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Novel Exact Soliton Solutions of Cahn–Allen Models with Truncated M-fractional Derivative

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Abstract: In recent research the effect of different type fractional derivative to nonlinear evolution equations plays a vital rule in the various branches of science and engineering. The nonlinear physical phenomena are expressed by nonlinear partial differential equations which are characteristics on the field of solid-state physics, plasma physics, fluid mechanics, chemical physics, mechanics, biology, chemistry, and so on. To visualize and identify their properties, it is essential to find the exact and multi solitons of the related nonlinear partial differential equation. In this work, we investigate more soliton solutions for novel truncated M-fractional Cahn–Allen (t-MfCA) models to secure different soliton solutions via the unified scheme. This model has significant in the area of mathematical physics and also known as reaction–diffusion. The obtained solutions are expressed in turns of trigonometric, hyperbolic and rational function solution under the condition on the free constraints. This work offers the kink, singular soliton, different type interaction of kink and lump wave for the numerical value of the free constraints. Form the obtained solution it is shown that the implement method is more informal, effective and reliable as compared to other methods. The calculation and all analytic solutions are verified by computational software MAPLE 18.

Keywords: Cahn–Allen Models, Unified Scheme, Reaction–Diffusion

1. Introduction

In recent years, the new exact soliton solutions to the nonlinear evolution equations (NLEEs) with the most general fractional derivative operator is widely used to describe many important phenomena and dynamic processes in plasma physics, biological dynamics, quantum mechanics, electro-magnetic waves and many others field. For the resolution, to govern the solutions of these models' numerous methods have been recognized. For example, the Homotopy Perturbation Method [1], Sumudu decomposition method [2], Extended tanh approach [3], New extended (G'/G)-expansion scheme [4], generalized exponential rational function method [5], extended (G′/G)-expansion approach [6], Hamiltonian formation [7], sine-Gordon expansion method [8], Modified Double Sub-equation Method [9], Bright and dark optical solitons method [10], EMSE method [11], inverse scattering method [12], Hirota bilinear approach [13-15], MSE scheme [16-19], multiple exp-function algorithm [20], improved (G′/G)-expansion approach [21-22], $\exp(-\varphi(\xi))$-expansion method [23] and so on.

In this work, exact soliton solutions are investigated from the Cahn–Allen equation. Here we start the Cahn–Allen equation in the following form:

$$D_{M,t}^{\alpha,\gamma}H - H_{xx} - H + H^3 = 0. \tag{1}$$

Some of the reliable and effective schemes named as: extended Sinh–Gordon equation expansion method [24], first integral method [25, 26] and Bernoulli sub-equation function method [27]. We successes to investigate novel exact soliton, we implement unified scheme [28] to solve Cahn–Allen equation.

2. Truncated M-fractional Derivative

Assume that $H:(0,\infty) \to \mathbb{R}$ the Truncated M-fractional derivative of H with order m exhibited is given by

$$D_{N,t}^{m,d}H(t) = \lim_{l\to 0}\frac{H\left(tF_d(lt^{1-m})\right)-H(t)}{l}.; 0 < m < 1, d > 0.$$

Here $F_d(\cdot)$ is a truncated Mittag-Leffler function of one parameter which defined as [29]:

$$F_d(z) = \sum_{j=0}^{i}\frac{z^j}{\Gamma(dj+1)}.$$

Properties of Truncated M-fractional derivative:

Suppose $0 < m < 1, d > 0, a, b \in \mathbb{R}$ and H, h $m-$differentiable at the point $t > 0$, then

$$D_{N,t}^{m,d}\left(aH(t) + bh(t)\right) = aD_{N,t}^{m,d}g(t) + bD_{N,t}^{m,d}h(t).$$

$$D_{N,t}^{n,d}\left(H(t)h(t)\right) = H(t)D_{N,t}^{m,d}h(t) + h(t)D_{N,t}^{m,d}H(t)$$

$$D_{N,t}^{m,d}\left(\frac{H(t)}{h(t)}\right) = \frac{H(t)\,D_{N,t}^{m,d}h(t)+h(t)D_{N,t}^{m,d}H(t)}{h(t)^2}$$

$D_{N,t}^{m,d}(c) = 0$ where $H(t) = c$ is constant function.

$$D_{N,t}^{m,d}H(t) = \frac{t^{1-n}}{\Gamma(d+1)}\frac{d}{dt}\left(H(t)\right).$$

3. The Unified Method

To describe the procedure of the unified method, consider a higher order nonlinear evolution equation:

$$P[u] = P(f, f_{xx}, D_{M,t}^{\lambda,d}f, D_{M,t}^{\lambda,d}(f_{xx}) \;\dots\dots\dots). \tag{2}$$

where $f = f(x,t)$ and P is a polynomial about f and its derivatives.

Step-01:

At first, we apply $\xi = kx - \omega\frac{\Gamma(d+1)}{\lambda}t^{\lambda}$ to obtain an ordinary differential equation from eq. (2) as:

$$Q[u] = Q(f, k^2, f_{\xi\xi}, -\omega\, f_{\xi}, -k^2\omega f_{\xi\xi\xi} \;\dots\dots\dots) \tag{3}$$

Step-02: Let the trail solution of eq. (3) be,

$$f(\xi) = a_0 + \sum_{i=-N}^{N}[a_i\psi(\xi)^i]. \tag{4}$$

where a_i $(i = 0, \pm 1, \pm 2, \dots, N)$ are constants to be investigated.

The eq. (4) satisfied the following condition,

$$\psi(\xi)' = \left(\psi(\xi)\right)^2 + \beta. \tag{5}$$

Step-03: We usually balance the highest-order derivatives and the nonlinear term in eq. (3), to find the positive integer N.

Step-04: Substituting $f(\xi), f_{\xi\xi}, f_{\xi}, f_{\xi\xi\xi}$ in eq. (3), we obtain a polynomial of $\psi^{\pm i}, i = 0,1,2 \dots.$

Step-05: The co-efficient of $\psi^{\pm i}$ equating to zero and solving them to estimate of the values a_i, k, ω Consequently, the deserved solutions will be obtained.

From eq. (4), the following solution are obtained:

The following hyperbolic function solutions occur for $\beta < 0$

$$\psi(\xi) = \begin{cases} \dfrac{\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\,cosh\left(2\sqrt{-\beta}(\xi+\chi)\right)}{msinh\left(2\sqrt{-\beta}(\xi+\chi)\right)+n}, \\ \dfrac{-\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\,cosh\left(2\sqrt{-\beta}(\xi+\chi)\right)}{msinh\left(2\sqrt{-\beta}(\xi+\chi)\right)+n}, \\ \sqrt{-\beta} + \dfrac{-2m\sqrt{-\beta}}{m+cosh\left(2\sqrt{-\beta}(\xi+\chi)\right)-sinh\left(2\sqrt{-\beta}(\xi+\chi)\right)}, \\ -\sqrt{-\beta} + \dfrac{2m\sqrt{-\beta}}{m+cosh\left(2\sqrt{-\beta}(\xi+\chi)\right)-sinh\left(2\sqrt{-\beta}(\xi+\chi)\right)}, \end{cases} \tag{6}$$

The following trigonometric function solutions occur for $\beta > 0$

$$\psi(\xi) = \begin{cases} \dfrac{\sqrt{(m^2-n^2)\beta} - m\sqrt{\beta}\,cos\left(2\sqrt{\beta}(\xi+\chi)\right)}{msin\left(2\sqrt{\beta}(\xi+\chi)\right)+n}, \\ \dfrac{-\sqrt{(m^2-n^2)\beta} - m\sqrt{\beta}\,cos\left(2\sqrt{\beta}(\xi+\chi)\right)}{msin\left(2\sqrt{\beta}(\xi+\chi)\right)+n}, \\ i\sqrt{\beta} + \dfrac{-2mi\sqrt{\beta}}{m+cos\left(2\sqrt{\beta}(\xi+\chi)\right)-i\,sin\left(2\sqrt{\beta}(\xi+\chi)\right)}, \\ -i\sqrt{\beta} + \dfrac{2li\sqrt{\beta}}{m+cos\left(2\sqrt{\beta}(\xi+\chi)\right)+i\,sin\left(2\sqrt{\beta}(\xi+\chi)\right)}, \end{cases} \tag{7}$$

The rational function solution occurs for $\beta = 0$

$$\psi(\xi) = -\frac{1}{\xi+\chi}. \tag{8}$$

4. Application of Unified Method in Time M-fractional Cahn–Allen Equation

Here we operate the unified methods with novel truncated M-fractional derivative to derive several types of solitons for Cahn–Allen equation. John W. Cahn and Sam Allen discovered the nonlinear Cahn–Allen equation in the concerning form [24]:

$$D_{M,t}^{\alpha,\gamma}u - u_{xx} - u + u^3 = 0. \tag{9}$$

Using Truncated M-fractional derivative, the transformation variable, $u(x,t) = u(\xi)$ and $\xi = kx - \omega\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}$ eq. (9) reduces to,

$$-\omega\frac{du}{d\xi} + k^2\frac{d^2u}{d\xi^2} + u - u^3 = 0. \tag{10}$$

According to step-3 and step-2 the solution of eq. (10) is

$$u = a_0 + a_1\varphi(\xi) + \frac{a_2}{\varphi(\xi)}. \tag{11}$$

Here a_0, a_1 and a_2 are the unknowns. Injecting eq. (11) into eq. (10), we accomplish the algebraic equations containing a_0, a_1, a_2 and other parameters.

$$-2k^2a_1 + a_1^3 = 0.$$

$$3a_0a_1^2 - \omega a_1 = 0.$$

$$-2\beta k^2a_1 + 3a_1a_0^2 + 3a_2a_1^2 - \mu_1 = 0.$$

$$-\omega\beta a_1 + a_0^3 + 6a_0a_1a_2 + \omega a_2 - a_0 = 0.$$

$$-2\beta k^2a_2 + 3a_2a_0^2 + 3a_1a_2^2 - a_2 = 0.$$

$$3a_0a_1^2 + \beta\omega a_2 = 0.$$

$$-2\beta^2k^2a_2 + a_2^3 = 0.$$

By solving the above equation with the help of commercial software MAPLE 18, we obtain the succeeding sets:

Set-01: $k = \sqrt{\frac{-1}{8\beta}}, \omega = \frac{3}{2}\sqrt{\frac{-1}{4\beta}}, a_0 = \frac{1}{2}, a_1 = \sqrt{\frac{-1}{4\beta}}, a_2 = 0.$

Set-02: $k = \sqrt{\frac{-1}{32\beta}}, \omega = -\frac{3}{2}\sqrt{\frac{-1}{16\beta}}, a_0 = -\frac{1}{2}, a_1 = \sqrt{\frac{-1}{16\beta}}, a_2 = -\beta\sqrt{\frac{-1}{16\beta}}.$

Solution for Set-I: We take the parameters as:

$$k = \sqrt{\frac{-1}{8\beta}}, \omega = \frac{3}{2}\sqrt{\frac{-1}{4\beta}}, a_0 = \frac{1}{2}, a_1 = \sqrt{\frac{-1}{4\beta}}, a_2 = 0$$

For the above parameters, with the help of Eq. (11), for $\beta < 0$ the solution of Eq. (10) becomes,

$$u_{1,1} = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{\frac{1}{-\beta}}\left(\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)\right)}{m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n}. \tag{12}$$

$$u_{1,2} = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{\frac{1}{-\beta}}\left(-\sqrt{-(m^2+n^2)\beta} - l\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)\right)}{m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n}. \tag{13}$$

$$u_{1,3} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{-1}{\beta}}\left(\sqrt{-\beta} + \frac{-2l\sqrt{-\beta}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)}\right). \tag{14}$$

$$u_{1,4} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{-1}{\beta}}\left(-\sqrt{-\beta} + \frac{2l\sqrt{-\beta}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)}\right). \tag{15}$$

Where $\xi = kx - \omega\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}$ and $m, n, C, \gamma, \beta, \alpha$ are arbitrary constant with $0 < \alpha < 1, \gamma > 0.$ Also, for the same parameters and $\beta > 0$, the solution of Eq. (10) becomes

$$u_{1,5} = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{\frac{-1}{\beta}}\left(\sqrt{(m^2-n^2)\beta} - l\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)\right)}{m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n}. \tag{16}$$

$$u_{1,6} = \frac{1}{2} + \frac{1}{2}\frac{\sqrt{\frac{-1}{\beta}}\left(-\sqrt{(m^2-n^2)\beta} - l\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)\right)}{m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n}. \tag{17}$$

$$u_{1,7} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{-1}{\beta}}\left(i\sqrt{\beta} + \frac{-2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)-i\sin\left(2\sqrt{\beta}(\xi+C)\right)}\right) \tag{18}$$

$$u_{1,8} = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{-1}{\beta}}\left(-i\sqrt{\beta} + \frac{2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)+i\sin\left(2\sqrt{\beta}(\xi+C)\right)}\right). \tag{19}$$

Where $\xi = kx - \omega\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}$ and $m, n, C, \gamma, \beta, \alpha$ are arbitrary constant with $0 < \alpha < 1, \gamma > 0.$

Solution for Set-II: Here we recall the parameters

$$k = \sqrt{\frac{-1}{32\beta}}, \omega = -\frac{3}{2}\sqrt{\frac{-1}{16\beta}}, \mu_0 = -\frac{1}{2}, \mu_1 = \sqrt{\frac{-1}{16\beta}}, \mu_2 = -\beta\sqrt{\frac{-1}{16\beta}}.$$

With the above parameters, for $\beta < 0$ the required hyperbolic solution becomes:

$$u_{2,1} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(\frac{\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)}{m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n} - \frac{\beta\, m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n}{\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)}\right). \tag{20}$$

$$u_{2,2} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(\frac{-\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)}{m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n} - \frac{\beta\, m\sinh\left(2\sqrt{-\beta}(\xi+C)\right)+n}{-\sqrt{-(m^2+n^2)\beta} - m\sqrt{-\beta}\cosh\left(2\sqrt{-\beta}(\xi+C)\right)}\right). \tag{21}$$

$$u_{2,3} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\sqrt{-\beta} - \frac{2m\sqrt{-\beta}\sqrt{\frac{-1}{16\beta}}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)} - \frac{\beta\sqrt{\frac{-1}{16\beta}}}{\sqrt{-\beta}+\frac{-2m\sqrt{-\beta}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)}}. \tag{22}$$

$$u_{2,4} = -\frac{1}{2} - \sqrt{\frac{-1}{16\beta}}\sqrt{-\beta} + \frac{2m\sqrt{-\beta}\sqrt{\frac{-1}{16\beta}}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)} - \frac{\beta\sqrt{\frac{-1}{16\beta}}}{\sqrt{-\beta}+\frac{-2m\sqrt{-\beta}}{m+\cosh\left(2\sqrt{-\beta}(\xi+C)\right)-\sinh\left(2\sqrt{-\beta}(\xi+C)\right)}}. \tag{23}$$

For $\beta > 0$, the required trigonometric solution becomes:

$$u_{2,5} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(\frac{\sqrt{(m^2-n^2)\beta}-m\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)}{m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n} - \frac{\beta\left(m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n\right)}{\sqrt{(m^2-n^2)\beta}-m\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)}\right). \tag{24}$$

$$u_{2,6} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(\frac{-\sqrt{(m^2-n^2)\beta}-m\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)}{m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n} - \frac{\beta\left(m\sin\left(2\sqrt{\beta}(\xi+C)\right)+n\right)}{-\sqrt{(m^2-n^2)\beta}-m\sqrt{\beta}\cos\left(2\sqrt{\beta}(\xi+C)\right)}\right). \tag{25}$$

$$u_{2,7} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(i\sqrt{\beta} - \frac{-2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)-i\sin\left(2\sqrt{\beta}(\xi+C)\right)}\right) - \frac{\beta\sqrt{\frac{-1}{16\beta}}}{i\sqrt{\beta}-\frac{-2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)-i\sin\left(2\sqrt{\beta}(\xi+C)\right)}}. \tag{26}$$

$$u_{2,8} = -\frac{1}{2} + \sqrt{\frac{-1}{16\beta}}\left(-i\sqrt{\beta} + \frac{2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)+i\sin\left(2\sqrt{\beta}(\xi+C)\right)}\right) - \frac{\beta\sqrt{\frac{-1}{16\beta}}}{-i\sqrt{\beta}+\frac{2mi\sqrt{\beta}}{m+\cos\left(2\sqrt{\beta}(\xi+C)\right)+i\sin\left(2\sqrt{\beta}(\xi+C)\right)}}. \tag{27}$$

Where $\xi = kx - \omega\frac{\Gamma(\gamma+1)}{\alpha}t^{\alpha}$ and $m, n, C, \gamma, \beta, \alpha$ are arbitrary constant with $0 < \alpha < 1, \gamma > 0$.

5. Effect of Truncated M Fractional Derivative on Numerical Solution

In this subdivision, we discuss the effect of truncated M fractional derivative on the obtained solution. For the special value of the parameters, we illustrate kink, different type interaction of kink and lump solution, soliton solution with 3-D graph and corresponding density plot. Here, we plotted the numerical solution with the parameter of truncated M fractional derivative $\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$.

For $\beta < 0$ the unified approach provides hyperbolic function solution in eq. (12)-(15). In figure 1, we illustrate the effect of truncated M fractional derivative parameter α on kink solution of eq. (12) for the parameter $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9]$ at $d = 0.5, \beta = -0.5, m = 4, n = -0.5, C = -1$. In figure 2, we show the interact of kink and lump wave of the solution eq. (12) for the parameter $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9]$ at $d = 1.5, \beta = -0.5, m = -4, n = 5, C = 1$.

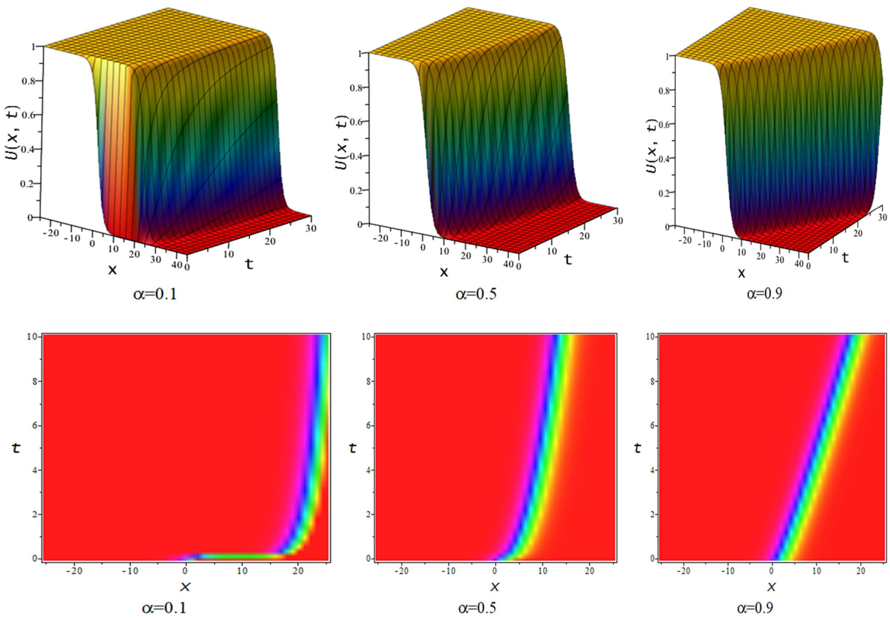

Figure 1. *3-D profile (upper three) and density plot (lower three) of the kink shape solution eq. (12) for the parameter* $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9]$ *at* $d =$

$0.5, \beta = -0.5, m = 4, n = -0.5, C = -1$.

For $\beta > 0$ the unified approach provides trigonometric function solution in eq. (16)-(19). In figure 3, we illustrate linked lump solution of the eq. (16) for the parameter $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9]$ at $d = 0.5, \beta = 0.1, m = 4, n = 0.5, C = 1$. In figure 4, the interaction of kink and lump solution of the eq. (16) for the parameter $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9]$ at $d = 0.5, \beta = 0.5, m = 4, n = -0.5, C = -1$.

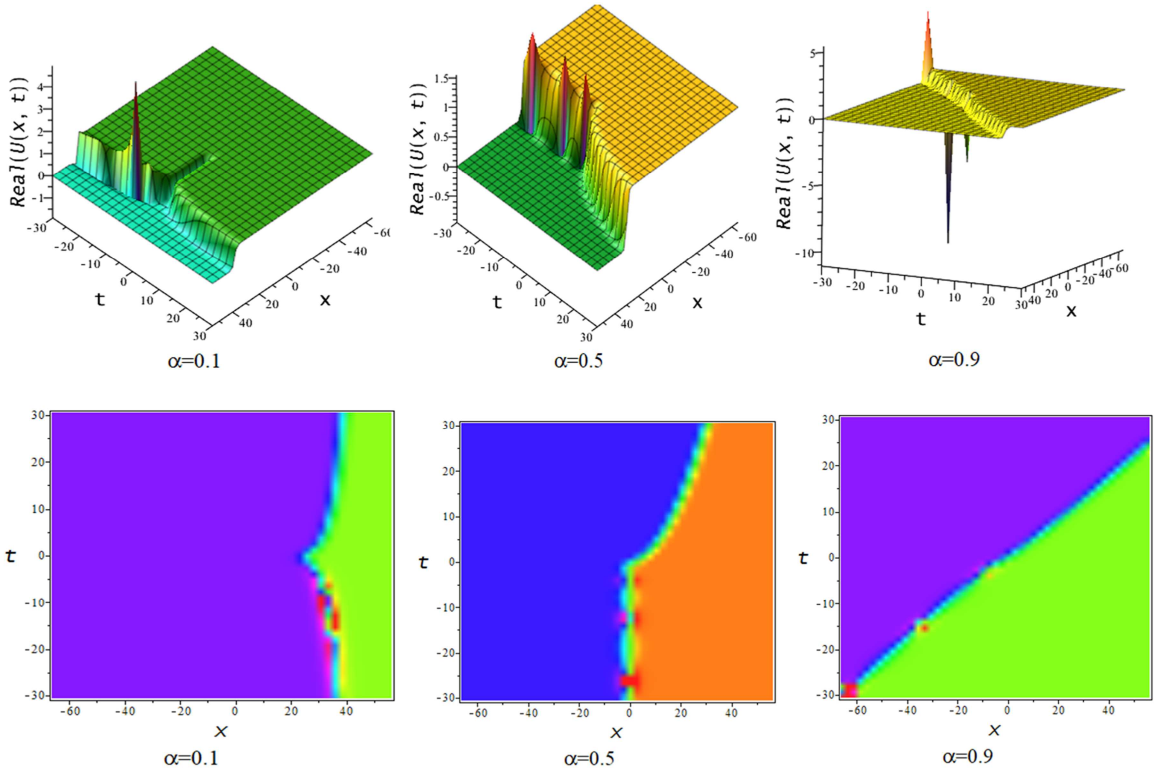

Figure 2. *3-D profile (upper three) and density plot (lower three) of the interaction solution eq. (12) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 1.5, \beta = -0.5, m = -4, n = 5, C = 1$.

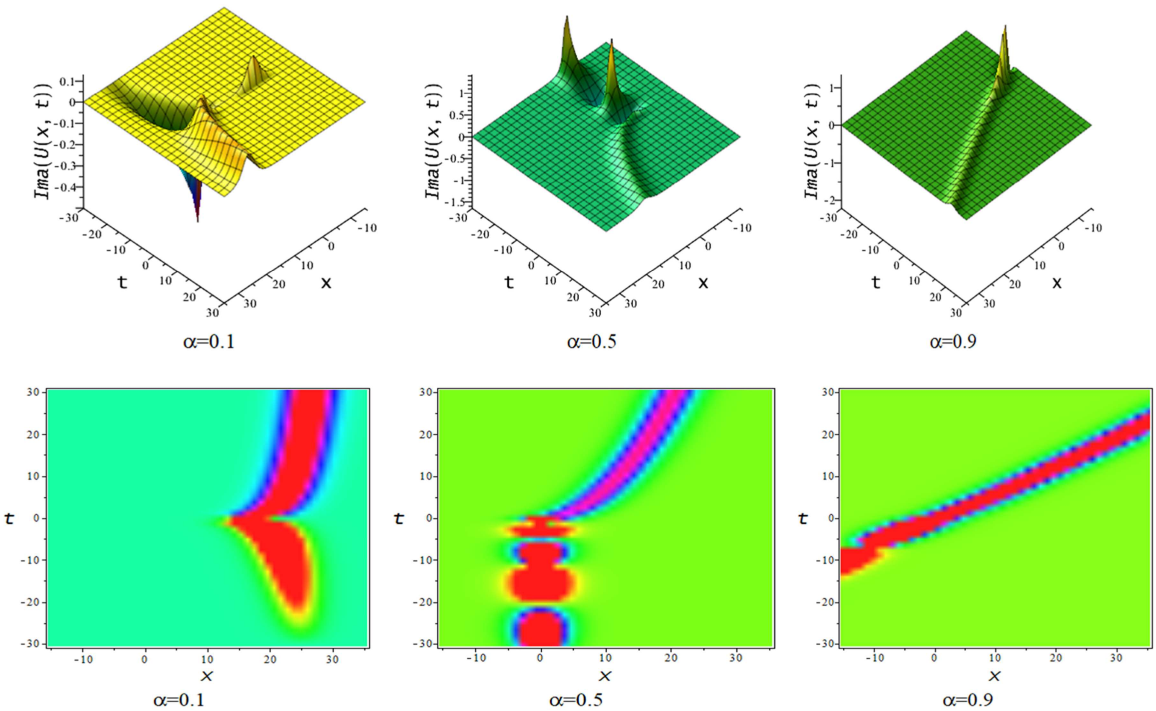

Figure 3. *3-D profile (upper three) and density plot (lower three) of the linked lump wave solution eq. (16) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at*

$d = 0.5, \beta = 0.1, m = 4, n = 0.5, C = 1$.

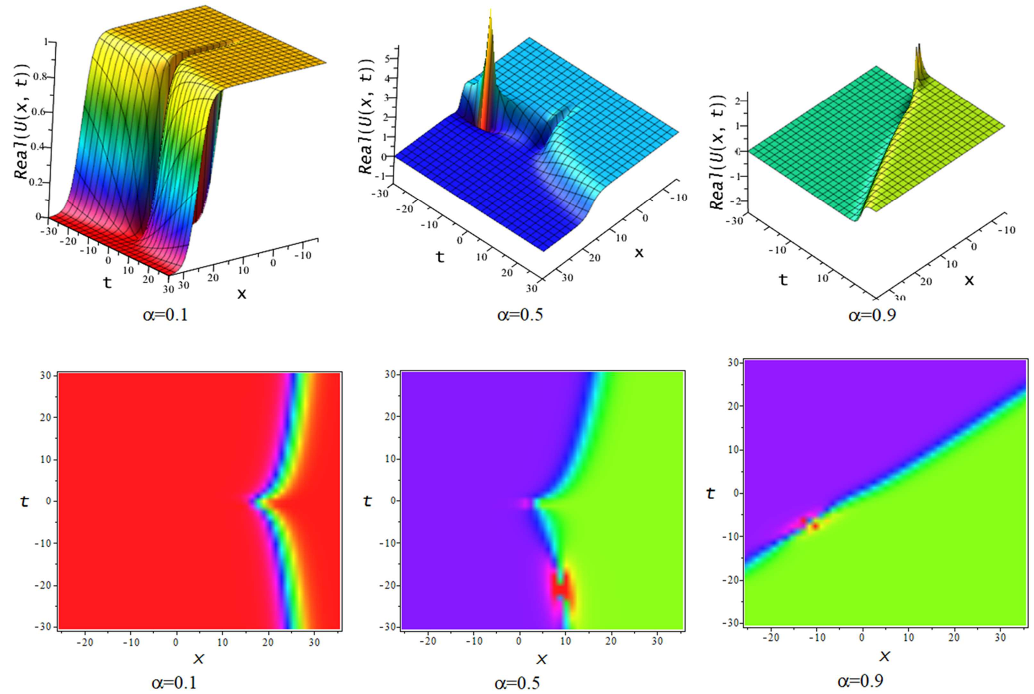

Figure 4. *3-D profile (upper three) and density plot (lower three) of the interaction of kink and lump solution eq. (16) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 0.5, \beta = 0.5, m = 4, n = -0.5, C = -1$.

For $\beta < 0$ the unified approach provides hyperbolic function solution in eq. (20)-(23). In figure 5, we show soliton solution of eq. (20) for the parameter $[\alpha = 0.1, \alpha = 0.5, \alpha = 0.9\ ]$ at $d = 1.5, \beta = -1, m = -1, n = -0.5, C = -1$. In figure 6, we show the feature of lump wave solution of eq. (20) $[\ \alpha = 0.1, \alpha = 0.5, \alpha = 0.9\ ]$ at $d = 1.5, \beta = -1, m = 1, n = -0.005$.

For $\alpha > 0$ the unified approach provides trigonometric function solution in eq. (24)-(27). In figure 7, we illustrate the interaction of lump and kink solution of the solution of eq. (25) with the influence of the parameter $[\ \alpha = 0.1, \alpha = 0.5, \alpha = 0.9\ ]$ at $d = 0.5, \beta = 0.25, m = 4, n = -2, C = -1$. In figure 8, we illustrate the interaction of lump and kink solution of the solution of eq. (26) with the influence of the parameter $[\ \alpha = 0.1, \alpha = 0.5, \alpha = 0.9\ ]$ at $d = 0.5, \beta = 0.1, m = -4, n = -2, C = -0.1$.

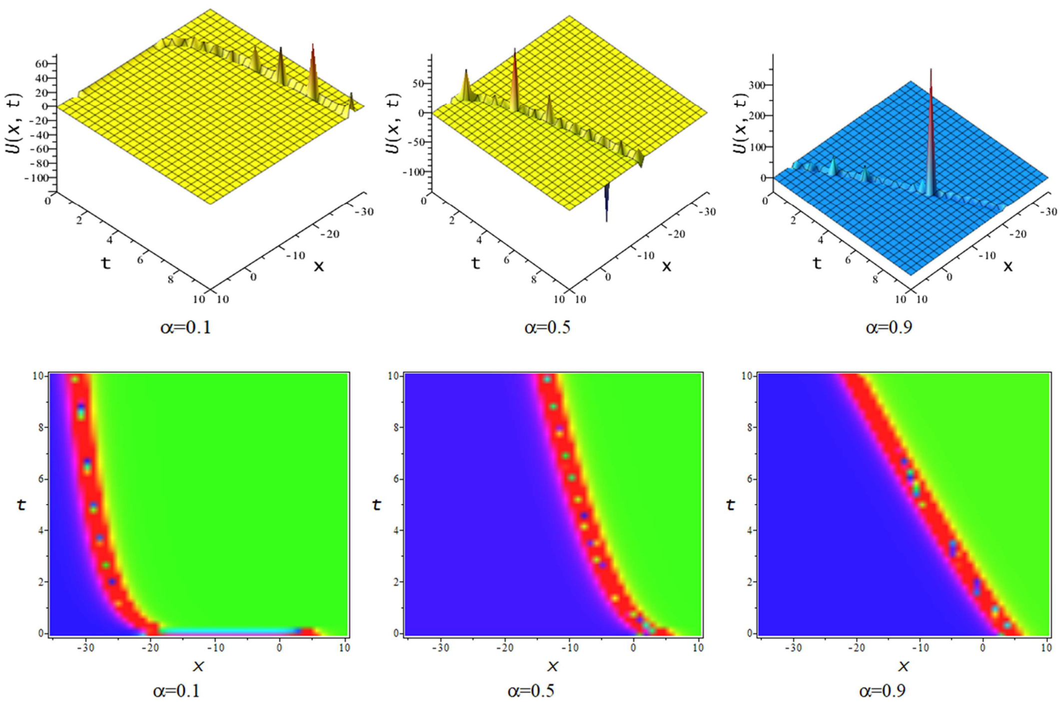

Figure 5. *3-D profile (upper three) and density plot (lower three) of the soliton solution of eq. (20) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 1.5, \beta = -1, m = -1, n = -0.5, C = -1$.

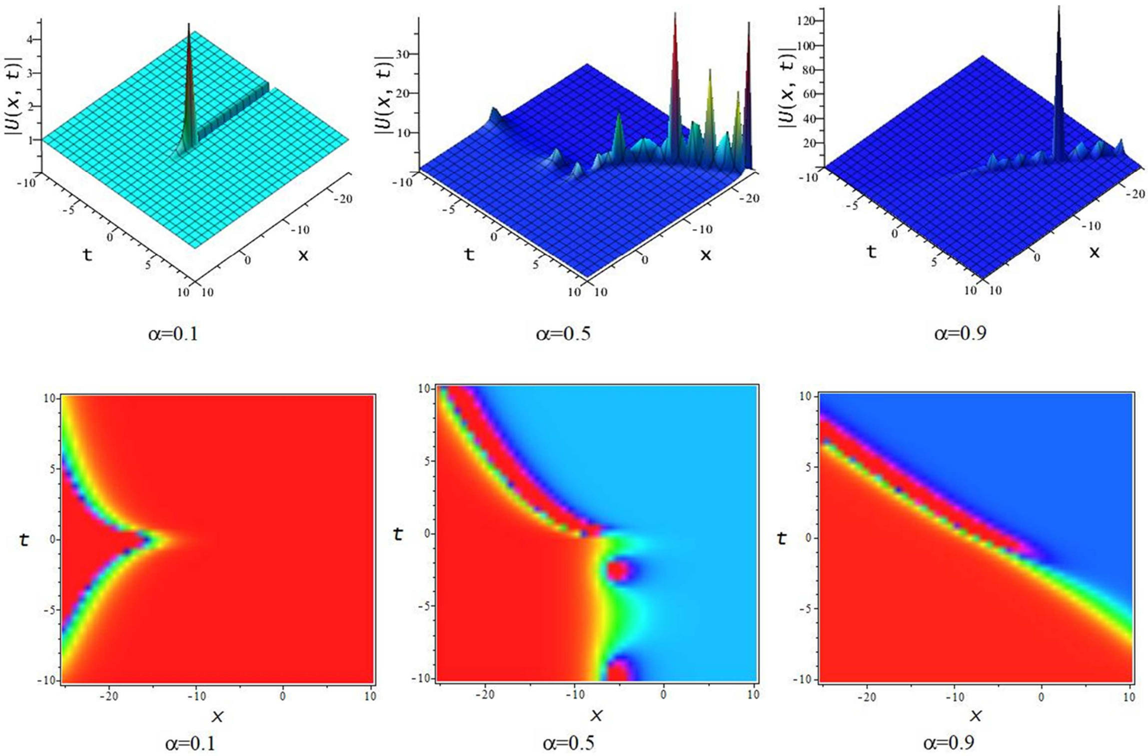

Figure 6. *3-D profile (upper three) and density plot (lower three) of the lump wave solution eq. (20) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 1.5, \beta = -1, m = 1, n = -0.005, C = -1$.

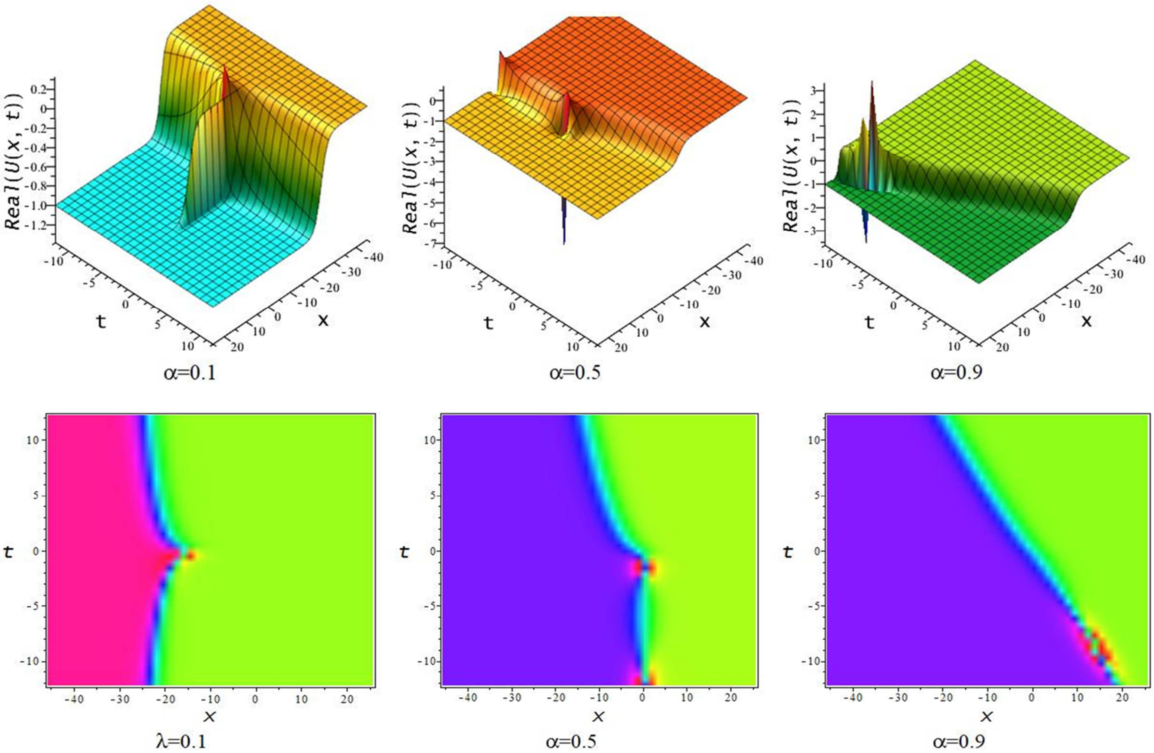

Figure 7. *3-D profile (upper three) and density plot (lower three) of interaction of kink and lump solution of the eq. (25) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 0.5, \beta = 0.25, m = 4, n = -2, C = -1$.

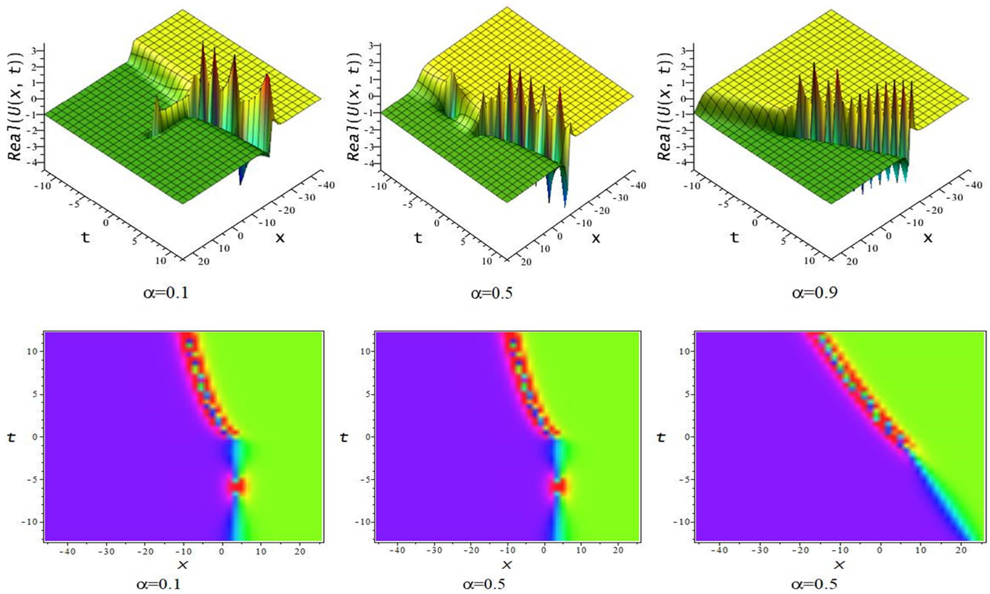

Figure 8. *3-D profile (upper three) and density plot (lower three) of interaction of kink and lump solution of the eq. (26) for the parameter [*$\alpha = 0.1, \alpha = 0.5, \alpha = 0.9$*] at* $d = 0.5, \beta = 0.1, m = -4, n = -2, C = -0.1$.

6. Conclusion

In this work, we successfully implement the unified scheme to obtain some new exact soliton solutions from truncated M-fractional Cahn–Allen equation. The numerical solution of the obtain solution are illustrated in figures 1 to 8. The effect of truncated M-fractional derivative on the obtained solutions are demonstrated with 3-D plot and corresponding density plot such as the kink soliton, different type interaction of kink and lump soliton solution, singular soliton. More indeed, the amplitude and shape of the wave are developed with deeply minor values of the truncated M-fractional derivative parameter α.

Author Contributions

Investigation, Idea maker, Methodology and Writing – original draft, Mohammad Mobarak Hossain; Writing Graphical representation and corrections, Md Mamunur Roshid; Supervision, Conceptualization, Software and funding acquisition, Md. Abu Naim Sheikh; Supervision and Finalization, Mohammad Abu Taher, Supervision and Modification and checking draft, Harun-Or-Roshid. All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest

The authors declare that they have no competing interests.

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