

Research Article

# Application of Fractal Dimension for Cardiac Arrhythmias Classification

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## Abstract

Fractal analysis is crucial for understanding complex, irregular patterns found in nature, finance, and various scientific fields. It helps to reveal self-similarity, where structures repeat at different scales, providing insights into chaotic systems like weather patterns, stock markets, and biological growth. By applying fractal analysis, researchers can model phenomena that traditional geometric methods cannot easily describe, enabling better predictions and deeper comprehension of dynamic systems. The Fractals are a fascinating mathematical tool for modeling the roughness of nature and understanding structure of such complex objects. They are considered a tool for understanding the world. In general, fractal objects are characterized by the fractal dimension. The application of fractal geometry to the analysis of ECG time series data is examined in this paper. A method based on the assessment of the Fractal Dimension (FD) of ECG recordings is suggested for the identification of cardiac diseases. In this work, and in order to exploit the fractal dimension to analyze fractal signals, the notion of fractal dimension is defined by presenting methods for calculating this dimension such as Higuchi algorithm, Katz method, regularization, box-counting etc... Each of them has its own advantages and disadvantages. This study has shown that the electrocardiogram (ECG) is a fractal signal. This allows to classify heartbeats founded on the concept of fractals. The main aim is to develop a digital technique to analyze ECG signals in order to make an accurate diagnosis of cardiovascular diseases.

## Keywords

Fractal Dimension, Fractal Signal, Electrocardiogram Signal, Classification of Heart Diseases, MIT/BIH Database

## 1. Introduction

The concept of fractal dimension finds extensive applications across diverse fields. In physics, it helps in understanding complex physical systems like turbulence, diffusion processes, and chaotic systems. In medicine, it aids in analyzing biological structures like blood vessels or the intricacies of the human brain. Fractal dimension also finds utility in computer graphics, where it is used to create visually appealing and realistic landscapes by generating intricate natural

patterns.

One of the key characteristics of fractal dimension is its non-integer nature. Unlike regular geometric objects, fractals possess fractional dimensions, indicating their intricate and self-similar structures. This property allows us to grasp the complexity and self-replicating nature of various natural phenomena, such as coastlines, clouds, and biological systems. [1, 2].

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In essence, fractal dimension goes beyond traditional Euclidean dimensions such as length, width, and depth, by quantifying the degree of irregularity or complexity within an object. It allows to describe and analyze complex phenomena that cannot be adequately captured by simple integers and introduces a more nuanced understanding of the intricate patterns and formations found in the world around us. [1]

Fractal dimension is a fundamental concept that plays a crucial role in various fields. Defined as the measure of complexity or self-similarity of an object, it provides a valuable tool for understanding the intricate structures found in nature, mathematics, and even computer graphics. The concept of fractal dimension originates from the study of fractals, which are geometric shapes that exhibit infinite detail and complexity at all levels of magnification.

The fractal dimension quantifies the degree of fluctuation in temporal data, enabling analysis. From there, we may discuss comparison, categorization, and prediction. Resistance to variations in time or space dilation is a crucial characteristic of a fractal signal. These signals can typically be one-dimensional (for example fractal time series) or multi-dimensional (for example fractals from natural terrain models). Additionally, they might have continuous or discrete amplitude as well as a discrete or continuous character. Our research aims to investigate the relationship between a fractal signal and a fractal dimension by defining the ways in which fractal properties might be utilized for analytical purposes.

For these reasons we have provided the electrocardiogram (ECG) as an example of a fractal signal. Since cardiovascular diseases are one of the leading causes of death worldwide, as reported by the annual statistical studies carried out at the level of the WHO (World Health Organization), we have chosen this example. The ECG signal continues to be one of the tests used most frequently to identify cardiovascular diseases; it is one of the most common and extensively utilized tools for the analysis and diagnosis of cardiac arrhythmias. The cardiac diseases are regarded as the prevalent cause of death globally. The data indicates a growing number of deaths, which contributes to cardiovascular failure. Globally, 17.3 million people died in 2008, were caused by this drawback [3], according to the World Health Organization. By 2030, this figure will rise to 23.3 million people. Furthermore, countries with moderate to low per capita income have accounted for roughly 80% of fatality cases, with the majority of these cases being the result of de-

layed or incorrect pathology diagnosis. The ECG signal is one of the most reliable and affordable methods for identifying cardiac conditions, hence it is useful for assessing the health of the heart [4]. The cardiac electrical activity is displayed on the electrocardiogram (ECG). This activity and T waves are directly associated. The T wave, which represents ventricular repolarization, the P wave, which indicates atrial depolarization, and the QRS complex, which reflects ventricular depolarization, are used to identify them.

These distinct waves typically occupy distinct temporal locations and frequency spectra for the identical signal originating from the same patient, whether they are in good health or have a heart condition. While many disorders can be identified and interpreted with ECG signal estimation, other anomalies cannot be detected with this method alone. Therefore, we require a more dependable approach, which is authorized to evaluate an ECG signal.

The purpose of this study is to classify heartbeats and to provide a way for optimizing the usage of ECG data. The heart's fractal structure makes this approach fundamentally reliant on the fractal idea.

## 2. Methodology

The two main parts of the process are the classification and the fractal dimension calculation. Finding the fractal dimension is the first step. The categorization of arrhythmias is the focus of the final phase.

### 2.1. Fractal Characterization of ECG Signal

#### 2.1.1. ECG Presentation

Heart electrical activity is shown on the electrocardiogram (ECG). The occurrence of mechanical and electrical activity is determined by electrical activation of a muscle cell. The cell surface quickly depolarizes in response to stimulation, creating an electric current that leads to contraction. At the base of the superior vena cava, at the so-called Keith and Flack node (denoted K and F in the figure), the ECG activation wave begins in the right atrium. The two ventricles (RV and VG) are ultimately reached by this wave after it passes via the two atria and the atrioventricular node, also known as the Aschoff-Tawara node (shown as AT in the figure) [5].

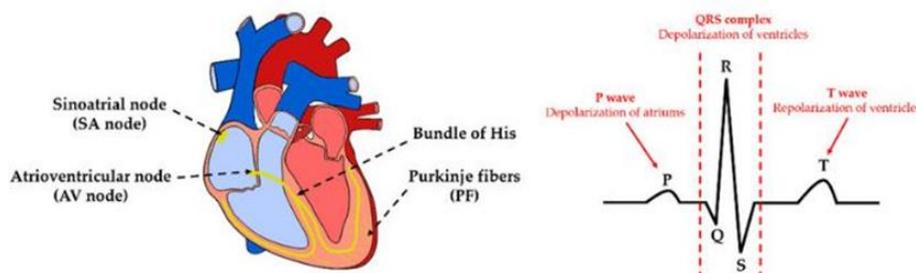


Figure 1. The ECG activation wave's trajectory.

Electrodes positioned at certain locations throughout the body can collect the processes of depolarization and repolarization that result from this activation wave in the cardiac cells. This results in the general ECG waveform that is depicted in the figure below:

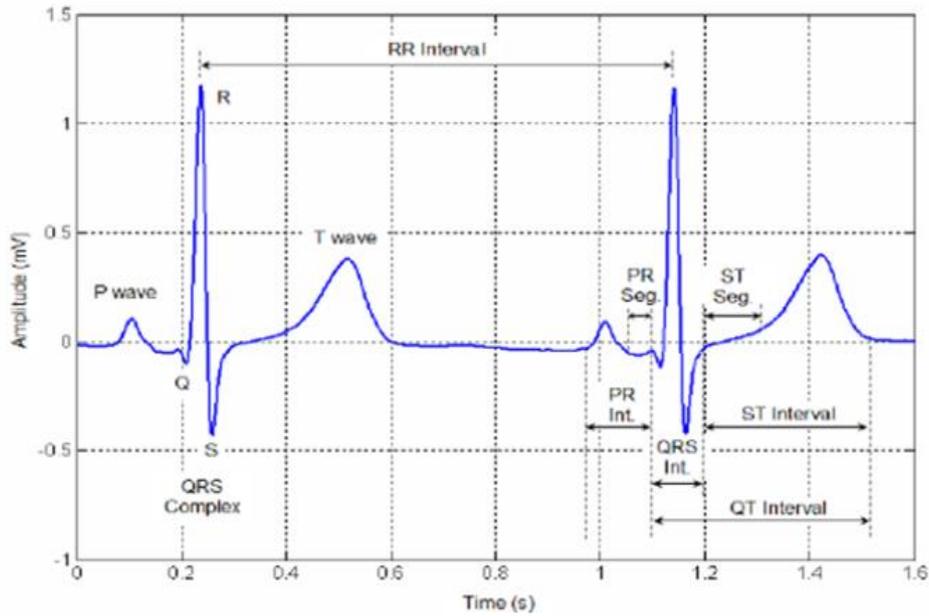


Figure 2. ECG signal presentation.

### 2.1.2. The Heart's Fractal Structure

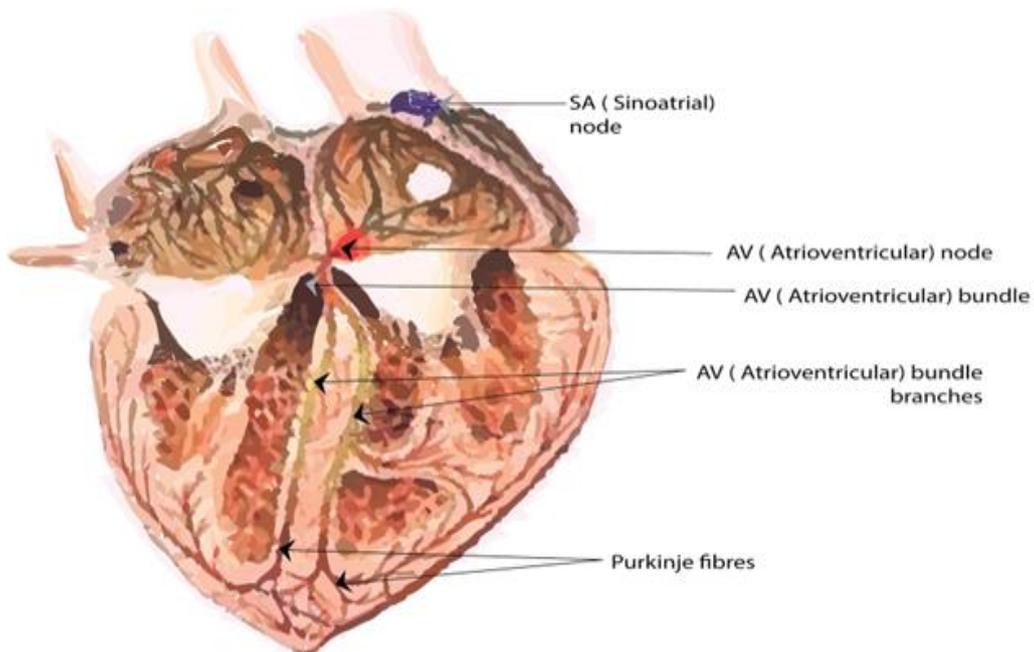


Figure 3. Fractal structure of the heart.

When the heart is beating normally, the bundle of His and Purkinje triggers the ventricular myocardium. The structure of fine Purkinje fibers is fractal and they are highly branching [6]. Research has demonstrated that Purkinje fiber activation of the ventricles causes fractal depolarization of the ventricles. Moreover, the QRS complex reflects this depolarization.

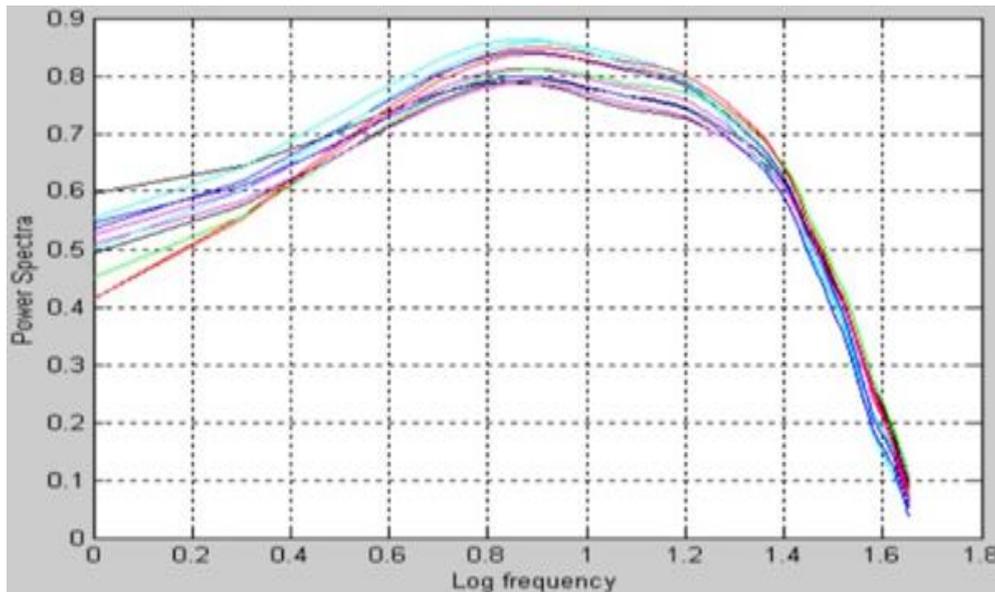


Figure 4. A normal QRS's power spectrum.

The average power spectrum of many QRS complexes from various individuals is plotted logarithmically in the above figure. The decay in  $1/f^\beta$  is displayed in this plot, where  $f$  is the frequency and  $\beta$  is the fractal dimension. It was determined that The QRS complex's spectrum exhibits an inverse power law, and that this spectrum has a fractional slope. The fractal structure of the cardiac conduction network, particularly the finely branching Purkinje fibers, provided justification for this characteristic.

## 2.2. MIT/BIH Database

The signals taken from the MIT/BIH database are utilized. It consists of many databases; It is regarded as the most popular one, which serves as the foundation for MIT/BIH arrhythmia [7]. There are 48 recordings of ECG signals in this database, each lasting roughly half an hour, from 47 distinct patients. Additionally, 200 samples per mV are the resolution and 360 Hz is the sampling frequency. Additionally, Every record contains around 109,000 beats, which are divided into 15 different types of heartbeats. According to the MIT-BIH, which forms the basis of the arrhythmia class, the various diseases are separated. Approximately 70% of the beats belong to the class of (NOR) regular rhythms, which is the most overflowing class.

## 2.3. Fractal Dimension Estimation

One of the key ideas for characterizing fractal objects is the fractal dimension. It looks at the ecumenical index that measures how complex they are. [8]. "If a fractal can be described as a union of sets, each of which is an exact reduced duplicate of the whole set (Koch flak, Sierpinski triangle), then it is rigorously self-similar. Even the most fractal-looking things in nature lack this exact form. Natural items aren't only

accurate reduced replicas of their original forms. Even while a magnified image of one portion won't exactly replicate the entire thing, it will still seem the same in terms of quality. This characteristic is known as semi-self-similarity or statistical self-similarity. Since the human heart's ECG signal is a self-similar item. Thus, a fractal dimension is required." [9]

## 2.4. Techniques to Compute the Fractal Dimension

Each table should have a Numerous approaches exist for determining the (FD) Fractal Dimension, including Higuchi [10], Katz [11], Regularization [12], box-counting [13] etc... Each of them has its own advantages and disadvantages.

### 1. The Higuchi algorithm

One descriptive metric that has been shown to be helpful in assessing the self-similarity or complexity of biological signals is fractal dimension. Since the ECG signal from a human heart is a self-similar object, it needs to have a fractal dimension that can be identified and distinguished between different states of cardiac pathological situations using mathematical techniques [12]. Numerous algorithms exist for determining the Fractal Dimension, as was mentioned in the preceding paragraph. Higuchi is a reliable technique and a good disease representative, according to the information given.

Presume a time sequence of  $x=\{x(1), x(2), \dots, x(N)\}$ :

The FD is determined in this way::

a) Create  $K$  new time series  $x_m^k$  are described as:

$$x_m^k = \{(m), \dots, (m + [(N - m)/k]k)\} \quad (1)$$

Where  $K$  represents the discrete time interval between points, and  $m=1, 2, \dots, K$  displays the beginning time value.

b) As previously specified, determine the length of every new time series.

$$L_m(k) = 1/K \{ (N-1) / [(N-m)/K] \sum |x(m+ik) - x(m+(i-1)k)| \} \quad (2)$$

where a normalization factor is  $(N-1) / [(N-m)/K]$ .

c) Determine the curve's length of  $K$  the interval of time.

$$L(k) = 1 / K \sum_{m=1}^k L_m(k) \quad (3)$$

d) Lastly,  $D$  is the representation of the Fractal Dimension curve in the following equation.

$$(L(k)) = D (1/K) + \bar{b} \quad (4)$$

### 2. Katz algorithm.

The time series is used to determine the fractal dimension, which has the following definition:

$$DF_K = (\text{Log}(L)) / (\text{Log}(d)) \quad (5)$$

Where  $d$  is the (Euclidean) distance between the series' first point and the point that offers the furthest distance in relation to the first point, and  $L$  is the length of the ECG time series overall. The entire length,  $L$ , of waveforms, which are ordered sets of pairs of points  $(x, y)$ , is equal to the sum of the distances between each subsequent point.

$$L = \text{sum} (\text{dist} (i, i + 1)) \quad (6)$$

When the distance between two pairs of coordinates,  $a = (x, y)$  and  $b = (u, v)$ , is denoted by the symbol  $\text{dist} (a, b)$ . Waveforms that advance monotonically have inherent beginnings. Thus, the distance that separates a waveform's starting point (point 1) from any other point (point  $i$ ) on the waveform is its plane extent, also known as its diameter; namely

$$d = \max (\text{dist} (1, i)) \quad (7)$$

### 3. Hausdorff algorithm

Covering the curve with balls  $B_i$  whose diameter ( $\text{diam } B_i$ ) is equal to or less than  $\epsilon$  is the method used. The approximate measurement of the entire object can be obtained by adding together the diameters of the balls. We take into consideration the minimum of these dimensions since these balls do not intersect in an empty space. We tend  $\epsilon$  towards 0 in order to get the best estimate of  $E$  by these overlaps. The Hausdorff dimension can be found using

$$DF_H = \lim_{\delta \rightarrow 0} (\ln N_\delta / \ln(1/\delta)) \quad (8)$$

Where  $N_\delta$  is the bare minimum of balls with a diameter of  $\epsilon$  required to pave the curve.

### 4. Approximation of the PSD of the QRS complex

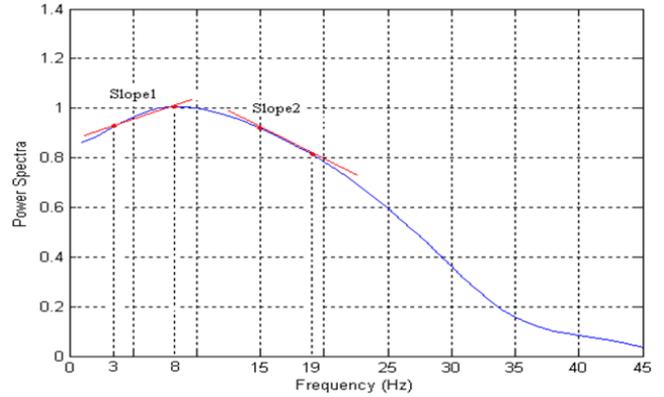


Figure 5. Approximation of the PSD of the QRS complex.

The preceding figure depicts the PSD of a QRS complex as well as these approximations by two straight within the two distinct frequency ranges of [15Hz, 19Hz] and [3Hz, 8Hz]. The two slopes of the straight lines are calculated in these two frequency intervals. After calculation we find the first slope equal to 0.02 noted  $p_1 = 0.02$  and the second equal to -0.02 noted  $p_2 = -0.02$ . Each slope corresponds to the fractal dimension.

## 3. Results and Discussion

The fractal dimension of an ECG refers to a measure of the irregularity or complexity present in the signal. This measure helps in measuring the ECG waveform's self-similarity at various scales. It can offer insightful knowledge on the underlying dynamics of the cardiac system and aid in diagnosing certain heart conditions. Several methods can be used to calculate the fractal dimension of an ECG signal.

These include algorithms like Katz, Higuchi, Petrosian, Maragos, and the amplitude scale approach [14]. By applying these algorithms, researchers can accurately estimate the fractal dimension of the ECG signal and classify them accordingly. Kouros Kiani and Farzane Maghsoudi have examined and reviewed these techniques as well as others [15]. It has been mentioned that the Higuchi approach is precise and an excellent representation of the disease. In this study, Matlab is used to determinate the fractal dimension applying the Higuchi technique. From the raw electrocardiogram (ECG) data, the estimated (FD) is used to distinguish between the signals of individuals in good health and those with Premature Ventricular Complex (PVC), Paroxysmal Supraventricular Tachycardia (PSVT), and Premature Atrial Contracture (PAC).

Therefore, it is feasible to differentiate between a disease and a normal instance according to the fractal dimension's value. Furthermore, it is possible to determine if it is a PVST, PAC, or PVA. Thus, the primary goal of this research is to accurately categorize a range of arrhythmias using a fractal dimension.

The ECG signals can be classified into categories that follow

using the fractal dimension and the findings:

PVC:  $1.3 < FD \leq 1.37$ ; Normal:  $FD > 1.56$ ;

PAC:  $1.37 < FD \leq 1.56$ ; PSVT:  $1 < FD \leq 1.3$

Therefore, it is conceivable to distinguish between a disease and a normal instance according to the value of the fractal dimension. Furthermore, this value can determine if it is a PVST, PAC, or PVA. Thus, the primary goal of this research is to use a fractal dimension to accurately classify a range of arrhythmias.

## 4. Conclusion

The fractal dimension of an ECG signal is a useful tool for analyzing and quantifying its complexity. This work aims to determine the (FD) fractal dimension for the purpose of categorizing various cardiac diseases. Fractal analysis tools are assumed for the analysis of irregular signals which are fractal signals and we take the example of ECG signals. It is well noted that the description and analysis of these signals depend heavily on the characterization of the local regularity.

## Abbreviations

FD	Fractal Dimension
ECG	Electrocardiogram
PVC	Vickers Hardness Ventricular Complex
PSVT	Paroxysmal Supraventricular Tachycardia
PAC	Premature Atrial Contracture
DS	Shrinkage According Diameter
HS	Shrinkage According Height

## Data Availability Statement

The MIT-BIH Arrhythmia Database, used in this study, is publicly available from PhysioNet (<https://physionet.org/content/mitdb/1.0.0/>). The MIT-BIH Arrhythmia Database is a useful resource. It consists of electrocardiogram (ECG) recordings with annotated rhythms. Researchers interested in replicating or extending our work can access the MIT-BIH Arrhythmia Database through the provided link. We urge researchers to follow PhysioNet's guidelines for data usage and to appropriately credit the original dataset as well as any pertinent articles used in conjunction with it.

## Conflicts of Interest

The authors declare no conflicts of interest.

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