

Research Article

Analytical Solution to One Problem of Cooling Fiber Mass

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Abstract

To analyze the cooling process of raw cotton, a mathematical model is proposed in the form of a boundary value problem for a system of thermal conductivity equations. The developed model makes it possible to describe the distribution and propagation of heat within a fibrous mass represented in the form of a ball. Such an approach provides an opportunity to investigate the thermal state of raw cotton during the cooling process and to determine the main regularities of heat transfer inside the fibrous material. The proposed mathematical formulation takes into account the thermal conductivity processes occurring within the fibrous mass and allows the temperature field to be determined as a function of spatial coordinates and time. The boundary and initial conditions are formulated in accordance with the physical characteristics of the cooling process. This makes it possible to obtain a mathematical description that adequately reflects the changes in temperature inside the considered fibrous medium during cooling. Using the method of separated variables, an analytical solution to the proposed initial-boundary value problem is obtained. The application of this method makes it possible to represent the solution in a convenient analytical form and to analyze the dependence of the temperature distribution on the main parameters of the process. The obtained analytical results provide a theoretical basis for studying the dynamics of heat propagation and evaluating the cooling efficiency of the fibrous mass. The results of the study can be used to improve and optimize technological processes associated with the cooling of raw cotton. In particular, the proposed mathematical model can contribute to the selection of appropriate cooling conditions and technological parameters aimed at reducing excessive temperature increases and minimizing possible losses in fiber quality. The obtained results may also be useful for developing more effective approaches to controlling thermal processes during the processing and storage of fibrous materials. The work is of interest to specialists in the fields of thermal physics, heat transfer, mathematical modeling, fibrous materials processing technology, cotton processing, and related scientific and engineering areas concerned with the analysis and control of thermal processes in fibrous materials.

Keywords

Heat Transfer, Cooling, Raw Cotton, Fiber Quality, Initial Temperature, Thermal Conductivity, Parabolic Type, Orthogonally of Function, Characteristic Equation

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1. Introduction

During cooling, a complex, non-stationary heat and mass transfer process occurs in a cotton sack, determining its external and internal states. External processes are characterized by mass transfer from the sack's surface to the surrounding environment and heat exchange between the fiber mass and the environment.

The rate of heat transfer within the cotton sack is crucial for maintaining the quality of the fiber and seeds during cooling [1-6]. Therefore, we examined the problem of heat transfer within a cotton sack during cooling as a sphere.

2. Method of Research

The problem of heat transfer in a fibrous mass can be expressed as follows: A spherical body of radius R is given, possessing the property of isotropy, with a known initial temperature distribution.

Convective heat exchange occurs within the body between the fibrous mass and the air according to Newton's law. It is necessary to find the radial temperature distribution of the fibrous mass and air at any given time.

Then, based on the laws of thermodynamics (Fourier's and Newton's laws of heat conduction, conservation of energy, etc.), this problem can be formulated as a system of parabolic differential equations [7-15]:

$$\begin{cases} c_1 \rho_1 \frac{\partial T_1}{\partial \tau} = \lambda_1 \left(\frac{\partial^2 T_1}{\partial r^2} + \frac{2}{r} \frac{\partial T_1}{\partial r} \right) + \alpha (T_2 - T_1) \\ c_2 \rho_2 \frac{\partial T_2}{\partial \tau} = \lambda_2 \left(\frac{\partial^2 T_2}{\partial r^2} + \frac{2}{r} \frac{\partial T_2}{\partial r} \right) - \alpha (T_2 - T_1) \end{cases} \quad (1)$$

with initial

$$T_1(r, 0) = T_{10}, \quad T_2(r, 0) = T_c$$

and boundary conditions

$$T_1(R, \tau) = \beta T_c, \quad T_2(R, \tau) = \beta T_c, (\beta > 1)$$

where c_i, λ_i, ρ_i are the heat capacity, thermal conductivity, and density of the fibrous mass and air, respectively; α is the heat transfer coefficient between the fibrous mass and air; T_1, T_2, T_c – is the temperature of the fibrous mass, air, and the external environment, respectively; and T_{10} – is the initial temperature of the fibrous mass [16-18]: .

Let's make a change of variables

$$\theta_1 = rT_1, \quad \theta_2 = rT_2.$$

Then, we obtain the problem

$$\begin{cases} \frac{\partial \theta_1}{\partial \tau} = \alpha_1 \frac{\partial^2 \theta_1}{\partial r^2} + \alpha_1 (\theta_2 - \theta_1) \\ \frac{\partial \theta_2}{\partial \tau} = \alpha_2 \frac{\partial^2 \theta_2}{\partial r^2} + \alpha_2 (\theta_2 - \theta_1) \end{cases} \quad (2)$$

where

$$\alpha_2 = \frac{\lambda_2}{c_2 \rho_2}, \quad \alpha_1 = \frac{\lambda_1}{c_1 \rho_1}, \quad \alpha_2 = \frac{\alpha}{c_2 \rho_2}, \quad \alpha_2 = \frac{\alpha}{c_1 \rho_1}$$

$$\theta_1(r, 0) = rT_{10}, \quad \theta_2(R, \tau) = rT_c$$

$$\theta_1(R, \tau) = \beta rT_1, \quad \theta_2(R, \tau) = rT_c, \quad \theta_1(0, \tau) = 0.$$

We seek the solution to problem (2) in the form

$$\theta_1(r, \tau) = \sum_{n=1}^{\infty} A_n \sin \frac{\pi n}{R} r + \beta rT_c,$$

$$\theta_2(r, \tau) = \sum_{n=1}^{\infty} B_n \sin \frac{\pi n}{R} r + rT_c.$$

Let's substitute them into the system (2)

$$\begin{aligned} \sum_{n=1}^{\infty} A'_n(\tau) \sin \frac{\pi n}{R} r &= -\alpha_1 \sum_{n=1}^{\infty} A_n \sin \frac{\pi n}{R} r \left(\frac{\pi n}{R} \right)^2 + \alpha_1 \sum_{n=1}^{\infty} [B_n(\tau) - A_n(\tau)] \sin \frac{\pi n}{R} r + \alpha_1 rT_c \cdot (1 - \beta) \\ \sum_{n=1}^{\infty} A'_n(\tau) \sin \frac{\pi n}{R} r &= -\alpha_1 \sum_{n=1}^{\infty} A_n \sin \frac{\pi n}{R} r \left(\frac{\pi n}{R} \right)^2 + \alpha_1 \sum_{n=1}^{\infty} [B_n(\tau) - A_n(\tau)] \sin \frac{\pi n}{R} r + \alpha_1 rT_c \cdot (1 - \beta) \\ \sum_{n=1}^{\infty} B'_n(\tau) \sin \frac{\pi n}{R} r &= -\alpha_2 \sum_{n=1}^{\infty} A_n \sin \frac{\pi n}{R} r \left(\frac{\pi n}{R} \right)^2 + \alpha_2 \sum_{n=1}^{\infty} [B_n(\tau) - A_n(\tau)] \sin \frac{\pi n}{R} r - \alpha_2 rT_c \cdot (1 - \beta) \end{aligned}$$

Considering the orthogonality of the function $\sin \frac{\pi n}{R} r$

$$\int_0^R \sin \frac{\pi n}{R} r \cdot \sin \frac{\pi k}{R} r dr = \begin{cases} \frac{R}{2}, & \text{if } n = k \\ 0, & \text{if } n \neq k \end{cases}.$$

We will receive

$$\begin{cases} A_n'(\tau) + \left[a_1 \left(\frac{\pi n}{R} \right)^2 + \alpha_1 \right] A_n(\tau) - \alpha_2 B_n(\tau) = \frac{2}{R} \alpha_1 T_c (1-\beta) \int_0^R r \sin \frac{\pi n}{R} r dr \\ B_n'(\tau) + \left[a_2 \left(\frac{\pi n}{R} \right)^2 + \alpha_2 \right] B_n(\tau) - \alpha_1 A_n(\tau) = -\frac{2}{R} \alpha_2 T_c (1-\beta) \int_0^R r \sin \frac{\pi n}{R} r dr \end{cases}$$

Because,

$$\int_0^R \sin \frac{\pi n}{R} r dr = \int_0^R r \left(\cos \frac{\pi n}{R} \right)' dr \left(-\frac{R}{\pi n} \right) = -\frac{R^2}{\pi n} (-1)^n = \frac{R^2}{\pi n} (-1)^{n+1}.$$

So, we have a system of ordinary differential equations in the form

$$\begin{cases} A_n'(\tau) + q_{1n} A_n(\tau) - \alpha_1 B_n(\tau) = F_{1n} \\ B_n'(\tau) + q_{2n} B_n(\tau) - \alpha_2 A_n(\tau) = F_{2n} \end{cases} \quad (3)$$

$$\lambda_n^2 + \lambda_n (q_{1n} + q_{2n}) + q_{1n} q_{2n} - \alpha_1 \alpha_2 = 0,$$

$$\lambda_{in} = \frac{-(q_{1n} + q_{2n}) \pm \sqrt{(q_{1n} - q_{2n})^2 + 4\alpha_1 \alpha_2}}{2}, i = 1, 2$$

where

$$q_{in} = \left[a_i \left(\frac{\pi n}{R} \right)^2 + \alpha_i \right] \quad i = 1, 2,$$

$$F_{1n} = \frac{2}{R} \alpha_1 T_c (1-\beta) (-1)^{n+1} \frac{R^2}{\pi n} (-1)^{n+1} = (-1)^{n+1} \frac{2\alpha_1 T_c (1-\beta) R}{\pi n},$$

$$F_{2n} = (-1)^{n+1} \frac{2\alpha_2 T_c (1-\beta) R}{\pi n}.$$

We seek the solution of the homogeneous system (3) in the form

$$A_n(\tau) = a_n e^{\lambda_n \tau}, \quad B_n = b_n e^{\lambda_n \tau}.$$

Then, the characteristic equations have the form

$$\begin{cases} (\lambda_n + q_{1n}) a_n - \alpha_1 b_n = 0 \\ (\lambda_n + q_{2n}) b_n - \alpha_2 a_n = 0 \end{cases}.$$

From here, we find

Since $\lambda_{in} < 0$, we introduce the notation $\mu_{in} = -\lambda_{in}$, $i = 1, 2$.

The solution of the homogeneous system (3) has the form

$$A_n(\tau) = a_n^{(1)} e^{-\mu_{1n} \tau} + a_n^{(2)} e^{-\mu_{2n} \tau},$$

$$B_n = k_{1n} a_n^{(1)} e^{-\mu_{1n} \tau} + k_{2n} a_n^{(2)} e^{-\mu_{2n} \tau},$$

where

$$k_{1n} = \frac{\lambda_{1n} + q_{2n}}{\alpha_1}, \quad k_{2n} = \frac{\lambda_{2n} + q_{2n}}{\alpha_1} = \frac{\alpha_2}{\lambda_{2n} + q_{2n}} \quad (4)$$

3. Research Results

We write the solution of the inhomogeneous system as

$$\begin{cases} A_n(\tau) = a_n^{(1)} e^{-\mu_{1n} \tau} + a_n^{(2)} e^{-\mu_{2n} \tau} + a_n^{(3)} \\ B_n = k_{1n} a_n^{(1)} e^{-\mu_{1n} \tau} + k_{2n} a_n^{(2)} e^{-\mu_{2n} \tau} + a_n^{(4)} \end{cases} \quad (5)$$

Substituting into the equations, we obtain

$$\begin{cases} q_{1n}a_n^{(3)} - \alpha_1 a_n^{(4)} = F_{1n} \\ q_{2n}a_n^{(4)} - \alpha_2 a_n^{(3)} = F_{2n} \end{cases}.$$

Solving this system, we find

$$a_n^{(4)} = \frac{\alpha_2 F_{1n} + q_{1n} F_{2n}}{q_{1n} q_{2n} - \alpha_2 \alpha_1}, \quad a_n^{(3)} = \frac{q_{2n} F_{1n} + \alpha_1 F_{2n}}{q_{1n} q_{2n} - \alpha_2 \alpha_1} \quad (6)$$

From the initial conditions, we find

$$\begin{cases} \sum_{n=1}^{\infty} [\alpha_n^{(1)} + \alpha_n^{(2)} + \alpha_n^{(3)}] \sin \frac{\pi n}{R} r = r(T_{20} - \beta T_c) \\ \sum_{n=1}^{\infty} [k_{1n} \alpha_n^{(1)} + k_{2n} \alpha_n^{(2)} + \alpha_n^{(4)}] \sin \frac{\pi n}{R} r = 0 \end{cases}.$$

Using the orthogonality condition

$$\begin{cases} \left[\alpha_n^{(1)} + \alpha_n^{(2)} + \alpha_n^{(3)} \right] \frac{R}{2} = (-1)^{n+1} \frac{R^2}{\pi n} (T_{20} - \beta T_c) \\ \left[k_{1n} \alpha_n^{(1)} + k_{2n} \alpha_n^{(2)} + \alpha_n^{(4)} \right] \frac{R}{2} = 0 \end{cases}.$$

From here, we find

$$\begin{aligned} a_n^{(1)} &= \frac{(-1)^{n+1} \frac{2R}{\pi n} [T_{10} - \beta T_c] k_{2n} - [k_{2n} a_n^{(3)} - a_n^{(4)}]}{k_{2n} - k_{1n}} \\ a_n^{(2)} &= \frac{(-1)^{n+1} \frac{2R}{\pi n} [T_{10} - \beta T_c] k_{1n} - [k_{1n} a_n^{(3)} - a_n^{(4)}]}{k_{1n} - k_{2n}} \end{aligned} \quad (7).$$

Therefore, the solution to problem (1) has the form

$$T_1(r, \tau) = \sum_{n=1}^{\infty} A_n(\tau) \frac{\sin \frac{\pi n}{R} r}{r} + \beta T_c,$$

$$T_2(r, \tau) = \sum_{n=1}^{\infty} B_n(\tau) \frac{\sin \frac{\pi n}{R} r}{r} + T_c,$$

where $A_n(\tau), B_n(\tau)$ are calculated using formulas (5) - (7).

Below in Tables 1 and 2 are presented the calculation results obtained for the following values of the parameters of the model under consideration [15].

For fiber: $T_{10} = 70^\circ \text{C}$; $\lambda_1 = 0,07$; $c_1 = 1600$; $R_2 = 0,025$; $\alpha = 2,5$; $T_c = 10^\circ \text{C}$.

Table 1 shows the experimental and calculated data on cooling raw cotton relative to the radius of the raw cotton lump at an initial fiber mass temperature of 70°C , lump radii of 200 mm, 150 mm, 30 mm, air temperature of 10°C and initial raw cotton moisture of 20%.

Table 1. Experimental and calculated data on cooling relative to the radius of a raw cotton lump.

R, mm		200		150		30	
τ , a second		Experimental	Calculations	Experimental	Calculations	Experimental	Calculations
0	T_1	70	70	70	70	70	70
50	T_1	62	63,36	60	61,81	56	57,71
100	T_1	46	47,15	41	41,98	24	25,18
150	T_1	30	30.98	27	28,31	38	38,96
200	T_1	21	21,81	19	19,95	16	16,92

Table 2 shows the experimental and calculated data on cooling raw cotton relative to the density of raw cotton at an initial fiber mass temperature of 70°C , lump density of 70; 160

va 200 kg/m^3 and an initial raw cotton moisture content of 15%. Air temperature 10°C .

Table 2. Experimental and calculated data on cooling relative to the density of raw cotton.

ρ , Density		70 kg/m ³		160 kg/m ³		200 kg/m ³	
τ , Time		Experimental	Calculations	Experimental	Calculations	Experimental	Calculations
0	T_1	70	70	70	70	70	70
20	T_1	46	46,96	58	59,01	68	69,11
40	T_1	28	28,75	40	41,05	57	58,08
60	T_1	19	19,91	24	24,61	50	50,82
80	T_1			20	19,82	41	41,65
100	T_1					38	38,52

An analysis of the results in Tables 1 and 2 shows that the lump diameter significantly influences the process intensity; i.e., as the lump diameter increases, the fiber cooling rate decreases sharply. The temperature difference in the seeds along the lump radius and the temperature difference between fibers and seeds located at equal distances from the lump surface are very large, leading to uneven cooling of the raw cotton components. To increase the seed cooling rate and achieve uniform drying, it is necessary to organize the cooling period for the raw cotton in a loosened state, i.e., to reduce the lump diameter whenever possible.

4. Conclusions

This paper presents an analytical study of the cooling problem of fibrous mass. Based on mathematical modeling of the heat transfer process, analytical solutions were obtained that allow us to describe the temperature distribution in the fibrous medium as a function of time and spatial coordinates.

The main results of the paper include:

1. Formulation of a mathematical model of the cooling process, taking into account the thermophysical properties of the fibrous mass, as well as the boundary and initial conditions.
2. Analytical solution of the problem using mathematical physics methods, such as separation of variables or integral transformations.
3. Analysis of the influence of parameters (thermal conductivity, heat capacity, material density) on cooling dynamics.
4. Practical significance of the obtained solutions, which can be used to optimize industrial processes related to the processing of fibrous materials.

The obtained results demonstrate that the analytical approach allows not only a qualitative description of the cooling process but also a quantitative assessment of the influence of

various factors on its dynamics. This opens up opportunities for developing more effective methods for controlling thermal processes in fibrous materials.

In the future, a promising direction of research may be the consideration of nonlinear effects, such as the dependence of thermophysical properties on temperature, as well as numerical modeling for more complex geometric configurations and boundary conditions.

Abbreviations

T_1, T_2, T_c – Temperature

Author Contributions

Mamatov Alisher Zulunovich: Methodology, Project administration, Supervision, Writing – review & editing

Jamshidbek Rakhmonov Turdaliyevich: Conceptualization, Data curation, Formal Analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing – original draft, Writing – review & editing

Conflicts of Interest

The authors declare no conflicts of interest.

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Biography



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Research Field

Mamatov Alisher Zulunovich: Mathematical Analysis, Differential Equations, Fourier Series, Numerical Methods, Mathematical Modeling.

Jamshidbek Rakhmonov Turdaliyevich: Mathematical Modeling, Differential Equations, Heat and Mass Transfer, Cotton Processing Technologies, Drying Processes.