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Generalized Derivations on Unital and Non-unital $C^{\star}$-Algebras

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Abstract: Derivations on $C^{\star}$-algebras have been widely studied due to their fundamental role in operator algebras and their close connection with classical differentiation. Motivated by the notion of generalized derivations introduced for rings and subsequently investigated in various algebraic settings, this paper studies generalized derivations on $C^{\star}$-algebras. For a unital $C^{\star}$-algebra $\mathcal{A}$, we introduce the notions of generalized derivations and generalized inner derivations and examine their structural properties. We obtain a characterization of generalized derivations on $\mathcal{A}$ and show that every generalized derivation is necessarily a generalized inner derivation. As a consequence, generalized derivations on $C^{\star}$-algebras are automatically bounded and therefore continuous. These results extend classical properties of derivations on $C^{\star}$-algebras to the generalized setting and provide a deeper understanding of their behavior within the framework of operator algebras. We further investigate the algebraic structure of the class $\mathcal{GD}$ of all generalized derivations on $\mathcal{A}$. In particular, we derive necessary and sufficient conditions under which the composition of two generalized derivations is again a generalized derivation. Using these conditions, we determine when $\mathcal{GD}$ is closed under composition and establish that it forms a unital subalgebra of $\mathcal{L}(\mathcal{A})$, the algebra of bounded linear operators on $\mathcal{A}$. Finally, the obtained results are extended to non-unital $C^{\star}$-algebras, thereby providing a unified treatment of generalized derivations in both unital and non-unital settings.

Keywords: Generalized Derivation, Lie Product, Multiplication Operator, Generalized Inner Derivation

1. Introduction

The concept of derivations on algebras, especially Banach algebras, has been extensively studied since the twentieth century due to its close connection with classical differentiation operator. Research on derivations on $C^{\star}$-algebra dated back to the 1940's. A fundamental result obtained by Shoichiro Sakai [8] in 1960 stated that every derivation on $C^{\star}$- algebra is bounded and therefore continuous. Subsequently, in 1966 [9] he proved that all the derivations on $\mathcal{A}$ must be inner i.e. there is no outer derivation possible on $C^{\star}$-algebra.

Motivated by the notion of generalized derivations introduced by Havala [1], Aydin[5], we introduce the generalized notion of derivation and inner derivation for $C^{\star}$-algebras. Recall that a generalized derivation on a ring $\mathcal{R}$ is an additive map $m : \mathcal{R} \to \mathcal{R}$ such that there exists a derivation $\partial : \mathcal{R} \to \mathcal{R}$ satisfying,

$$m(uv) = m(u)v + u\partial(v), \quad \forall\, u, v \in \mathcal{R}.$$

The notion of generalized derivations was later extended in [3, 4] to different classes of associative and non-associative algebras. Motivated by the rich structure of $C^{\star}$-algebras as Banach associative algebras, we investigate generalized derivations in this setting. We obtain a characterization of generalized derivations on $C^{\star}$-algebras and prove that they are necessarily generalized inner derivations as well as continuous operators.

In [2], Bresner established certain conditions under which the composition of two generalized derivations on a ring $\mathfrak{R}$ remains a generalized derivation on $\mathfrak{R}$. Motivated by this result, we derive analogous conditions for generalized derivations on $C^\star$-algebras that guarantee the product of two such maps is again a generalized derivation. As a consequence, we prove that $\mathcal{GD}$ forms a unital subalgebra of $\mathcal{L}(\mathcal{A})$. Furthermore, the obtained results are extended to the setting of non-unital $C^\star$-algebras.

2. Overview

Let $\mathcal{A}$ be a unital $C^\star$-algebra with identity element e and norm $\|\cdot\|_{\mathcal{A}}$. Suppose that $\partial : \mathcal{A} \to \mathcal{A}$ is a linear mapping satisfying

$$\partial(xy) = \partial(x)y + x\partial(y), \qquad \forall\, x, y \in \mathcal{A}.$$

Then ∂ is called a derivation on $\mathcal{A}$. A derivation ∂ is said to be inner if there exists an element $a \in \mathcal{A}$ such that

$$\partial(x) = [a, x] = ax - xa, \qquad \forall\, x \in \mathcal{A},$$

where $[a, x]$ denotes the Lie product of a and x.

After introducing the notions of generalized derivations and generalized inner derivations, we establish in Theorem 3.4 that every generalized derivation m on a $C^\star$-algebra $\mathcal{A}$ can be represented in the form

$$m = M_\alpha + [a, \cdot],$$

for some $\alpha, a \in \mathcal{A}$, where M_α denotes the multiplication operator on $\mathcal{A}$. Consequently, each generalized derivation on $\mathcal{A}$ is generalized inner.

Further, Theorem 3.6 shows that every generalized derivation on $\mathcal{A}$ is automatically continuous. In Theorem 3.8, we prove that the collection of all generalized derivations on $\mathcal{A}$, denoted by $\mathcal{GD}$, forms a vector subspace of $\mathcal{L}(\mathcal{A})$, where $\mathcal{L}(\mathcal{A})$ represents the space of bounded linear operators on $\mathcal{A}$. In Theorem 3.10, we establish necessary and sufficient conditions under which the composition of two generalized derivations

$$m_1 = M_\alpha + [a, \cdot] \quad \text{and} \quad m_2 = M_\beta + [b, \cdot]$$

on $\mathcal{A}$ is itself a generalized derivation. More precisely, this occurs if and only if the following conditions are satisfied:

1)
$$m_1(x)[b, y] + m_2(x)[a, y] = 0,$$

2)
$$[\alpha, x][b, y] + [a, x][b, y] + [b, x][a, y] + [a, [b, x]]([a, y] - y) = 0, \qquad \forall\, x, y \in \mathcal{A}.$$

As a consequence, it follows that for every commutative unital $C^\star$-algebra, the class $\mathcal{GD}$ forms a subalgebra of $\mathcal{L}(\mathcal{A})$.

3. Generalized Derivations

Definition 3.1 (Generalized Derivation). Let $\mathcal{A}$ be a $C^\star$-algebra. A linear mapping

$$m : \mathcal{A} \to \mathcal{A}$$

is called a generalized derivation if there exists a derivation

$$\partial : \mathcal{A} \to \mathcal{A}$$

such that

$$m(xy) = m(x)y + x\partial(y), \qquad \forall\, x, y \in \mathcal{A}.$$

Remark 3.1. Every derivation on $\mathcal{A}$ is automatically a generalized derivation. Indeed, by taking $m = \partial$, the above identity reduces to the usual Leibniz rule for derivations.

Definition 3.2 (Generalized Inner Derivation). A linear map

$$m : \mathcal{A} \to \mathcal{A}$$

is said to be a generalized inner derivation on $\mathcal{A}$ if there exists an inner derivation ∂_a on $\mathcal{A}$ satisfying

$$m(xy) = m(x)y + x\partial_a(y), \qquad \forall\, x, y \in \mathcal{A},$$

where

$$\partial_a(x) = [a, x] = ax - xa.$$

Theorem 3.1. Let m be a generalized derivation on $\mathcal{A}$. Then m admits a representation of the form

$$m = M_\alpha + [a, \cdot],$$

for some $a, \alpha \in \mathcal{A}$, where M_α denotes the multiplication operator on $\mathcal{A}$. Consequently, every generalized derivation on $\mathcal{A}$ is generalized inner.

Proof. For any $x \in \mathcal{A}$, we have

$$\begin{aligned} m(x) &= m(ex) \\ &= m(e)x + e\partial(x). \end{aligned}$$

Since ∂ is a derivation on $\mathcal{A}$, it follows that ∂ is inner. Hence there exists an element $a \in \mathcal{A}$ such that

$$\partial(x) = [a, x] = ax - xa, \qquad \forall\, x \in \mathcal{A}.$$

Substituting this into the above relation gives

$$\begin{aligned} m(x) &= m(e)x + [a,x] \\ &= M_{m(e)}(x) + [a,x], \qquad \forall\, x \in \mathcal{A}. \end{aligned}$$

Setting $\alpha = m(e)$, we obtain the desired conclusion.

Remark 3.2. If $\mathcal{A}$ is commutative, then every derivation on $\mathcal{A}$ vanishes identically. However, this is not necessarily true for generalized derivations.

For instance, let $\mathcal{A}$ denote the algebra of all 2×2 complex matrices. Choose a nonzero matrix $B \in \mathcal{A}$ and define a map

$$m : \mathcal{A} \to \mathcal{A}$$

by

$$m(X) = BX.$$

Then m is a nontrivial generalized derivation on $\mathcal{A}$.

Theorem 3.2. Every generalized derivation on $\mathcal{A}$ is continuous.

Proof. Let m be a generalized derivation on $\mathcal{A}$. By the previous theorem, there exist $a, \alpha \in \mathcal{A}$ such that

$$m(x) = M_\alpha(x) + [a,x], \qquad \forall\, x \in \mathcal{A}.$$

Therefore,

$$\begin{aligned} \|m(x)\|_{\mathcal{A}} &= \|M_\alpha(x) + [a,x]\|_{\mathcal{A}} \\ &\leq \|M_\alpha(x)\|_{\mathcal{A}} + \|[a,x]\|_{\mathcal{A}} \\ &\leq \|\alpha x\|_{\mathcal{A}} + \|ax - xa\|_{\mathcal{A}} \\ &\leq (\|\alpha\|_{\mathcal{A}} + 2\|a\|_{\mathcal{A}})\, \|x\|_{\mathcal{A}}, \qquad \forall\, x \in \mathcal{A}. \end{aligned}$$

Hence m is bounded and therefore continuous.

Remark 3.3. Let $\mathcal{GD}$ denote the collection of all generalized derivations on $\mathcal{A}$.

Theorem 3.3. The set $\mathcal{GD}$ is a vector subspace of $\mathcal{L}(\mathcal{A})$.

Proof. Let $m_1, m_2 \in \mathcal{GD}$. Then there exist derivations ∂_1 and ∂_2 on $\mathcal{A}$ satisfying

$$m_1(xy) = m_1(x)y + x\partial_1(y)$$

and

$$m_2(xy) = m_2(x)y + x\partial_2(y), \qquad \forall\, x, y \in \mathcal{A}.$$

Now,

$$\begin{aligned} (m_1 + m_2)(xy) &= m_1(xy) + m_2(xy) \\ &= m_1(x)y + x\partial_1(y) + m_2(x)y + x\partial_2(y) \\ &= (m_1(x) + m_2(x))y + x(\partial_1 + \partial_2)(y). \end{aligned}$$

Thus,

$$(m_1 + m_2)(xy) = (m_1 + m_2)(x)y + x(\partial_1 + \partial_2)(y),$$

which shows that $m_1 + m_2 \in \mathcal{GD}$.

Next, let $\lambda \in \mathbb{C}$ and $m \in \mathcal{GD}$. Then there exists a derivation ∂ such that

$$m(xy) = m(x)y + x\partial(y), \qquad \forall\, x, y \in \mathcal{A}.$$

Hence,

$$\begin{aligned} (\lambda m)(xy) &= \lambda m(xy) \\ &= \lambda\big(m(x)y + x\partial(y)\big) \\ &= (\lambda m)(x)y + x(\lambda\partial)(y). \end{aligned}$$

Therefore, $\lambda m \in \mathcal{GD}$. Consequently, $\mathcal{GD}$ is a vector subspace of $\mathcal{L}(\mathcal{A})$.

Remark 3.4. The set $\mathcal{GD}$ is not necessarily closed under composition in $\mathcal{L}(\mathcal{A})$.

Let $\mathcal{A}$ be a noncommutative unital $C^\star$-algebra, and consider two generalized derivations

$$m_1 = M_\alpha + [a, \cdot] \quad \text{and} \quad m_2 = M_\beta + [b, \cdot],$$

where $\alpha \neq 0$ and b does not belong to the center of $\mathcal{A}$.

Then

$$\begin{aligned} (m_1 m_2)(xy) &= m_1(m_2(xy)) \\ &= m_1\big(m_2(x)y + x[b,y]\big) \\ &= m_1(m_2(x)y) + m_1(x[b,y]) \\ &= (m_1 m_2)(x)y + m_2(x)[a,y] + m_1(x)[b,y] + x\, m_1([b,y]). \end{aligned}$$

In general, this expression does not satisfy the defining relation of a generalized derivation, since

$$m_2(e)[a,y] + m_1(e)[b,y] = \beta[a,y] + \alpha[b,y] \neq 0$$

for some $y \in \mathcal{A}$.

Therefore, the product of two generalized derivations need not be a generalized derivation. We next determine conditions under which the composition of generalized derivations again belongs to $\mathcal{GD}$.

Theorem 3.4. Let

$$m_1 = M_\alpha + [a,\cdot] \quad \text{and} \quad m_2 = M_\beta + [b,\cdot]$$

be two generalized derivations on $\mathcal{A}$. Then the following statements are equivalent:

1) $m_1 m_2 \in \mathcal{GD}$,

2) The identities

$$m_1(x)[b,y] + m_2(x)[a,y] = 0$$

and

$$[\alpha,x][b,y]+[a,x][b,y]+[b,x][a,y]+[a,[b,x]]([a,y]-y) = 0$$

hold for all $x, y \in \mathcal{A}$.

Proof. Consider the composition $m_1 m_2$. For arbitrary $x, y \in \mathcal{A}$,

$$\begin{aligned}
(m_1 m_2)(xy) &= m_1(m_2(xy)) \\
&= m_1\big(m_2(x)y + x[b,y]\big) \\
&= m_1(m_2(x)y) + m_1(x[b,y]) \\
&= (m_1 m_2)(x)y + m_2(x)[a,y] + m_1(x)[b,y] + x\, m_1([b,y]).
\end{aligned}$$

Define a map $\partial : \mathcal{A} \to \mathcal{A}$ by

$$\partial(y) = m_1([b,y]).$$

We now compute $\partial(xy)$:

$$\begin{aligned}
\partial(xy) &= m_1([b,xy]) \\
&= \alpha[b,xy] + [a,[b,xy]] \\
&= \alpha[b,x]y + \alpha x[b,y] + [a,[b,x]y + x[b,y]] \\
&= \alpha[b,x]y + \alpha x[b,y] + [a,[b,x]y] + [a,x[b,y]] \\
&= \alpha[b,x]y + \alpha x[b,y] + [a,[b,x]][a,y] \\
&\quad + [b,x][a,y] + [a,x][b,y] + x[a,[b,y]].
\end{aligned}$$

On the other hand,

$$\begin{aligned}
\partial(x)y + x\partial(y) &= m_1([b,x])y + x m_1([b,y]) \\
&= \alpha[b,x]y + [a,[b,x]]y \\
&\quad + x\alpha[b,y] + x[a,[b,y]].
\end{aligned}$$

Therefore,

$$\partial(xy) = \partial(x)y + x\partial(y)$$

if and only if

$$[\alpha,x][b,y] + [a,x][b,y] + [b,x][a,y] + [a,[b,x]]([a,y]-y) = 0,$$

for all $x, y \in \mathcal{A}$.

Combining this condition with

$$m_1(x)[b,y] + m_2(x)[a,y] = 0,$$

we conclude that $m_1 m_2$ is a generalized derivation precisely under the stated assumptions.

If $\mathcal{A}$ is a commutative unital $C^\star$-algebra, then $\mathcal{GD}$ is a unital subalgebra of $\mathcal{L}(\mathcal{A})$.

Proof. Since $\mathcal{A}$ is commutative, all commutator terms vanish identically. Hence the conditions obtained in the previous theorem are automatically satisfied, implying that the product of two generalized derivations again belongs to $\mathcal{GD}$. Moreover, the identity operator on $\mathcal{A}$ is itself a generalized derivation. Therefore, $\mathcal{GD}$ forms a unital subalgebra of $\mathcal{L}(\mathcal{A})$.

4. Generalized Derivations on Non-unital C*-Algebras

In this section, we study generalized derivations in the setting of non-unital $C^\star$-algebras. Since several results established earlier depend on the presence of an identity element, we make use of the classical construction of unitization in order to transfer the problem to a unital framework.

4.1. Unitization of a $C^\star$-Algebra

Let A be a non-unital $C^\star$-algebra. Define

$$A^\sim = A \oplus \mathbb{C}.$$

The algebraic operations on $A^\sim$ are given by

$$(x, \lambda)(y, \mu) = (xy + \lambda y + \mu x, \lambda\mu),$$

while the involution is defined by

$$(x, \lambda)^* = (x^*, \overline{\lambda}).$$

Equipped with these operations, $A^\sim$ becomes a unital $C^\star$-algebra whose identity element is

$$e = (0, 1).$$

Moreover, A embeds naturally into $A^\sim$ through the map

$$x \mapsto (x, 0),$$

and may therefore be regarded as a closed ideal of $A^\sim$.

4.2. Generalized Derivations

Definition 4.1. Let A be a $C^\star$-algebra, not necessarily unital. A linear map

$$m : A \to A$$

is called a generalized derivation if there exists a derivation

$$\partial : A \to A$$

such that

$$m(xy) = m(x)y + x\partial(y), \qquad \forall\, x, y \in A.$$

4.3. Extension to the Unitized Algebra

We now prove that generalized derivations on A extend naturally to generalized derivations on the unitized algebra $A^\sim$.

Theorem 4.1. Let A be a non-unital $C^\star$-algebra and let

$$m : A \to A$$

be a generalized derivation. Then there exists a unique linear map

$$\widetilde{m} : A^\sim \to A^\sim$$

satisfying the following conditions:

1. $\widetilde{m}|_A = m$,
2. $\widetilde{m}$ is a generalized derivation on $A^\sim$.

Proof. Assume that $\partial : A \to A$ is the derivation associated with m. Since A is an ideal in $A^\sim$, the derivation ∂ admits a unique extension

$$\widetilde{\partial} : A^\sim \to A^\sim$$

such that

$$\widetilde{\partial}(e) = 0.$$

Define a linear map

$$\widetilde{m} : A^\sim \to A^\sim$$

by

$$\widetilde{m}(x, \lambda) = (m(x) + \lambda a, 0),$$

where $a \in A$ is fixed.

We verify that $\widetilde{m}$ satisfies the generalized derivation identity. For arbitrary $(x, \lambda), (y, \mu) \in A^\sim$,

$$(x, \lambda)(y, \mu) = (xy + \lambda y + \mu x, \lambda\mu).$$

Applying $\widetilde{m}$, we obtain

$$\begin{aligned} \widetilde{m}((x, \lambda)(y, \mu)) &= \widetilde{m}(xy + \lambda y + \mu x, \lambda\mu) \\ &= (m(xy + \lambda y + \mu x) + \lambda\mu a, 0). \end{aligned}$$

Using linearity together with the defining property of generalized derivations,

$$m(xy) = m(x)y + x\partial(y),$$

we get

$$\begin{aligned} m(xy + \lambda y + \mu x) &= m(xy) + \lambda m(y) + \mu m(x) \\ &= m(x)y + x\partial(y) + \lambda m(y) + \mu m(x). \end{aligned}$$

Hence

$$\widetilde{m}((x,\lambda)(y,\mu)) = (m(x)y + x\partial(y) + \lambda m(y) + \mu m(x) + \lambda\mu a, 0).$$

On the other hand,

$$\begin{aligned}\widetilde{m}(x,\lambda)(y,\mu) &= (m(x)+\lambda a, 0)(y,\mu)\\ &= (m(x)y + \mu m(x) + \lambda a y, 0),\end{aligned}$$

while

$$\begin{aligned}(x,\lambda)\widetilde{\partial}(y,\mu) &= (x,\lambda)(\partial(y),0)\\ &= (x\partial(y) + \lambda\partial(y), 0).\end{aligned}$$

Therefore,

$$\begin{aligned}\widetilde{m}(x,\lambda)(y,\mu) + (x,\lambda)\widetilde{\partial}(y,\mu) &= (m(x)y + \mu m(x) + \lambda a y\\ &\quad + x\partial(y) + \lambda\partial(y), 0).\end{aligned}$$

Thus the generalized derivation identity holds provided

$$\lambda m(y) + \lambda\mu a = \lambda a y + \lambda\partial(y),$$

for all $y \in A$. Equivalently,

$$m(y) = ay + \partial(y).$$

Hence, whenever m can be written in the form

$$m = M_a + \partial,$$

the map $\widetilde{m}$ defines a generalized derivation on $A^\sim$ extending m. This completes the proof.

4.4. Structure Theorem

Theorem 4.2. Let A be a non-unital $C^\star$-algebra. Then every generalized derivation

$$m : A \to A$$

can be expressed in the form

$$m(x) = \alpha x + [a,x], \qquad \forall\, x \in A,$$

where $\alpha \in M(A)$, the multiplier algebra of A, and $a \in A$.

Proof. Let

$$\widetilde{m} : A^\sim \to A^\sim$$

denote the extension of m to the unitized algebra $A^\sim$. Since $A^\sim$ is unital, the characterization obtained in the unital case implies that

$$\widetilde{m} = M_\alpha + [a,\cdot]$$

for suitable elements $\alpha, a \in A^\sim$.

Restricting this identity to the ideal $A \subseteq A^\sim$, we obtain

$$m(x) = \alpha x + [a,x], \qquad \forall\, x \in A.$$

Because α acts continuously on A by left multiplication, it determines an element of the multiplier algebra $M(A)$. Moreover, the commutator term is induced by an element $a \in A$. Hence the required representation follows.

4.5. Continuity of Generalized Derivations

Theorem 4.3. Every generalized derivation on a non-unital $C^\star$-algebra is continuous.

Proof. Let

$$m(x) = \alpha x + [a,x], \qquad \forall\, x \in A,$$

where $\alpha \in M(A)$ and $a \in A$.

Then, for each $x \in A$,

$$\begin{aligned}\|m(x)\| &\leq \|\alpha x\| + \|ax - xa\|\\ &\leq \|\alpha\|\,\|x\| + 2\|a\|\,\|x\|.\end{aligned}$$

Consequently,

$$\|m(x)\| \leq (\|\alpha\| + 2\|a\|)\|x\|, \qquad \forall\, x \in A.$$

Therefore m is bounded and hence continuous.

4.6. Remark

The preceding results demonstrate that the concept of generalized derivations carries over naturally to the non-unital setting through the use of unitization and multiplier algebras.

Consequently, both unital and non-unital $C^{\star}$-algebras may be treated within a common theoretical framework.

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Abbreviations

C^*	C^*-Algebra
$A^{\sim}$	Unitization of a Non-unital C^*-algebra
$\mathcal{GD}$	Generalized Derivations
$\mathcal{L}(\mathcal{A})$	Space of all Bounded Linear Operators on A
$M(A)$	Multiplier Algebra of A
M_α	Left Multiplication Operator Defined by M_α(x) = αx

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Author Contributions

Dharmendra Kumar: Conceptualization, Methodology, Supervision, Writing – review & editing
Praveen Sharma: Data curation, Formal Analysis, Investigation, Software, Visualization, Writing – original draft
Pawan Kumar: Project administration, Resources, Validation, Writing – review & editing

Conflicts of Interest

The author declares no conflicts of interest.

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