International Journal of Transportation Engineering and Technology
2016; 2(1): 8-14
http://www.sciencepublishinggroup.com j/ijtet
doi: 10.11648/j.ijtet.20160201.12

Determination of Added Mass and Inertia Moment of Marine Ships Moving in 6 Degrees of Freedom

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To cite this article:
Do Thanh Sen, Tran Canh Vinh. Determination of Added Mass and Inertia Moment of Marine Ships Moving in 6 Degrees of Freedom. *International Journal of Transportation Engineering and Technology*. Vol. 2, No. 1, 2016, pp. 8-14. doi: 10.11648/j.ijtet.20160201.12

Received: March 16, 2016; **Accepted**: March 30, 2016; **Published**: April 25, 2016

Abstract: When a ship moves in water with acceleration or deceleration, quantities of fluid moving around the hull creating additional hydrodynamic forces acting on the hull. It is imagined as the added mass which increases the total system mass and inertia moment. In order to establish the mathematical model for ship motion, the added components need to be determined. For a particular ship, these hydrodynamic components can be obtained by experiment. However, for ship simulation especially at the initial design stage it is necessary to calculate and estimate by theoretical method. This study aims to find out a general method to calculate all components of added mass and inertia moment in 6 degrees of freedom for simulating ship movement.

Keywords: Added Mass, Hydrodynamic Coefficient, Mathematical Modeling, Ship Simulation

1. Introduction

According to potential flow theory, no force exists when a rigid body moves at a steady speed through ideal fluid. However, once it accelerates, an hydrodynamic force experiences proportionally to the acceleration. The hydrodynamic force (F_H) can be described [1]:

$$F_H = M[\dot{u} \quad \dot{v} \quad \dot{w} \quad \dot{p} \quad \dot{q} \quad \dot{r}]^T \tag{1.1}$$

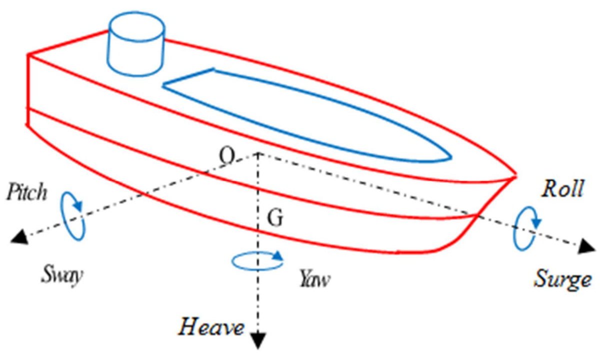

Figure 1. *Motions in six degrees of freedom (6DOF).*

Where, $\upsilon = [u, v, w, p, q, r]^T$ is rigid-body velocity matrix in 6 degrees of freedom (6DOF) (see fig. 1 and table 1):

Table 1. *Table of degrees of freedom DOF.*

DOF	description	velocities	
1	*surge - motion in x direction*	*u*	*Linear velocities*
2	*sway - motion in y direction*	*v*	
3	*heave - motion in z direction*	*w*	
4	*roll – rotation about the x axis*	*p*	*Angular velocities*
5	*pitch - rotation about the y axis*	*q*	
6	*yaw - rotation about the z axis*	*r*	

M is system inertia matrix of the rigid body and added mass. When a ship accelerates or decelerates on water, quantities of surrounding water moving together with the hull increase the system inertia matrix:

$$M = M_S + M_A \tag{1.2}$$

Where M_S is mass and inertia moment matrix of the ship which is derived:

$$M_s = \begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_x & -I_{xy} & -I_{xz} \\ -mz_g & 0 & -mx_g & -I_{yx} & I_y & -I_{yz} \\ -my_g & mx_g & 0 & -I_{zx} & -I_{zy} & I_z \end{bmatrix} \tag{1.3}$$

M_A is added mass and added inertia moment matrix. Assuming that m_{ij} is a component in the i^{th} direction caused by acceleration in direction j. It is defined as hydrodynamic force due to unit acceleration and consists of 36 components:

$$M_A = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} \\ m_{21} & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} \\ m_{31} & m_{32} & m_{33} & m_{34} & m_{35} & m_{36} \\ m_{41} & m_{42} & m_{43} & m_{44} & m_{45} & m_{46} \\ m_{51} & m_{52} & m_{53} & m_{54} & m_{55} & m_{56} \\ m_{61} & m_{62} & m_{63} & m_{64} & m_{65} & m_{66} \end{bmatrix} \quad (1.4)$$

Added mass was first introduced by Dubua in 1776 [9] then was expressed mathematically and exactly by Green in 1883 and Stokes in 1843 [10] by expression of added mass of a sphere. Later many researchers generalized the notion of added mass to an arbitrary body moving in different regimes [4].

In the past, there were many studies determining the added mass including experiment and theoretical prediction.

For a particular ship, it can be obtained by experimental method. However, the experimental method is limited and cannot solve all added mass components. For simulating the ship motion especially in initial design stage, M has to be calculated by estimation method.

The principal for calculating of added mass is based on work of Ursell in 1949 [11] and Frank in 1967 [12] for arbitrary symmetric cross section. Then Keil introduced method for any arbitrary water depth based on a variation of the method of Ursell with Lewis conformal mapping in 1974 [13]. In 1967, Frank described the pulsating source method for deep water [12].

Nils Salvesen, E. O. Tuck and Odd Faltisen introduced new method to predict heave, pitch, sway, roll and yaw motions as well as wave-induced vertical and horizontal shear forces, bending moments, and torsional moments for a ship advancing at constant speed with arbitrary heading in regular waves in 1970 [8].

Previous studies showed that mij could not be determined by a unique method. A total way to determine all components of added mass and added moment of inertia is necessary. It is suggested to combine some methods to determine all the remaining components. This study aims to generalize and propose an optimal combination of different methods to calculate total components of the added mass matrix with computer.

2. Methods to Determine Added Mass

2.1. Elimination of Added Mass Components Due to Symmetry of Ship Hull and Added Mass Matrix

With marine ships, the body is symmetric on port-starboard (xy plane), it can be concluded that vertical motions due to heave and pitch induce no transversal force:

$$m_{32} = m_{34} = m_{36} = m_{52} = m_{54} = m_{56} = 0.$$

Due to the symmetry of the added mass matrix, it can concluded that mij = mji,:

$$m_{23} = m_{43} = m_{63} = m_{25} = m_{45} = m_{65} = 0.$$

The same consideration is applied for the longitudinal motions caused by acceleration in direction j = 2, 4, 6:

$$m_{12} = m_{14} = m_{16} = 0, \text{ and}$$

$$m_{21} = m_{41} = m_{61} = 0$$

Thus, for a ship moving in 6DOF, 36 components of added mass are reduced to 18:

$$M_A = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 & m_{15} & 0 \\ 0 & m_{22} & 0 & m_{24} & 0 & m_{26} \\ m_{31} & 0 & m_{33} & 0 & m_{35} & 0 \\ 0 & m_{42} & 0 & m_{44} & 0 & m_{46} \\ m_{51} & 0 & m_{53} & 0 & m_{55} & 0 \\ 0 & m_{62} & 0 & m_{64} & 0 & m_{66} \end{bmatrix} \quad (2.1)$$

The problem is to determine all remaining components of the matrix MA by predictive method.

2.2. Method of Equivalent Ellipsoid

Basing on theory of kinetic energy of fluid mij is determined from the formula:

$$m_{ij} = -\rho \oint_S \varphi_i \frac{\partial \varphi_j}{\partial n} dS \quad (2.2)$$

Where S is the wetted ship area, ρ is water density, φ_i is potentials of the flow when the ship is moving in i^{th} direction with unit speed. Potentials φ_i satisfy the Laplace equation [3].

To calculate m_{ij}, the ship can be relatively assumed as a 3D body such as sphere, spheroid, ellipsoid, rectangular, cylinder etc. (fig. 2)

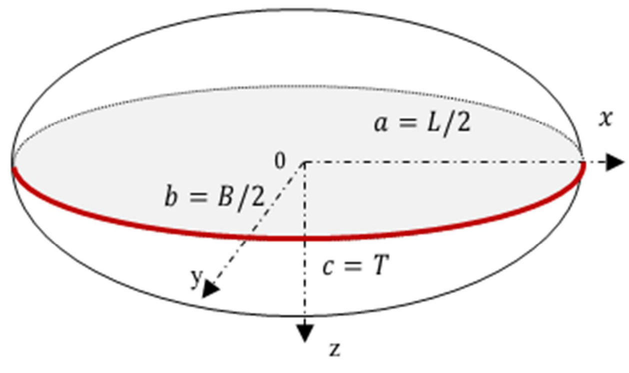

Figure 2. *Ship assumed as an Ellipsoid*

With a marine ship, the most representative of the hull is elongated ellipsoid with c/b = 1 and r = a/b. Where a, b are semi axis of the ellipsoid. Basing on theory of hydrostatics, m_{11}, m_{22}, m_{33}, m_{44}, m_{55}, m_{66} can be described [4], [7]:

$$m_{11} = mk_{11} \quad (2.3)$$

$$m_{22} = mk_{22} \quad (2.4)$$

$$m_{33} = mk_{33} \quad (2.5)$$

$$m_{44} = k_{44} I_{xx} \quad (2.6)$$

$$m_{55} = k_{55} I_{yy} \quad (2.7)$$

$$m_{66} = k_{66} I_{zz} \quad (2.8)$$

For simplifying the calculation, each component can be represented by correspondent k_{ij} called hydrodynamic coefficient.

$$k_{11} = \frac{A_0}{2-A_0} \quad (2.9)$$

$$k_{22} = \frac{B_0}{2-B_0} \quad (2.10)$$

$$k_{33} = \frac{C_0}{2-C_0} \quad (2.11)$$

$$k_{44} = 0 \quad (2.12)$$

$$k_{55} = \frac{(L^2-4T^2)^2(A_0-C_0)}{2(c^4-a^4)+(C_0-A_0)(4T^2+L^2)^2} \quad (2.13)$$

$$k_{66} = \frac{(L^2-B^2)^2(B_0-A_0)}{2(L^4-B^4)+(A_0-B_0)(L^2+B^2)^2} \quad (2.14)$$

where:

$$A_0 = \frac{2(1-e^2)}{e^3}\left[\frac{1}{2} ln\left(\frac{1+e}{1-e}\right) - e\right] \quad (2.15)$$

$$B_0 = C_0 = \frac{1}{e^2} - \frac{1-e^2}{2e^3} ln\left(\frac{1+e}{1-e}\right) \quad (2.16)$$

$$\text{with } e = \sqrt{1-\frac{b^2}{a^2}} = \sqrt{1-\frac{d^2}{L^2}} \quad (2.17)$$

d and L are maximum diameter and length overall. Inertia moment of the displaced water is approximately the moment of inertia of the equivalent ellipsoid:

$$I_{xx} = \frac{1}{120}\pi\rho LBT(4T^2+B^2) \quad (2.18)$$

$$I_{yy} = \frac{1}{120}\pi\rho LBT(4T^2+L^2) \quad (2.19)$$

$$I_{zz} = \frac{1}{120}\pi\rho LBT(B^2+L^2) \quad (2.20)$$

The limitation of this method is that the calculating result is only an approximation. The more equivalent to the elongated ellipsoid it is, the more accurate the result is obtained. Moreover, this method cannot determine component m_{24}; m_{26}, m_{35}; $m_{44,}$ m_{15} and m_{51}.

2.3. Strip Theory Method

The "Ordinary Strip Theory Method" was introduced by Korvin-Kroukovsky and Jacobs in 1957. Then it was developed by Tasai in 1969 with a *"Modified Strip Theory Method"* [6].

Basing on this method a ship can be made up of a finite number of transversal 2D slices (fig. 3). Each slice has a form closely resembling the segment of the representative ship and its added mass can be easily calculated. The added mass of the whole ship is obtained by integration of the 2D value over the length of the hull. Components m_{ij} are determined:

$$m_{22} = \int_{L_1}^{L_2} m_{22}(x)dx \quad (2.21)$$

$$m_{33} = \int_{L_1}^{L_2} m(x)dx \quad (2.22)$$

$$m_{24} = \int_{L_1}^{L_2} m_{24}(x)dx \quad (2.23)$$

$$m_{44} = \int_{L_1}^{L_2} m_{44}(x)dx \quad (2.24)$$

$$m_{26} = \int_{L_1}^{L_2} m_{22}(x)xdx \quad (2.25)$$

$$m_{35} = -\int_{L_1}^{L_2} m_{33}(x)xdx \quad (2.26)$$

$$m_{46} = \int_{L_1}^{L_2} m_{24}(x)xdx \quad (2.27)$$

$$m_{66} = \int_{L_1}^{L_2} m_{22}(x)x^2dx \quad (2.28)$$

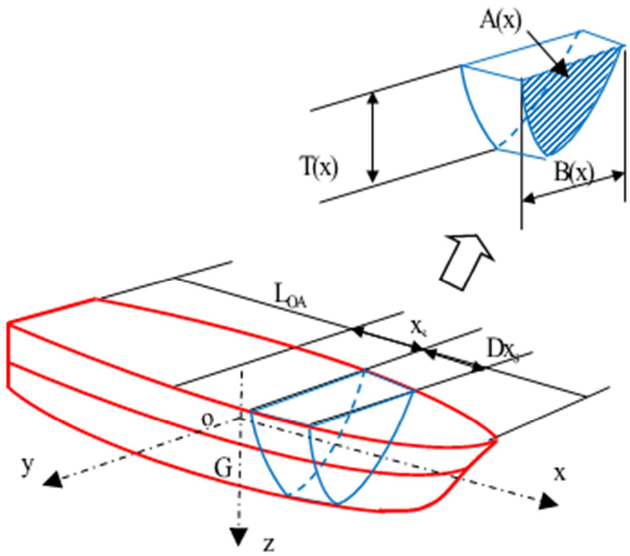

Figure 3. *The hull is divided into slices.*

Where $m_{ij}(x)$ is added mass of 2D cross section at location x_s.

Lewis transformation mapping:

In practice the form of each slice is various and complex. For numbering and calculating in computer Lewis transformation is the most proper solution.

With this method a cross section of hull is mapped conformably to the unit semicircle (ζ-plane) which is derived in details in [2], [4], [6], [7] (fig. 4):

$$\zeta = y + iz = ia_0\left(\sigma + \frac{p}{\sigma} + \frac{q}{\sigma^3}\right) \quad (2.29)$$

and the unit semicircle is derived:

$$\sigma = e^{i\varphi} = cos\theta + isin\theta \quad (2.30)$$

Where $i^2 = -1$; $a_0 = \frac{T(x)}{1+p+q}$. By substituting into the formula (2.15), descriptive parameters of a cross section can be obtained:

$$\begin{cases} y = [(1+p)sin\theta - bsin3\theta]\frac{B(x)}{2(1+p+q)} \\ z = -[(1-p)cos\theta + bcos3\theta]\frac{B(x)}{2(1+p+q)} \end{cases} \quad (2.31)$$

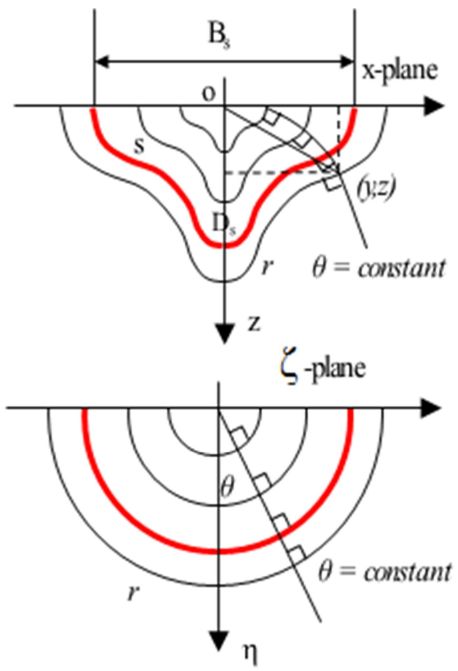

Figure 4. *Mapping relation between x- and ζ –plane.*

Where B(x), T(x) are the breadth and draft of the cross section s. Parameter p, q are described by means of the ratio H(x) and β(x).

$$H(x) = \frac{B(x)}{2T(x)} = \frac{1+p+q}{1-p+q} \quad (2.32)$$

$$\beta(x) = \frac{A(x)}{B(x)T(x)} = \frac{\pi}{4}\frac{1-p^2-3q^2}{(1+q)^2-p^2} \quad (2.33)$$

Parameter θ is physical meaningless. It corresponds to the polar angle of given point prior to conformal transformation from a semicircle: π/2 ≥ θ ≥ -π/2.

$$q = \frac{\frac{3}{4}\pi + \sqrt{\left(\frac{\pi}{4}\right)^2 - \frac{\pi}{2}\alpha(1-\gamma^2)}}{\pi + \alpha(1-\gamma^2)} - 1\,;\; p = (q+1)q \quad (2.34)$$

$$\alpha = \beta - \frac{\pi}{4}\,;\; \gamma = \frac{H-1}{H+1} \quad (2.35)$$

The component $m_{ij}(x)$ of each section is determined by formulas:

$$m_{22}(x) = \frac{\rho\pi T(x)^2}{2}\frac{(1-p)^2+3q^2}{(1-p+q)^2} = \frac{\rho\pi T(x)^2}{2}k_{22}(x) \quad (2.36)$$

$$m_{33}(x) = \frac{\rho\pi B(x)^2}{8}\frac{(1+p)^2+3q^2)}{(1+p+q)^2} = \frac{\rho\pi B(x)^2}{8}k_{33}(x) \quad (2.37)$$

$$m_{24}(x) = \frac{\rho T(x)^3}{2}\frac{1}{(1-p+q)^2}\left\{-\frac{8}{3}P(1-p) + \frac{16}{35}q^2(20-7p) + q\left[\frac{4}{3}(1-p)^2 - \frac{4}{5}(1+p)(7+5p)\right]\right\} = \frac{\rho T(x)^3}{2}k_{24}(x) \quad (2.38)$$

$$m_{44}(x) = \rho\frac{\pi B(x)^4}{256}\frac{16[p^2(1+q)^2+2q^2]}{(1-p+q)^4} = \frac{\rho\pi B(x)^4}{256}k_{44}(x) \quad (2.39)$$

Then, total m_{ij} is calculated:

$$m_{22} = \mu_1(\lambda = \frac{L}{2T})\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)dx \quad (2.40)$$

$$m_{33} = \mu_1(\lambda = \frac{L}{B})\frac{\rho\pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)dx \quad (2.41)$$

$$m_{24} = \mu_1(\lambda = \frac{L}{2T})\frac{\rho}{2}\int_{L_1}^{L_2} T(x)^3 k_{24}(x)dx \quad (2.42)$$

$$m_{44} = \mu_1(\lambda = \frac{L}{2T})\frac{\rho\pi}{256}\int_{L_1}^{L_2} B(x)^4 k_{44}(x)dx \quad (2.43)$$

$$m_{26} = \mu_2(\lambda = \frac{L}{2T})\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)xdx \quad (2.44)$$

$$m_{35} = -\mu_2\left(\lambda = \frac{L}{B}\right)\frac{\rho\pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)xdx \quad (2.45)$$

$$m_{46} = \mu_2\left(\lambda = \frac{L}{2T}\right)\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^3 k_{24}(x)xdx \quad (2.46)$$

$$m_{66} = \mu_2\left(\lambda = \frac{L}{2T}\right)\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)x^2dx \quad (2.47)$$

Where $\mu_1(\lambda)$, $\mu_2(\lambda)$ are corrections related to fluid motion along x-axis due to elongation λ of the body [4]. $\mu_1(\lambda)$ is correction for the added mass which can be used the Pabst correction as the most well-known experimental correction:

$$\mu_1(\lambda) = \frac{\lambda}{\sqrt{1+\lambda^2}}\left(1 - 0.425\frac{\lambda}{\sqrt{1+\lambda^2}}\right) \quad (2.48)$$

$\mu_2(\lambda)$ is correction for the added moment of inertia using the theoretical formula:

$$\mu_2(\lambda) = k_{66}(\lambda, q)q\left(1 + \frac{1}{\lambda^2}\right) \quad (2.49)$$

It also necessary to note that specific forms of ships consisting of re-entrant forms and asymmetric forms are not acceptable for applying Lewis forms [4], [6].

To have a section accepted for Lewis forms, the area coefficient β is bounded by a lower limit to omit re-entrant Lewis forms and by an upper limit to omit asymmetric Lewis forms:

For H(x) ≥ 1.0:

$$\frac{3\pi}{32}(2 - \frac{1}{H(x)}) < \beta < \frac{\pi}{32}(10 + H(x) + \frac{1}{H(x)}) \quad (2.50)$$

For H(x) ≤ 1.0:

$$\frac{3\pi}{32}(2 - H(x)) < \beta < \frac{\pi}{32}(10 + \frac{1}{H(x)} + H(x)) \quad (2.51)$$

2.4. Determining Remaining Added Mass

It is understandable that m_{13} are relatively small in comparison with total added mass and can be ignored. Thus, $m_{13} = m_{31} \approx 0$.

The Ellipsoid and Strip theory methods do not determine component m_{15}. It is approximately considered that the component m_{15} and m_{24} are caused by the hydrodynamic force due to m_{11} and m_{22} with the force center at the center of buoyancy of the hull Z_B [2]. Therefore:

$$m_{15} = m_{51} = m_{11}Z_B \quad (2.52)$$

$$m_{24} = m_{42} = -m_{22}Z_B \tag{2.53}$$

Thus, the formula to calculate m_{15} and m_{51} is obtained:

$$m_{15} = m_{51} = -m_{11}\frac{m_{42}}{m_{22}} \tag{2.54}$$

When m_{24} and m_{42} can be obtained by the Strip theory method.

3. Calculation and Assessment

For comparison and assessment of results, the ship model experimented by Motora [1960] (see fig. 5) is used [2], [14]:

- L: 170.0m
- B: 22.8m
- T: 9.3m
- Displacement: 20.876 MT

The ship is divided longitudinally into 20 slices with ratio B(x)/2T(x) and A(x)/B(x) as in table 2:

Table 2. *Sectional presentation of H(x) and β(x).*

Slice	Sta	dx	X	H=B/2T	β=A/B
21	10.000	4.146	95.366	0.000	0.000
20	9.750	4.146	91.220	0.160	0.540
19	9.500	4.146	87.073	0.275	0.562
18	9.250	4.146	82.927	0.366	0.548
17	9.000	8.293	74.634	0.448	0.556
16	8.500	8.293	66.341	0.650	0.669
15	8.000	16.585	49.756	0.860	0.679
14	7.000	16.585	33.171	1.136	0.782
13	6.000	16.585	16.585	1.226	0.870
12	5.000	16.585	0.000	1.226	0.921
11	4.000	16.585	-16.585	1.226	0.885
10	3.000	16.585	-33.171	1.211	0.774
9	2.000	8.293	-41.463	0.980	0.674
8	1.500	8.293	-49.756	0.799	0.529
7	1.000	4.146	-53.902	0.605	0.410
6	0.750	4.146	-58.049	0.486	0.440
5	0.500	4.146	-62.195	0.404	0.470
4	0.250	4.146	-66.341	0.299	0.500
3	0.000	2.073	-68.415	0.910	0.572
2	-0.125	2.073	-70.488	1.028	0.500
1	-0.250	0.000	-70.488	0.000	0.000

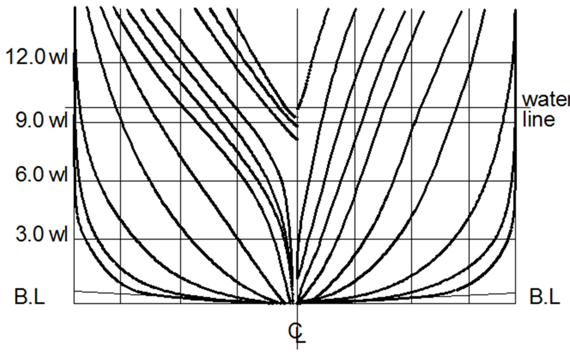

Figure 5. *Actual body plan of the sample ship.*

Numbering values of mapping are calculated and displayed in curves on computer in figure 6, 7 and 8. The results indicate that the transformation is relatively proper in comparison with the actual forms.

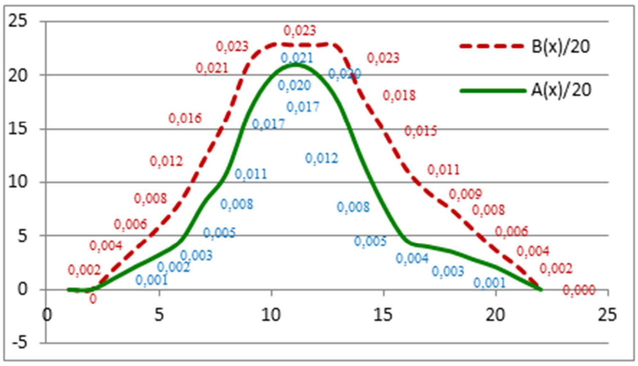

Figure 6. *B(x) and A(x).*

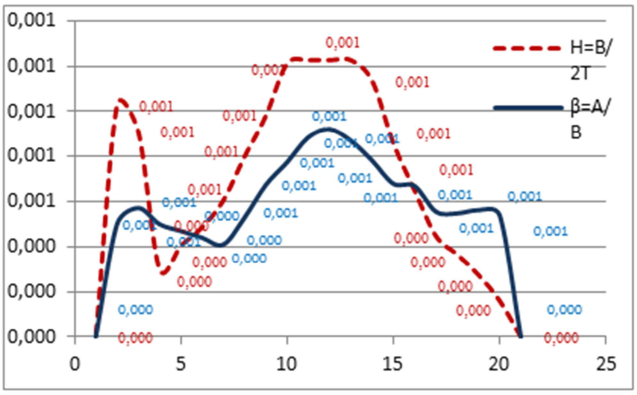

Figure 7. *H(x) and β(x).*

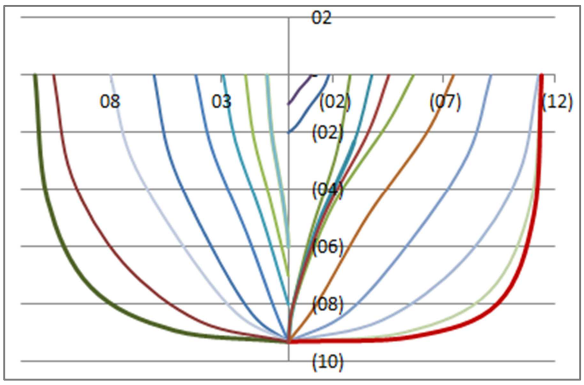

Figure 8. *Body plan presented using Lewis forms.*

The calculating results of above suggested methods and experimental data are summed up and presented in table 3. The column "proposed" is the value suggested for application.

Table 3. *Comparing results of m_{ij}.*

m_{ij}	Ellipsoid	Lewis	experiment	Proposed
m_{11}/m	0.033		0.032	0.033
m_{22}/m	0.939	0.986	1.040	0.986
m_{33}/m	0.939	1.004		1.004
m_{44}/mB^2		0.010		0.010
m_{55}/mL^2	0.039			0.039
m_{66}/mL^2	0.039	0.045	0.056	0.045
m_{24}/m		0.628		0.628
m_{26}/mL		0.023		0.023
m_{35}/mB		-0.042		-0.042
m_{46}/mL		0.107		0.107
m_{15}/m	-0.021			-0.021

Basing on the above results, it is concluded that Strip theory method with Lewis forms can determine most component m_{ij} with high accuracy due to equivalent transformation. However, this method cannot determine component m_{11}, m_{55} which can be solved by considering the ship as an elongated ellipsoid.

As for the component $m_{15} = m_{51}$, this value is not so high, the estimation in the formula (2.54) is satisfied and acceptable.

4. Suggestion

All components of added mass and added moment of inertia can be calculated by combination of different methods. This study suggests a method to determine all components m_{ij} of the added mass matrix as in table 4 below:

Table 4. *Suggestion for determining m_{ij}.*

m_{ij}	Formula	Method
m_{11}	mk_{11}	(*)
m_{22}	$\mu_1(\lambda = \frac{L}{2T})\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)dx$	(**)
m_{33}	$\mu_1(\lambda = \frac{L}{B})\frac{\rho\pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)dx$	(**)
m_{44}	$\mu_1(\lambda = \frac{L}{2T})\frac{\rho\pi}{256}\int_{L_1}^{L_2} B(x)^4 k_{44}(x)dx$	(**)
m_{55}	$k_{55}I_{yy}$	(*)
m_{66}	$\mu_2\left(\lambda = \frac{L}{2T}\right)\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)x^2dx$	(**)
m_{24}, m_{42}	$\mu_1\left(\lambda = \frac{L}{2T}\right)\frac{\rho}{2}\int_{L_1}^{L_2} T(x)^3 k_{24}(x)dx$	(**)
m_{26}, m_{62}	$\mu_2(\lambda = \frac{L}{2T})\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^2 k_{22}(x)xdx$	(**)
m_{35}, m_{53}	$-\mu_2\left(\lambda = \frac{L}{B}\right)\frac{\rho\pi}{8}\int_{L_1}^{L_2} B(x)^2 k_{33}(x)xdx$	(**)
m_{46}, m_{46}	$\mu_2\left(\lambda = \frac{L}{2T}\right)\frac{\rho\pi}{2}\int_{L_1}^{L_2} T(x)^3 k_{24}(x)xdx$	(**)
m_{15}, m_{51}	$m_{15} = m_{51} = -m_{11}\frac{m_{42}}{m_{22}}$	(***)
m_{13}, m_{31}	0	(****)

(*) determined by assuming the ship as an equivalent elongated ellipsoid.
(**) determined by "Strip theory" with Lewis transformation.
(***) determined by formula (2.54).
(****) can be ignored.

5. Conclusion

In order to determine added mass and added moment of inertia, various methods are necessary to use for particular groups of component mij. The suggested combination method can determine all mij necessary to establish the mathematical model for motion of marine ships in 6DOFs used for computerizing simulation especially at the initial design stage.

However, for a hull with port-starboard asymmetry or re-entrant forms the Lewis shape of ship frame is inapplicable, further assessment for proper calculation should be taken into consideration for obtaining satisfied results.

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