infty^3}{3 k^* k_{hnf}}$$

Deploying the wall shear stress definition $\tau = \mu \frac{\partial u}{\partial y}$ and Fourier law $q = k \frac{\partial T}{\partial y}$, the non-dimensional form of skin friction (C_f), Nusselt (Nu_x) and Reynold Re_x numbers respectively are determined as

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho_{hnf} U_o^2}, \tau_w = \mu_{hnf} \frac{\partial u}{\partial y}, Re_x = \frac{x U_o}{\nu_{hnf}}$$

$$C_f Re_x^{\frac{1}{2}} = \frac{2 \mu_{hnf}}{\rho_{hnf}} f'' \quad (23)$$

$$Nu_x = \frac{x q_x}{k_f (T_w - T_\infty)}, q_x = -k_{hnf} \frac{\partial T}{\partial y}$$

$$Nu_x Re_x^{-\frac{1}{2}} = -\frac{k_{hnf}}{k_{bf}} \theta' \quad (24)$$

3. Numerical Procedure

The states variables are introduced to linearize the nonlinear ODEs of equations (20)-(21) together with the constrains in equation (22) as follows

$$f_1 = f, f_2 = f', f_3 = f'', \theta_1 = \theta, \theta_2 = \theta' \quad (25)$$

$$f'_1 = f_2 \quad (26)$$

$$f'_2 = f_3 \quad (27)$$

$$f'_3 = -0.5 f_1 f_3 + f_2^2 - \lambda \theta_1 + M f_2 - \delta f_2 \quad (28)$$

$$\theta'_1 = \theta_2 \quad (29)$$

$$\theta'_2 = \frac{1}{1+Rd} [-Pr(0.5 f_1 \theta_2 - f_2 \theta_1) - Ec M f_2^2] \quad (30)$$

The constraints

$$\left. \begin{aligned} f_1 = 0, f_2 = 1, \theta_1 = 1 \text{ at } \eta = 0 \\ f_2 = 1, \theta_1 = 0 \text{ at } \eta = \infty \end{aligned} \right\} \quad (31)$$

The shooting technique is employed by guessing targets s_1 and s_2 . The Runge-Kutta fourth order scheme is performed using a computer software MATLAB for numerical analysis.

where $M, Pr, Ec, \lambda, \delta, Rd$, are Hartman, Prandtl, Eckert, Thermal Grashof numbers, Darcy and Radiation.

Parameters respectively and are stated as follows

4. Results and Discussion

The effect of pertinent parameters on dimensionless velocity and temperature profiles, skin friction coefficient and local Nusselt number are depicted graphically and in tabular form and discussed in details.

4.1. Effects of Parameter Variation on Velocity Profile

From Figures 2-6 the maximum fluid velocity occurs at the surface of the plate and it decays to zero free stream value far away from the plate satisfying the given boundary conditions. Figure 2 shows increase in magnetic parameter raises Lorentz force in the flow. Motion of the flow retards. Figure 3 illustrates escalation of Prandtl number thins velocity due to the kinematic viscosity and the internal resistance. Figure 4 denotes the velocity of the fluid increases with Darcy number. The rise of Darcy number increments porosity of the medium making the fluid to flow more easily. Figure 5, demonstrates radiation ascents with the velocity of the fluid. Thermal radiation consequently improves temperature and convective currents. Figure 6, depicts Grashof number climbs with velocity profiles. Upthrust force in the flow prompts the fluid to move more swiftly leading to elevated velocity.

$$\lambda = 7, \delta = 0.3, Rd = 0.7, Ec = 0.1$$

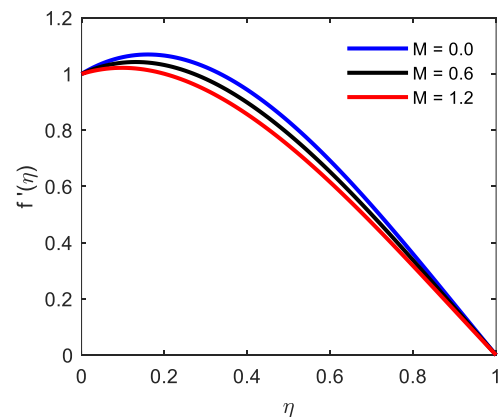


Figure 2. Velocity profiles for different values of Magnetic parameter.

$$\lambda = 0.5, \delta = 0.3, Rd = 0.4, Ec = 0.6, M = 0.8$$

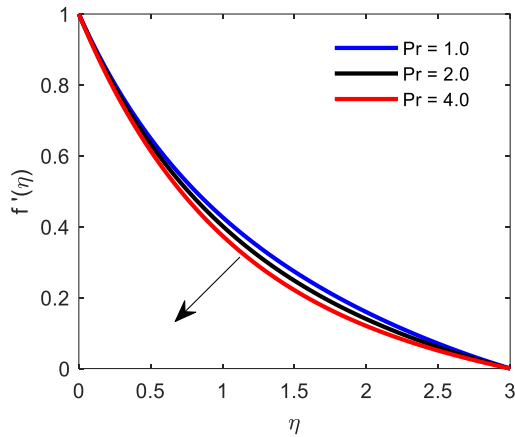


Figure 3. Velocity profiles for different values of Prandtl number.

$$\lambda = 0.5, Rd = 0.4, Ec = 0.6, M = 0.8, Pr = 0.9$$

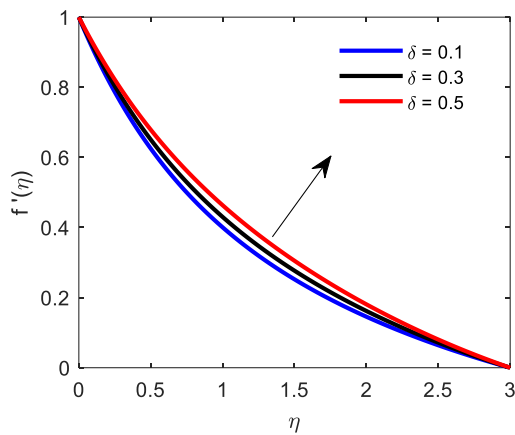


Figure 4. Velocity profiles for different values of Darcy number.

$$\lambda = 5, \delta = 0.5, Ec = 0.6, M = 0.5, Pr = 0.9$$

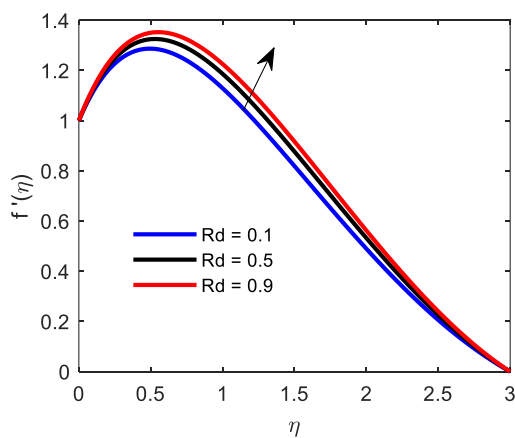


Figure 5. Velocity profiles for different values of radiation parameter.

$$\delta = 0.5, Ec = 0.8, M = 1, Pr = 3, Rd = 0.3$$

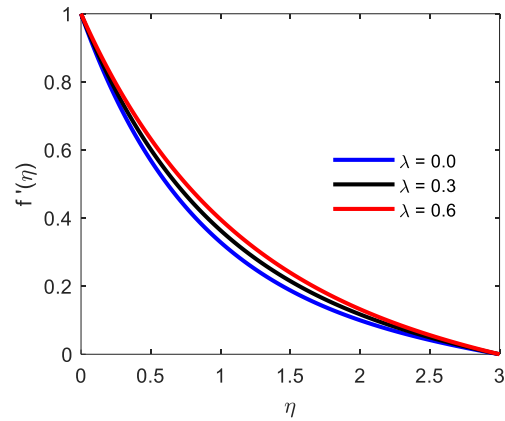


Figure 6. Velocity profiles for different values of Grashof number.

4.2. Effects of Parameter Variation on Temperature Profiles

The effect of different parameters on dimensionless temperature profile is presented in Figures 7-11. The maximum fluid temperature occurs on the plate surface and it decays to zero far away from the plate surface. Figure 7 depicts increase in fluid temperature with magnetic parameter. Magnetic strength induces resistive force in the flow building up heat between the layers of the fluid caused by the soiling viscous drag. On Figure 8 the temperature dwindles with increase in Prandtl number which lowers thermal diffusivity. Figure 9 depicts the effect of radiation on temperature profile. Radiation adds energy into the fluid. Figure 10 gives the effect of Darcy number on the temperature profile of the fluid. Large Darcy number means more space for passage of the fluid thus quick and efficient distribution of heat. Figure 11 presents upsurge of Eckert number with temperature. The friction between layers and particles of the fluid cause significant growth of heat.

$$\lambda = 0.3, \delta = 0.5, Rd = 0.7, Pr = 8, Ec = 0.8$$

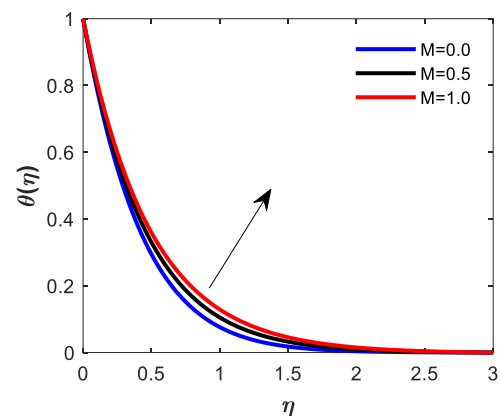


Figure 7. Temperature profiles for different values of magnetic parameter.

$$\lambda = 0.3, \delta = 0.5, Rd = 0.7, Ec = 0.03, M = 0.03$$

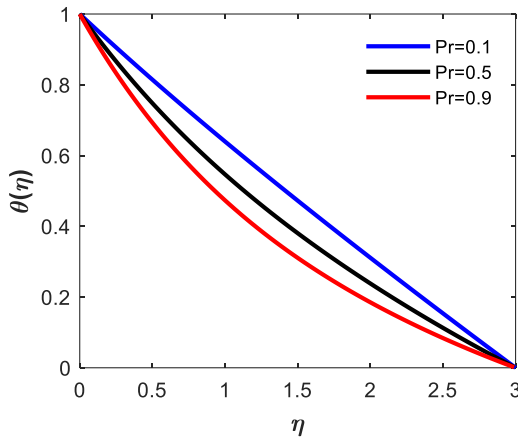


Figure 8. Temperature profiles for different values of Prandtl number.

$$\lambda = 0.3, \delta = 0.5, Ec = 0.03, M = 0.03, Pr = 8$$

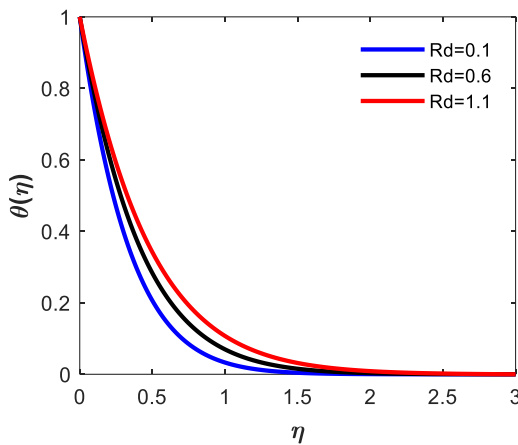


Figure 9. Temperature profiles for different values of Radiation parameter.

$$\lambda = 0.3, Ec = 0.03, M = 0.8, Pr = 8, Rd = 0.7$$

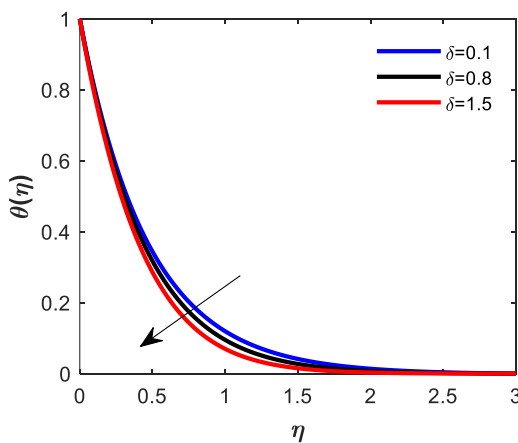


Figure 10. Temperature profiles for different values of Darcy number.

$$M = 0.8, Pr = 8, Rd = 0.4, \delta = 0.6, \lambda = 0.3$$

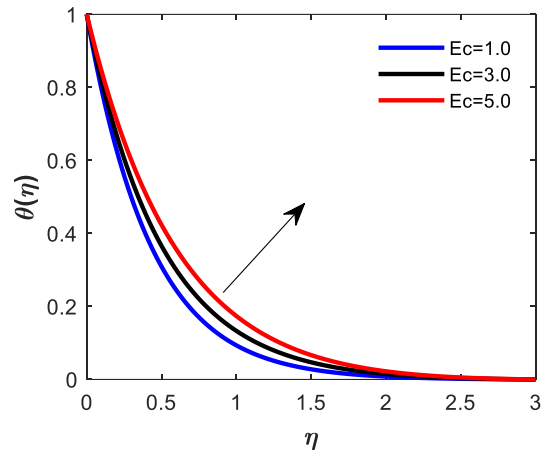


Figure 11. Temperature profiles for different values of Eckert number.

4.3. Effects of Magnetic Field, Radiation and Porosity on Skin Friction and Rate of Energy Transfer

Table 2. Effect of Magnetic field on local skin friction and rate of energy transfer.

M	$Cf_x Re_x^{\frac{1}{2}} \times 10^{-2}$	$Nu_x Re_x^{-\frac{1}{2}}$
0.0	1.785	0.5068
0.3	1.731	0.5128
0.6	1.678	0.5189
0.9	1.624	0.5250

Table 3. Effect of Rd on local skin friction and rate of energy transfer.

Rd	$Cf_x Re_x^{\frac{1}{2}} \times 10^{-2}$	$Nu_x Re_x^{-\frac{1}{2}}$
0.1	1.767	0.50879
0.2	1.749	0.51082
0.3	1.731	0.51284
0.4	1.714	0.51487

Tables 2-4 reveals the effect of magnetic field parameter, Radiation parameter and porosity on local skin friction and rate of energy transfer. The skin friction coefficient decreases with increase in magnetic field strength, radiation energy and porosity. The rate of energy transfer is elevated with a rise of magnetic field, radiation and porosity.

Table 4. Effect of porosity on local skin friction and rate of energy transfer.

δ	$Cf_x Re_x^{\frac{1}{2}} \times 10^{-2}$	$Nu_x Re_x^{-\frac{1}{2}}$
0.01	1.782	0.50697
0.02	1.781	0.50717
0.03	1.780	0.50737
0.04	1.778	0.50757

5. Conclusion

This study examined the magnetohydrodynamic flow of a (Cu/Ag – glycerine) hybrid nano-coolant based on the Tiwari-Das nanofluid model. The partial differential equations governing the flow were formulated and transformed into ordinary differential system of equations by applying appropriate transformation variables. The resultant ODEs were solved numerically using the fourth-order Runge-Kutta method with a shooting technique. The results were indicated graphically and in tabular form. Matlab ode45 function was deployed. The following results were obtained from the study:

- 1) The velocity lowers with the growth of magnetic strength and Prandtl number while it escalates with Darcy, radiation and Grashoff number.
- 2) While Darcy and Prandtl numbers diminish temperature, magnetic strength, radiation and Eckert numbers extends it.
- 3) The positive deviation of magnetic strength, radiation and porosity contracted the local skin coefficient.
- 4) The rate of energy transfer was elevated with the expansion of magnetic field, radiation and porosity.

For use of this nanofluid as a coolant time contact with the heat source is increased with magnetic field and Prandtl number at low radiation, Darcy and Grashof numbers. Larger Darcy and Prandtl numbers minimize overheating of a system. Opposite trend is registered with the hike of magnetic strength, radiation and viscosity. While the reduction of these parameters improve the slipperiness of the nanofluid, their rise enhances the rate of energy migration. Further study can be extended by investigating unsteady flow past a vertical sheet using different nanoparticles.

Abbreviations

MHD	Hydromagnetic
ODE	Ordinary Differential Equation
MATLAB	Matrix Laboratory
PV	Photovoltaic Cell

Conflicts of Interest

The authors declare no conflicts of interest.

Appendix: Greek Letters and Symbols

Cu	Copper
Ag	Silver
k_c, k_a	Thermal conductivity of copper and silver ($Wm^{-1}K^{-1}$)
k_{hnf}, k_{bf}	Thermal conductivity of hybrid nanofluid and base fluid ($Wm^{-1}K^{-1}$)
$k^*, \bar{k}_{bf}$	Mean absorption coefficient of combines and base fluid (m^{-1})
C_c, C_a	Fitting constants of copper and silver (m^{-1})
q	Heat flux (W/m^2)
Nu_x	Nusselt number
C_f	Skin friction N/m^2
Re_x	Reynolds number
M	Hartman number
Pr	Prandtl number
Rd	Radiation (J)
Ec	Eckert number
u, v	Velocity in the x and y directions (m/s)
u_∞	Far field velocity (m/s)
T, T_∞	Temperature near and far the plate (K)
B_o	Initial magnetic field (A/m)
k_w, k_g	Thermal conductivity of water and silver ($Wm^{-1}K^{-1}$)
T_o	Initial temperature (K)
x	Distance along x (m)
L	Length of the plate (m)
$C_{p,bf}$	Specific heat capacity of the base fluid (Jkg^{-1}, K^{-1})
$\rho_{bf}, \rho_w, \rho_g, \rho_{hnf}$	Density of the base fluid, water, glycerin and hybrid nanofluid (kgm^{-3})
μ_{bf}, μ_w, μ_g	Viscosity of base fluid, water and glycerin (Pa.s)
ϕ_c, ϕ_a	Volume Fraction of Copper and Silver
ρ_c, ρ_a	Density of Copper and Silver
$(C_p)_{hnf}$	Specific Heat Capacity of hybrid nanofluid
ϕ_{hnf}	Volume Fraction of Hybrid Nanofluid
σ_c, σ_a	Electrical conductivity of copper and silver (S/m)
$\beta_{hnf}, \beta_{bf}, \beta_c, \beta_a$	Thermal Expansion of Hybrid, Base, Copper and Silver (K^{-1})
ψ	Free Stream Function
η	Similarity Variable

θ	Temperature Variable
τ	Shear Wall Stress (Nm^{-2})
γ	Kinematic Viscosity (m^2s^{-1})
δ	Darcy Number
λ	Grashof Number

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