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A Comprehensive Survey on the Metric Dimension Problem of Graphs and Its Types

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Abstract: Consider a robot that is navigating a graph-based environment and trying to figure out where it is at the moment. It can send a signal to determine how far away it is from every set of fixed landmarks. We address the problem of finding exactly the minimum number of landmarks required and their perfect placement to make sure the robot can always locate itself. The graph's metric dimension is the quantity of landmarks, and the graph's metric basis is the set of nodes on which they are distributed. The metric dimension of a graph is the smallest set of nodes needed to uniquely identify every other node using the shortest path distances. Optimization, network theory, navigation, pattern recognition, image processing, locating the origin of a spread in a network, canonically labeling graphs, and embedding symbolic data in low-dimensional Euclidean spaces are a few examples of applications for metric dimension. Also, Due to its many and varied applications in fields like social sciences, communications networks, algorithmic designs, and others, the study of dominance is the kind of metric dimension that is developing at the fastest rate. This survey provides a self-contained introduction to the metric dimension and an overview of several metric dimension results and applications. We also present algorithms for computing the metric dimension of families of graphs.

Keywords: Metric Dimension, Resolving Set, Double Resolving Set, Edge Metric Dimension

1. Introduction

Let's assume that the connected, undirected, simple graph $G=(V,E)$ has the vertex set V and the edge set E and $d(u,v)$ be the shortest path between two vertices $u,v \in V(G)$. An ordered vertex set $B=\{x_1,x_2,\ldots,x_k\}\subseteq V(G)$ is a resolving set of G if the representation

$$r(v|\,B) = (d(v, x_1), d(v, x_2),\ldots,d(v, x_k))$$

is unique for every $v\in V(G)$. The metric dimension of G, abbreviated $\dim(G)$, is the cardinality of minimum resolving set of G [1].

In order to uniquely identify the location of an intruder in a network and a minimum number of sonar units must be deployed as part of the sonar system defending the coast line., Slater [2, 3] developed the idea of a minimum resolving set as a locating set of G and uses the cardinality of B as a locating number. The concepts of the smallest resolving set as a metric foundation of G and the cardinality of B as the metric dimension of G were independently introduced by Harary and Melter in [4].

2. Metric Dimension

Computing the metric dimension of graphs using the metric dimension problem (*MDP*) is a difficult combinatorial optimization problem. The metric dimension of a connected graph G is the minimum number of vertices in a subset B of G such that all other vertices are uniquely determined by their distances to the vertices in B. In this case, B is called a *metric basis* for G. The *basic distance* of a metric two-dimensional graph G is the distance between the elements of B. Giving a characterization for those graphs whose metric dimensions are two, they enumerated the number of n vertex metric two-dimensional graphs with the basic distance 1 [5].

Pan et al. [6] computed the metric dimension of the splitting graphs $S(P_n)$ and $S(C_n)$ of a path and cycle. They proved that the metric dimension of these graphs varies and depends on the number of vertices of the graph. Hussain et al.

[7] introduced a line graph of honeycomb network and then they calculated the metric dimension on line graph of honeycomb network. Murdiansyah et al. [8] presented a PSO (Particle Swarm Optimization) algorithm for determining the metric dimension of graphs. They choosed PSO because of its simplicity, robustness, and adaptability for various optimization problems. Mulyono et al. [9] devoted to determine the metric dimension of friendship graph F_n, lollipop graph $L_{m,n}$ and Petersen graph $P_{n,m}$. Fernau et al. [10] presented a linear-time algorithm for computing the metric dimension for chain graphs, which are bipartite graphs whose vertices can be ordered by neighborhood inclusion. Garces et al. [11] computed the metric dimension of truncated wheels. Chuanjun et al. [12] showed that the metric dimension of the join of two path graphs is unbounded because of its dependence on the size of the paths. Mohamed et al. [13] studied the metric dimension of subdivisions of several graphs, including the Lilly graph, the Tadpole graph and the special trees star tree, bistar tree and coconut tree.

The metric dimensions of path powers three and four are unbounded, as demonstrated by Nawaz et al. [14]. They also showed multiple results about the edges of the power of path and power of total graph. Ahmad et al. [15] found the metric dimension of Kayak paddles graph and cycles with chord. Rehman et al. [16] computed the metric dimension of Arithmetic Graph A_m, when m has exactly two distinct prime divisors. Imran et al. [17] studied the metric dimension of some classes of convex polytopes which are obtained by the combinations of two different graphs of convex polytopes. Mladenovic et al. [18] proposed a variable neighborhood search approach for solving the metric dimension and minimal doubly resolving set problems. Kratica et al. [19] computed the metric dimension of graphs by a genetic algorithm that used the binary encoding and the standard genetic operators adapted to the problem. Jäger et al. [20] found that the metric dimension of $Z_n \times Z_n \times Z_n$, $n \geq 2$ is $\lfloor \frac{3n}{2} \rfloor$. Imran et al. [21] investigated the metric dimension of the barycentric subdivision of Möbius ladders, the generalized Petersen multigraphs $P(2n, n)$ and proved that they have metric dimension 3 when n is even and 4 when n is odd. Nadeem et al. [22] discussed the metric dimension of toeplitz graphs with two and three generators. Korivand et al. [23] presented the metric dimension threshold of some families of graphs and a characterization of graphs of order for which the metric dimension threshold equals 2, n-2 and n-1. Munir et al. [24] discussed a new regular family of constant metric dimension. Nazeer et al. [25] computed the metric dimension of some new graphs and named them middle graphs, -total graphs, symmetrical planar pyramid graph, reflection symmetrical planar pyramid graph, middle tower path graph and reflection middle tower path graph.

3. Types of Metric Dimension

In this section, we discuss the types of metric dimension of some graphs.

Stephen et al. [26] determined the total metric dimension of paths, cycles, grids, and of the 3-cube and the Petersen graph. Kratica et al. [27] presented genetic algorithm for determining the strong metric dimension of graphs that used binary encoding and standard genetic operators adapted to the problem. Zafari et al. [28] determined the cardinality of minimal doubly resolving sets and strong metric dimension of jellyfish graph and cocktail party graph. Ameen et al. [29] discussed the localization problem in Kayak paddle graphs KP(l,m,n) for $l,m \geq 3$ and $n \geq 2$ by computing edge version of metric and double metric dimensions. Zafari [30] determined the minimal resolving set, doubly resolving set, and strong metric dimension for a class of Cayley graphs. Chitra et al. [31] introduced the concept of non-isolated resolving set and non-isolated resolving number and presented several basic results. Okamoto et al. [32] presented the exact value of local metric dimension of some graphs. Gómez et al. [33] studied the problem of finding exact values for the local metric dimension of corona product of graphs. Wei et al. [34] studied the edge metric dimension problem for certain classes of planar graphs. Ramírez et al. [35] studied study the problem of finding exact values or bounds for the local metric dimension of strong product of graphs. Meera et al. [36] studied the radiatic dimension of some standard graphs and characterize graphs of diameter 2 that are radio graceful. Marsidi et al. [37] gave the local metric dimension of some operation graphs such as joint graph P_n+C_m, amalgamation of parachute, amalgamation of fan. Feng et al. [38] studied the (fractional) metric dimension for the hierarchical product of rooted graphs. Velázquez et al. [39] showed that the computation of the local metric dimension of a graph with cut vertices is reduced to the computation of the local metric dimension of the so-called primary subgraphs. The main results are applied to specific constructions including bouquets of graphs, rooted product graphs, corona product graphs, block graphs and chain of graphs. Budianto et al. [40] computed the local metric dimension of starbarbell graph, $K_m \odot P_n$ graph and M¨obius ladder graph for even positive integers $n \geq 6$. Moreno et al. [41] obtained tight bounds and closed formulae for the k-metric dimension of the lexicographic product of graphs in terms of the k-adjacency dimension of the factor graphs. Cynthia et al. [42] investigated the local metric basis and local metric dimension of Cyclic Split Graph. Laihonen [43] studied the problem of the ℓ-set-metric dimension in two infinite classes of graphs, namely, the two dimensional grid graphs and the n-dimensional binary hypercubes.

Mohamed et al. [44] computed the exact value of the secure resolving set of some networks such as trapezoid network, Z-(P_n) network, open ladder network, tortoise network and also determined the domination number of the networks such as the twig network, double fan network, bistar network and linear kc_4–snake network. Mohamed et al. [45] presented the first attempt to compute heuristically the minimum connected dominant resolving set of graphs by a binary version of the equilibrium optimization algorithm. Moreno et al. [46] studied the simultaneous metric dimension of families composed by lexicographic product graphs. Mohamed et al. [47] presented the first attempt to compute heuristically the minimum connected resolving set of graphs by a binary version of the Enhanced Harris Hawks Optimization. Cabaro et al. [48] investigated the 2-resolving dominating set in the join, corona and lexicographic product of two graphs and determined the bounds of the 2-resolving dominating number of these graphs. In [49] If a resolving set induces a star, it is said to be a star resolving set, and if it induces a route, it is said to be a path resolving set. The star resolving number and path resolving number are the minimal cardinality of these sets. They investigated these resolving parameters for the hypercube networks.

4. Applications of Metric Dimension

Robots that are in motion can send signals to a set of fixed landmarks to calculate their distance from them. Finding out how many landmarks and where to put them so that the robot can always identify its location is an issue that is essential to its ability to know where it is right now [50, 51]. The number of landmarks is referred to as the graph's metric dimension, and the set of nodes on which they are distributed is known as the graph's metric basis.

The metric dimension problem has recently been widely applied to resolve several practical problems. It is used in chemistry to distinguish between different molecular compounds [52]. It is also used to create reliable sensor networks [53], where network invaders are verified using the networks' metric base, connected joins in graphs [54] and coin weighing problems [55]. It can be used to determine the source of information circulating on networks in addition to recognizing intruders. It is used, for instance, to find patient 0 or other items in complex networks.

5. Conclusion

In this paper, we have introduced an introduction to the metric dimension and an overview of several metric dimension results and applications and we have presented algorithms for computing the metric dimension of families of graphs.

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