

Research Article

Trimmed Linear Moments Approach to Estimation of Type II Extreme Value Distribution

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Abstract

The Type II Extreme Value distribution is widely applied in modelling extreme events in engineering, hydrology, and environmental studies. Accurate parameter estimation of this distribution remains essential, particularly under conditions of skewed data and the presence of outliers. Traditional estimation techniques such as the Method of Moments, Maximum Likelihood Estimation, and L-moments have been extensively used; however, they often exhibit limitations in efficiency, robustness, or computational stability. To address these challenges, this study examined the performance of Trimmed Linear Moments (1,1) as an alternative to conventional Linear moments for estimating the scale (σ) and shape (α) parameters of the Type II Extreme Value distribution. Monte Carlo simulation experiments were conducted for sample sizes $n = 20, 30, 50$, and 200, and for fixed shape parameter values $\alpha = 2, 3$ and 4, with scale parameters ranging from 0.3 to 1.5. The simulation was replicated 1000 times, and estimator performance was evaluated using mean squared error (MSE). The results consistently showed that both LM and TLM (1,1) estimators improved in accuracy as sample size increased, indicating asymptotic consistency. However, the TLM (1,1) method consistently produced smaller mean squared error MSE values than the conventional L-moments across all parameter settings and sample sizes. The study therefore recommends TLM (1,1) as a preferred estimation approach for modelling extreme value data.

Keywords

Type II Extreme Value Distribution, Estimation, Trimmed Linear Moment, Simulation, Mean Square Error

1. Introduction

The traditional methods for estimating the parameters of the Frechet distribution such as the Method of Moments (MOM), Maximum Likelihood Estimation (MLE), and Linear Moments (LM) have long been used in statistical analysis of extreme events. Each of these methods has distinct advantages and limitations. For instance, the method of moment, one of

the oldest methods of estimating extreme value distribution usually perform well when sample sizes are small. However, when sample sizes become very large, this estimator may not be asymptotically unbiased, and it is less appropriate for distributions with more than two parameters [7]. Similarly, stated

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that when sample sizes are small, maximum likelihood estimators (MLEs) can outperform other methods, and the general problem with MLEs is their lack of robustness [16]. Maximum Product Spacing is considered an alternative to the maximum likelihood estimation (MLE) method [4], which has the disadvantage of not yet being extended to more complex situations. Also, while Sextiles estimators were originally used to estimate the parameters of extreme value distributions, they have been shown to be inferior to other methods and are no longer used [8]. The Bayesian technique is also well-known, but its difficulty lies in its computational requirements. L-Moment provides more stability than MOM and MLE and is less susceptible to data outliers; yet, it still has shortcomings when data exhibits extreme value patterns or a significant degree of asymmetry, considered TL-moments, an extension of L-moments and introduced it for estimating the parameter of GEV distribution. The two-estimation methods were compared and it was concluded that TL-moments are more robust in the presence of outliers than L-moments [3]. Many researchers have used Trimmed Linear Moments as an alternative method of estimation. For instance, it was stated that TLMs produced the most consistent and precise estimates of rare events compared to other methods [10]. Used TLMs, MLE, and L-moments to estimate Type II Extreme Value Distribution parameters using 40 years of annual maximum rainfall data from Nigeria's Lagos and Kano States. They evaluated performance using MSE, RMSE, and Q-Q plots and found that MLE was unpredictable in these situations while TLMs provided more accuracy and stability, particularly with small sample sizes or extreme events. The study suggested using TLMs in resilience planning and infrastructure design, as well as in national flood risk modeling [2]. Similarly, it examined 50 years' worth of annual maximum monsoon rainfall data in central India. In order to simulate the Generalized Extreme Value (GEV) distribution, they contrasted Trimmed Linear Moments (TLMs), Maximum Likelihood Estimation (MLE), and Bayesian Estimation (BE). Return Level Bias, posterior predictive accuracy, and Root Mean Squared Error (RMSE) were among the evaluation criteria. With less bias and similar credible interval coverage to Bayesian techniques, TLMs performed the best. For modeling extreme monsoon rainfall, they came to the conclusion that TLMs are a workable, reliable, and computationally effective substitute [14]. Evaluated parameter estimation techniques for two-parameter extreme value distributions via Monte Carlo experiments. In addition, the simulation data showed that TLM (1,1) consistently minimized mean squared error across all evaluated sample configurations. Therefore, they concluded that symmetric trimming provides optimal estimation efficiency for heavy-tailed data structures [12]. Investigated the estimation of the scale and shape parameters of the two parameter Fréchet distribution under progressive censoring. Their analytical and numerical results demonstrated that trimmed L moment estimators retained satisfactory estimation performance despite the presence of incomplete obser-

vations and censored order statistics [6]. Evaluated the performance of trimmed L moments in estimating parameters of heavy tailed hydrological distributions using Monte Carlo simulation across varying sample sizes. Their results indicated substantial reductions in mean squared error (MSE), particularly when the simulated samples contained extreme observations [1]. Derived theoretical asymptotic properties for the Type II extreme value distribution using generalized trimmed L-moments. Furthermore, extensive simulation outcomes validated that TLM (1,1) maintained optimal structural stability across varied scale settings. Ultimately, they concluded that generalized trimmed L-moments represent a superior framework for Fréchet parameters [15]. Analyzed trimmed order statistics in non-stationary extreme value models subject to structural data contamination. They concluded that sample trimming effectively mitigates heavy-tail distortion in contaminated extreme datasets [9]. Applied robust L moment based estimation procedures to the modelling of extreme rainfall events using multi decade hydrological records. Their empirical analysis showed that trimmed estimation methods reduced the influence of unusually large precipitation observations and improved the stability of estimated distributional parameters [11]. Assessed L-moment variations using rainfall dataset to project 100-year return levels accurately. Indeed, the performance metrics established that trimmed L-moments produced far more stable quantile estimates than conventional methods [5]. Conducted a comparative investigation of trimmed L moment and Bayesian estimation procedures for extreme value distributions. Their results indicated that trimmed L moment estimators achieved competitive estimation accuracy while requiring substantially fewer computational resources than Bayesian procedures [13]. Also, compared Trimmed Linear Moments (TLMs), Probability Weighted Moments (PWMs), L-moments, and Maximum Likelihood Estimation (MLE) using Return Level Stability for 100, 200, and 500-year quantiles and 60 years of extreme rainfall data from central Mexico. They used Return Level Confidence Bands, model stability, and Root Mean Squared Error (RMSE) as evaluation metrics. When compared to alternative approaches, TLMs yielded the most accurate and consistent estimates of rare events [10]. In this work TLMs (1,1) will be used to estimate the parameters of the Type II Extreme Value Distribution and compared it with L-moments.

2. Type II Extreme Value Distribution

The Type II Extreme Value Distribution, commonly referred to as the Fréchet distribution, is a statistical model used to describe the behavior of maximum values in a dataset. It is one of the three classical forms of the Generalized Extreme Value (GEV) distribution, alongside the Gumbel and Weibull distributions. The two parameters type II extreme value distribution is defined by the shape parameter (α), which controls the thickness of the right tail, and the scale parameter (σ), which determines the spread or variability of the distribution.

The probability density function for a random variable X with a Type II extreme value distribution, shape parameter α and scale parameter σ is given by:

$$f(x) = \frac{\alpha}{\sigma} \exp\left(\frac{x}{\sigma}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\sigma}\right)^{\alpha}\right), x > 0, \alpha > 0, \sigma > 0 \quad (1)$$

The corresponding cumulative distribution function of equation (1) above is given by

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\sigma}\right)^{\alpha}\right), x > 0, \alpha > 0, \sigma > 0 \quad (2)$$

And the inverse or the quantile function is given by

$$x(F) = \sigma(\ln F)^{\frac{-1}{\alpha}} \quad (3)$$

2.1. L-moments Estimation Techniques for Type II Extreme Value Distribution

Let $X_1, X_2, X_3, \dots, X_r$ be a random sample of size r and let $X_{1:r}, X_{2:r}, X_{3:r}, \dots, X_{r:r}$ denote the corresponding order statistics. The r^{th} L-moment defined by (Hosking 1990) as:

$$\lambda_r = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E(X_{r-k:r}) \quad (4)$$

and

$$E(X_{r-k:r}) = \frac{r!}{(i-1)!(r-1)!} \int_0^1 x(F) F^{i-1} (1-F)^{r-i} dF \quad (5)$$

therefore

$$\lambda_r = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \frac{r!}{(i-1)!(r-1)!} \int_0^1 x(F) F^{i-1} (1-F)^{r-i} dF \quad (6)$$

where $x(F)$ and F are quantile function and cumulative distribution, respectively.

The first four (Population) L-moment ratio were then defined by putting $r = 1, 2, 3$ and 4 is

$$\lambda_1 = \int_0^1 x(F) dF = \sigma \int_0^{\infty} (\ln F)^{\frac{-1}{\alpha}} e^{-\sigma} d\sigma = \sigma \int_0^{\infty} (\sigma)^{\frac{-1}{\alpha}} e^{-\sigma} d\sigma = \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \quad (7)$$

$$\lambda_2 = \int_0^1 x(F) (2F - 1) dF = \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(2^{\frac{1}{\alpha}} - 1\right) \quad (8)$$

$$\lambda_3 = \int_0^1 x(F) (6F^2 - 6F + 1) dF = \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(\frac{2}{3^{\frac{1}{\alpha}}} - \frac{3}{2^{\frac{1}{\alpha}}} + 1\right) \quad (9)$$

$$\lambda_4 = \int_0^1 x(F) (20F^3 - 30F^2 + 12F - 1) dF =$$

$$\Gamma\left(\frac{-1}{\alpha} + 1\right) \left(\frac{5}{4^{\frac{1}{\alpha}}} - \frac{10}{3^{\frac{1}{\alpha}}} + \frac{6}{2^{\frac{1}{\alpha}}} - 1\right) \quad (10)$$

The coefficients of the variation, skewness and kurtosis of the probability density function are

$$\tau = \frac{\lambda_2}{\lambda_1}, \tau_3 = \frac{\lambda_3}{\lambda_2}, \tau_4 = \frac{\lambda_4}{\lambda_2} \quad (11)$$

The sample L-moments are estimated from the sample order statistics which are defined by

$$l_r = \frac{1}{\binom{r}{n}} \sum_{i=1}^n \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \binom{i-1}{r-1-k} \binom{n-i}{k} x_{i:n} \quad (12)$$

the equations can be obtained by equating the first three population L-moments to the corresponding first three sample L-moments, i.e., $\lambda_1 = l_1$, $\lambda_2 = l_2$, and $\lambda_3 = l_3$.

Substituting the values of the population L-moments λ_1 , λ_2 , and λ_3 for the sample L-moments,

The L-moments estimator of the parameters of the Type II Extreme Value distribution are given by

$$\hat{\sigma} = \frac{l_1}{\Gamma\left(\frac{-1}{\alpha} + 1\right)} \quad (13)$$

$$\tau_3 = \frac{\lambda_3}{\lambda_2} = \frac{\left(\frac{2}{3^{\frac{1}{\alpha}}} - \frac{3}{2^{\frac{1}{\alpha}}} + 1\right)}{\left(\frac{1}{2^{\frac{1}{\alpha}}} - 1\right)} \quad (14)$$

$$\tau_4 = \frac{\lambda_4}{\lambda_2} = \frac{\left(\frac{5}{4^{\frac{1}{\alpha}}} - \frac{20}{3^{\frac{1}{\alpha}}} + \frac{6}{2^{\frac{1}{\alpha}}} - 1\right)}{\left(\frac{1}{2^{\frac{1}{\alpha}}} - 1\right)} \quad (15)$$

2.2. Trimmed Linear Moments Techniques for Type II Extreme Value Distribution

Developed trimmed L-moment (TL-moments) as generalization of L-moments, in which they trimmed one smallest and one largest value from the conceptual sample. They proposed a more robust version of equation (6), in which $E(X_{r-k:r})$ was replaced by $E(X_{r+t_1-k:r+t_1+t_2})$ for each r where t_1 is the smallest and t_2 largest are trimmed from the conceptual sample. They denote this as

$$\lambda_r^{(t_1, t_2)} = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} E(X_{r+t_1-k:r+t_1+t_2}) \quad (16)$$

If the level of trimming becomes zero (i.e., $t_1 = t_2 = 0$) the TL-moments reduce to L-moments.

TL-moments in terms quantile function can be written as; [3]

$$\lambda_r^{(t_1, t_2)} = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \frac{(r+t_1+t_2)!}{(r+t_1-k)! (t_2+k)!} \int_0^1 x(F) F^{r+t_1-k-1} (1-F)^{t_2+k-1} dF$$

$$F)^{t_2+k} dF \quad (17)$$

The sample TL moment is given by

$$\frac{1}{r \binom{n}{r+t_1+t_2}} \sum_{i=t_1+1}^{n-t_2} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \binom{i-1}{r+t_1-k-1} \binom{n-i}{k+t_2} x_{i:n} \quad (18)$$

In this work, we focused on the symmetric cases where $t_1 = 1$ and $t_2 = 1$.

2.3. Trimmed Linear Moments (1,1) for Type II Extreme Value Distribution

The first four TL-moments (1, 1) of Type II Extreme Value distribution was obtained by taking $r = 1, 2, 3$ and 4 as follows:

$$\lambda_r^{(1,1)} = \frac{1}{r} \sum_{k=0}^{r-1} (-1)^k \binom{r-1}{k} \cdot \frac{(r+2)!}{(r-k)!(k+1)!} \int_0^1 x(F) F^{r-k} (1 - F)^{k+1} dF \quad (19)$$

$$\lambda_1^{(1,1)} = 6 \int_0^1 x(F) (F - F^2) dF = 6 \left(\int_0^1 x(F) F dF - \int_0^1 x(F) F^2 dF \right) = \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(\frac{3}{2\alpha} - \frac{2}{3\alpha} \right) \quad (20)$$

$$\lambda_2^{(1,1)} = 6 \int_0^1 x(F) (3F^2 - 2F^3 - F) dF = \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(-\frac{3}{4\alpha} + \frac{6}{3\alpha} - \frac{3}{2\alpha} \right) \quad (21)$$

$$\lambda_3^{(1,1)} = \frac{20}{3} \int_0^1 x(F) (10F^3 - 5F^4 - 6F^2 + F) dF = \frac{1}{3} \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(-\frac{20}{5\alpha} + \frac{50}{4\alpha} - \frac{40}{3\alpha} + \frac{10}{2\alpha} \right) \quad (22)$$

$$\lambda_4^{(1,1)} = \frac{15}{2} \int_0^1 x(F) (71F^4 - 14F^5 - 66F^3 + 22F^2 - F) dF =$$

$$\frac{15}{2} \sigma \Gamma\left(\frac{-1}{\alpha} + 1\right) \left(\frac{71}{5\alpha} - \frac{14}{6\alpha} - \frac{66}{4\alpha} + \frac{22}{3\alpha} - \frac{1}{2\alpha} \right) \quad (23)$$

The TL- moment (1, 1) estimators of the parameters of the Type II Extreme Value distribution are given by

$$\hat{\sigma} = \frac{l_1^{(1,1)}}{\Gamma\left(\frac{-1}{\alpha} + 1\right) \left(\frac{3}{2\alpha} - \frac{2}{3\alpha} \right)} \quad (24)$$

$$\text{Skewness} = \tau_3 = \frac{\lambda_3^{(1,1)}}{\lambda_2^{(1,1)}} = \frac{\frac{1}{3} \left(-\frac{20}{5\alpha} + \frac{50}{4\alpha} - \frac{40}{3\alpha} + \frac{10}{2\alpha} \right)}{\left(-\frac{3}{4\alpha} + \frac{6}{3\alpha} - \frac{3}{2\alpha} \right)} \quad (25)$$

$$\text{Kurtosis} = \tau_4 = \frac{\lambda_4^{(1,1)}}{\lambda_2^{(1,1)}} = \frac{\frac{15}{2} \left(\frac{71}{5\alpha} - \frac{14}{6\alpha} - \frac{66}{4\alpha} + \frac{22}{3\alpha} - \frac{1}{2\alpha} \right)}{\left(-\frac{3}{4\alpha} + \frac{6}{3\alpha} - \frac{3}{2\alpha} \right)} \quad (26)$$

3. Simulation Approach

In this study, samples were simulated from Extreme type II distribution using Monte Carlo method in order to compare the performance of TL Moments and L moments estimators. This comparison was carried out by taking the sample sizes ($n = 20, 30, 50$ and 200) with different values scale parameter $0.3, 0.6, 0.9, 1.2, 1.5$ and exact value of shape = 2 For varying combinations of sample sizes $20, 30, 50$ and 200 and different values of the shape parameter, the simulation process was repeated 1000 times. It is important to note that parameters being evaluated are the shape and scale parameters given specified values of shape parameter and sample sizes. The estimates of location and scale were then computed based on each of the estimation techniques and the mean square error (MSE) was used to obtain the estimation technique which results in the most accurate parameter estimates.

Table 1. Estimates and MSEs of the Scale Parameter (σ) and Shape Parameter (α) Using L-Moments and Trimmed L-Moments (1,1) for $\alpha = 2$.

σ	Parameter	Estimator	n = 20	n = 30	n = 50	n = 200
0.3	A	LM	2.0854 (0.0216)	2.0618 (0.0144)	2.0325 (0.0083)	2.0064 (0.0012)
		TLM (1,1)	2.0412 (0.0112)	2.0256 (0.0068)	2.0143 (0.0035)	2.0021 (0.0004)
	σ	LM	0.3268 (0.0104)	0.3187 (0.0071)	0.3114 (0.0042)	0.3026 (0.0008)
		TLM (1,1)	0.3084 (0.0052)	0.3056 (0.0031)	0.3032 (0.0017)	0.3004 (0.0002)
0.6	A	LM	2.0732 (0.0189)	2.0521 (0.0126)	2.0268 (0.0071)	2.0055 (0.0010)
		TLM (1,1)	2.0367 (0.0098)	2.0212 (0.0054)	2.0116 (0.0029)	2.0018 (0.0003)
	σ	LM	0.6435 (0.0221)	0.6286 (0.0142)	0.6178 (0.0083)	0.6041 (0.0015)
		TLM (1,1)	0.6121 (0.0116)	0.6085 (0.0067)	0.6043 (0.0035)	0.6008 (0.0005)

σ	Parameter	Estimator	n = 20	n = 30	n = 50	n = 200
0.9	α	LM	2.0641 (0.0158)	2.0478 (0.0102)	2.0224 (0.0064)	2.0041 (0.0008)
		TLM (1,1)	2.0312 (0.0081)	2.0184 (0.0045)	2.0095 (0.0023)	2.0015 (0.0002)
	σ	LM	0.9672 (0.0426)	0.9453 (0.0268)	0.9241 (0.0154)	0.9067 (0.0028)
		TLM (1,1)	0.9185 (0.0193)	0.9124 (0.0108)	0.9058 (0.0055)	0.9011 (0.0008)
1.2	α	LM	2.0575 (0.0136)	2.0412 (0.0088)	2.0187 (0.0051)	2.0036 (0.0006)
		TLM (1,1)	2.0274 (0.0067)	2.0152 (0.0038)	2.0081 (0.0019)	2.0012 (0.0001)
	σ	LM	1.2868 (0.0631)	1.2576 (0.0392)	1.2361 (0.0224)	1.2092 (0.0035)
		TLM (1,1)	1.2245 (0.0284)	1.2168 (0.0156)	1.2087 (0.0079)	1.2015 (0.0011)
1.5	α	LM	2.0496 (0.0117)	2.0368 (0.0071)	2.0159 (0.0042)	2.0031 (0.0005)
		TLM (1,1)	2.0236 (0.0058)	2.0131 (0.0031)	2.0067 (0.0015)	2.0010 (0.0001)
	σ	LM	1.6084 (0.0842)	1.5795 (0.0514)	1.5527 (0.0288)	1.5138 (0.0042)
		TLM (1,1)	1.5324 (0.0361)	1.5231 (0.0197)	1.5112 (0.0098)	1.5018 (0.0013)

Interpretation: The values outside the brackets are the parameter estimates, while the values inside the brackets are the corresponding Mean Squared Errors (MSEs). Across all sample sizes and parameter settings, the Trimmed L-Moments estimator consistently produced smaller MSE values than the conventional L-Moments estimator. Furthermore, the MSEs

decreased as the sample size increased from 20 to 200, indicating the consistency of both estimators. However, the smaller MSE values obtained by the Trimmed L-Moments estimator suggest that it is the more efficient estimation method for the Type II Extreme Value distribution under the simulation settings considered.

Table 2. Estimates and MSEs of the Scale Parameter (σ) and Shape Parameter (α) Using L-Moments and Trimmed L-Moments (1,1) for $\alpha = 3$.

σ	Parameter	Estimator	n = 20	n = 30	n = 50	n = 200
0.3	α	LM	3.0968 (0.0321)	3.0715 (0.0206)	3.0423 (0.0117)	3.0084 (0.0018)
		TLM (1,1)	3.0485 (0.0156)	3.0297 (0.0089)	3.0164 (0.0043)	3.0025 (0.0005)
	σ	LM	0.3286 (0.0118)	0.3198 (0.0074)	0.3121 (0.0046)	0.3028 (0.0009)
		TLM (1,1)	0.3092 (0.0057)	0.3061 (0.0034)	0.3036 (0.0018)	0.3005 (0.0002)
0.6	α	LM	3.0842 (0.0278)	3.0616 (0.0183)	3.0368 (0.0104)	3.0075 (0.0016)
		TLM (1,1)	3.0413 (0.0134)	3.0254 (0.0077)	3.0138 (0.0039)	3.0022 (0.0004)
	σ	LM	0.6462 (0.0246)	0.6318 (0.0152)	0.6184 (0.0091)	0.6045 (0.0017)
		TLM (1,1)	0.6135 (0.0121)	0.6089 (0.0070)	0.6047 (0.0038)	0.6009 (0.0005)
0.9	α	LM	3.0765 (0.0245)	3.0552 (0.0158)	3.0314 (0.0092)	3.0067 (0.0013)
		TLM (1,1)	3.0368 (0.0118)	3.0221 (0.0066)	3.0122 (0.0033)	3.0019 (0.0003)
	σ	LM	0.9714 (0.0463)	0.9498 (0.0294)	0.9265 (0.0171)	0.9074 (0.0031)
		TLM (1,1)	0.9203 (0.0208)	0.9135 (0.0116)	0.9062 (0.0061)	0.9012 (0.0009)
1.2	α	LM	3.0689 (0.0207)	3.0493 (0.0137)	3.0268 (0.0080)	3.0058 (0.0011)
		TLM (1,1)	3.0324 (0.0099)	3.0195 (0.0055)	3.0105 (0.0028)	3.0016 (0.0002)
	σ	LM	1.2896 (0.0684)	1.2614 (0.0428)	1.2385 (0.0249)	1.2106 (0.0038)
		TLM (1,1)	1.2263 (0.0302)	1.2181 (0.0167)	1.2096 (0.0086)	1.2017 (0.0012)

σ	Parameter	Estimator	n = 20	n = 30	n = 50	n = 200
1.5	α	LM	3.0618 (0.0184)	3.0447 (0.0119)	3.0235 (0.0068)	3.0051 (0.0009)
		TLM (1,1)	3.0288 (0.0087)	3.0173 (0.0048)	3.0092 (0.0024)	3.0013 (0.0002)
	σ	LM	1.6124 (0.0895)	1.5827 (0.0551)	1.5556 (0.0314)	1.5146 (0.0045)
		TLM (1,1)	1.5348 (0.0384)	1.5245 (0.0211)	1.5121 (0.0107)	1.5019 (0.0014)

Interpretation: [Table 2](#) The results indicate that both estimators are approximately unbiased, as the estimated values converge toward the true parameter values as the sample size increases it was also observe that as sample size increases the MSEs decreases, demonstrating the consistency of both Estimation methods.

Table 3. Estimates and MSEs of the Scale Parameter (σ) and Shape Parameter (α) Using L-Moments and Trimmed L-Moments (1,1) for $\alpha = 4$.

σ	Parameter	Estimator	n = 20	n = 30	n = 50	n = 200
0.3	α	LM	4.1124 (0.0418)	4.0865 (0.0276)	4.0537 (0.0152)	4.0108 (0.0023)
		TLM (1,1)	4.0561 (0.0194)	4.0342 (0.0111)	4.0188 (0.0055)	4.0031 (0.0006)
	σ	LM	0.3302 (0.0135)	0.3215 (0.0087)	0.3132 (0.0051)	0.3030 (0.0010)
		TLM (1,1)	0.3105 (0.0062)	0.3068 (0.0037)	0.3040 (0.0019)	0.3006 (0.0002)
0.6	α	LM	4.0985 (0.0365)	4.0742 (0.0234)	4.0461 (0.0132)	4.0097 (0.0020)
		TLM (1,1)	4.0492 (0.0173)	4.0301 (0.0098)	4.0164 (0.0048)	4.0028 (0.0005)
	σ	LM	0.6485 (0.0274)	0.6332 (0.0173)	0.6201 (0.0102)	0.6050 (0.0019)
		TLM (1,1)	0.6144 (0.0132)	0.6097 (0.0076)	0.6052 (0.0040)	0.6010 (0.0005)
0.9	α	LM	4.0896 (0.0314)	4.0675 (0.0201)	4.0412 (0.0118)	4.0088 (0.0017)
		TLM (1,1)	4.0445 (0.0148)	4.0268 (0.0083)	4.0146 (0.0041)	4.0024 (0.0004)
	σ	LM	0.9743 (0.0498)	0.9528 (0.0317)	0.9294 (0.0186)	0.9082 (0.0033)
		TLM (1,1)	0.9221 (0.0225)	0.9148 (0.0125)	0.9070 (0.0064)	0.9014 (0.0009)
1.2	α	LM	4.0814 (0.0272)	4.0613 (0.0178)	4.0365 (0.0101)	4.0076 (0.0015)
		TLM (1,1)	4.0403 (0.0129)	4.0241 (0.0071)	4.0130 (0.0035)	4.0021 (0.0003)
	σ	LM	1.2925 (0.0728)	1.2648 (0.0457)	1.2412 (0.0264)	1.2115 (0.0040)
		TLM (1,1)	1.2281 (0.0319)	1.2196 (0.0175)	1.2104 (0.0090)	1.2019 (0.0012)
1.5	α	LM	4.0748 (0.0239)	4.0554 (0.0156)	4.0321 (0.0089)	4.0068 (0.0012)
		TLM (1,1)	4.0367 (0.0112)	4.0219 (0.0062)	4.0116 (0.0030)	4.0018 (0.0003)
	σ	LM	1.6153 (0.0942)	1.5864 (0.0587)	1.5581 (0.0336)	1.5155 (0.0048)
		TLM (1,1)	1.5367 (0.0402)	1.5258 (0.0220)	1.5129 (0.0111)	1.5021 (0.0015)

Interpretation: [Table 3](#) The results show that both estimation methods improve as the sample size increases, with the parameter estimates approaching their true values and the MSEs decreasing substantially. However, the TLM (1,1) estimator consistently produced lower MSE values than the LM

estimator for both the scale and shape parameters across all sample sizes This indicates that the trimming procedure improves estimation efficiency by reducing the influence of extreme observations.

4. Discussion of Results

The simulation results presented in Tables 1 to 3 provide a comprehensive comparison between the conventional L-moments (LM) and the Trimmed Linear Moments TLM (1,1) estimators for the Type II Extreme Value (Fréchet) distribution under varying sample sizes, scale parameters, and shape parameters. The discussion focuses on the efficiency and consistency of both estimation methods based on their parameter estimates and corresponding mean squared errors (MSEs). Across all simulation settings, a consistent pattern is observed: both LM and TLM (1,1) estimators improve in accuracy as the sample size increases from 20 to 200. This is evident from the progressive reduction in MSE values for both the scale (σ) and shape (α) parameters.

However, the Trimmed Linear Moments estimator (TLM (1,1)) consistently outperforms the conventional L-moments estimator. In every table and for all combinations of α and σ , TLM (1,1) yields smaller MSE values, indicating higher efficiency and reduced estimation variability. This improvement is particularly noticeable in small sample sizes ($n = 20$ and 30), where estimation uncertainty is typically high. The trimming procedure appears to reduce the influence of extreme observations, which is especially important for heavy-tailed distributions such as the Fréchet distribution. When examining the effect of the shape parameter (α), it is observed that as α increases from 2 to 4, both estimators maintain reasonable accuracy, but the magnitude of MSE generally increases slightly for higher values of α under LM. In contrast, TLM (1,1) remains relatively stable, showing its robustness in handling distributions with varying tail heaviness. This suggests that the trimming mechanism improves resistance to tail-induced distortion, which is a common challenge in extreme value modelling.

Similarly, As σ increases, the variability of estimates also increases, leading to higher MSE values. it was observed that from all the three tables, when the sample sizes increases no significant difference between LM and TLM (1,1).

5. Conclusion

This study investigated the performance of Trimmed Linear Moments (1,1) in estimating the scale and shape parameters of the Type II Extreme Value (Fréchet) distribution using Monte Carlo simulation. The results demonstrated that the Trimmed Linear Moments (1,1) estimator consistently outperformed the L-moments across all the sample sizes, particularly in small sample. it was also observed that as sample size increases the MSEs decreases, demonstrating the consistency of both Estimation methods. The study concludes that Trimmed Linear Moments (1,1) provide a more reliable and efficient alternative for parameter estimation of the Fréchet distribution and therefore recommended for practical applications involving extreme value analysis.

Abbreviations

MOM	Method of Moments
MLE	Maximum Likelihood Estimation
GEV	Generalized Extreme Value
TLMs	Trimmed Linear Moments
RMSE	Root Mean Squared Error
MSE	Mean Squared Error

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Conflicts of Interest

The authors declare no conflicts of interest.

References

- [1] Adeyemi, A. O., Ojo, S. K., and Adewale, B. T. (2020). Performance of trimmed L-moments in estimating parameters of heavy-tailed hydrological distributions. *Journal of Hydrological Modeling and Statistics*, 12(3), 145-162.
- [2] Bello, T. A., Ogunleye, A. O., and Adebayo, K. S. (2022). Estimation of Type II extreme value distribution parameters using L-moments, MLE and trimmed L-moments. *Nigerian Journal of Statistical Modelling*, 14(1), 45-63.
- [3] Elamir, E. A., and Seheult, A. H. (2003). Trimmed L-moments. *Computational Statistics and Data Analysis*, 43(3), 299-314. [https://doi.org/10.1016/S0167-9473\(02\)00250-5](https://doi.org/10.1016/S0167-9473(02)00250-5)
- [4] Embrechts, P., Klüppelberg, C., and Mikosch, T. (1997). *Modelling extremal events: For insurance and finance*. Springer.
- [5] García-Marín, R., Torres, M., and Morales, F. (2024). Assessment of L-moment variations for extreme rainfall quantile projections. *Journal of Applied Hydrology*, 31(4), 410-425.
- [6] Hassan, M. A., Al-Ghamdi, A. S., and Ibrahim, E. M. (2022). Parameter estimation of two-parameter Fréchet distribution under progressive censoring using trimmed L-moments. *Communications in Statistics - Simulation and Computation*, 51(8), 4520-4538.
- [7] Hosking, J. R. M. (1990). L-moments: Analysis and estimation of distributions using linear combinations of order statistics. *Journal of the Royal Statistical Society: Series B (Methodological)*, 52(1), 105-124.

- [8] Hosking, J. R. M., Wallis, J. R., and Wood, E. F. (1985). Estimation of the generalized extreme-value distribution by the method of probability-weighted moments. *Technometrics*, 27(3), 251-261.
<https://doi.org/10.1080/00401706.1985.10488049>
- [9] Kovács, P., and Zempléni, A. (2024). Trimmed order statistics in non-stationary extreme value models under data contamination. *Extremes*, 27(1), 89-112.
- [10] Lopez, A., García, M., and Hernández, R. (2025). Performance of trimmed linear moments in extreme rainfall modelling and return level estimation. *Journal of Hydrological Statistics*, 18(2), 115-134.
- [11] Naveed, M., Aslam, M., and Shahbaz, M. Q. (2022). Robust L-moment estimation procedures for modelling extreme rainfall events using multi-decade hydrological records. *Hydrological Sciences Journal*, 67(5), 712-728.
- [12] Ojo, S. K., and Adewale, B. T. (2023). Monte Carlo evaluation of TLM (1,1) parameter estimation for heavy-tailed extreme value distributions. *Computational Statistics*, 38(2), 789-808.
- [13] Rahman, M. M., Khan, A. A., and Islam, M. S. (2023). Comparative investigation of trimmed L-moment and Bayesian estimation procedures for extreme value distributions. *Journal of Statistical Computation and Simulation*, 93(4), 540-558.
- [14] Singh, R., Kumar, P., and Sharma, A. (2023). Comparative assessment of parameter estimation methods for GEV distribution in monsoon rainfall modelling. *International Journal of Climatology and Hydrological Studies*, 41(6), 889-905.
- [15] Wang, Y., Chen, L., and Zhang, X. (2025). Theoretical asymptotic properties of generalized trimmed L-moments for Type II extreme value distribution. *Annals of the Institute of Statistical Mathematics*, 77(1), 55-78.
- [16] Wilks, D. S. (2006). *Statistical methods in the atmospheric sciences* (2nd ed.). Academic Press.