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Existence and Stability of Solutions for Semilinear Timoshenko System with Damping and Source Terms

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Abstract: In this paper, we are concerned with one-dimensional Timoshenko model for a beam with nonlinear damping and source terms. Under suitable conditions on the initial data, the theorem of global existence is proved by potential well method combined Galerkin procedure, and decay estimates of the energy is established by means of Nakao's inequality.

Keywords: Timoshenko System, Source Term, Damping Term, Global Existence, Stability

1. Introduction

In this paper, we study the semilinear Timoshenko system

$$u_{tt}-u_{xx}+k(u+v_x)+|u_t|^{p-1}u_t=f_1(u,v), \tag{1}$$

$$v_{tt}-k(u+v_x)_x+|v_t|^{q-1}v_t=f_2(u,v), \tag{2}$$

in $(0,1)\times(0,\infty)$, under the following boundary conditions

$$u(0,t)=u(1,t)=0, \quad t\geq 0, \tag{3}$$

$$v(0,t)=v(1,t)=0, \quad t\geq 0, \tag{4}$$

and initial conditions

$$u(x,0)=u_0(x), u_t(x,0)=u_1(x), \quad in \quad (0,1), \tag{5}$$

$$v(x,0)=v_0(x), v_t(x,0)=v_1(x), \quad in \quad (0,1). \tag{6}$$

The function u is the rotation angle of a filament of the beam and v is the transverse displacement of the beam, k is a strict positive constant. The nonlinear function $f_1(u,v)$ and $f_2(u,v)$ act as strong source terms, $p,q\geq 1$. For the corresponding linearized system of (1)-(2)

$$u_{tt}-u_{xx}+k(u+v_x)=0, \tag{7}$$

$$v_{tt}-k(u+v_x)_x=0, \tag{8}$$

which is given by Timoshenko [1] as a simple model describing vibration of a beam, this model for Timoshenko beams have attracted vast interest during the last thirty years. Systems (7)-(8) has been studied by many authors and results concerning existence and asymptotic behavior have been established. The stabilization of the Timoshenko system has been studied with different type of dampings, we refer the reader to [2, 3, 4, 5, 6, 7] and their references.

Let us mention some known results for semilinear Timoshenko system. Parente et.al [9] treated the existence and uniqueness for the problem

$$u_{tt}-u_{xx}+k(u+v_x)+f(u)=0, \tag{9}$$

$$v_{tt}-k(u+v_x)_x+g(v)=0, \tag{10}$$

with differential boundary conditions, where f,g are Lipschitz continuous functions. Araruna et.al [10] investigated the existence and uniqueness of strong and weak solution of the one-dimensional Timoshenko model (9)-(10) for beams with a nonlinear external forces and a boundary damping mechanics. They also proved that the energy of solution decays exponentially. Chueshov and Lasiecka [11] studied the existence of a compact global attractor for (9)-(10) with nonlinearities of f and g being locally Lipschitz in the 2-dimensional case. Gorgi and Vegni [12] gave the uniform energy estimate and the estimate of an absorbing set for the Timoshenko beam with memory and Dirichlet boundary condition. Messaoudi and Soufyane [8, 13] established a general decay result for a nonlinear Timoshenko system with a boundary control of memory type

$$\rho_1(x)u_{tt} = \Delta u + \alpha\sum_{i=1}^{n}\frac{\partial v}{\partial x_i} - \beta u - \alpha(x)f_1(u,v), \tag{11}$$

$$\rho_2(x)v_{tt} = \Delta v + \alpha\sum_{i=1}^{n}\frac{\partial u}{\partial x_i} - \alpha(x)f_2(u,v), \tag{12}$$

However, there has been less focus on the Timoshenko system with nonlinear source terms. Recently, Pei et al. [14, 15] studied the global well-posedness and long-term behavior of the Reissner Mindlin-Timoshenko plate systems, focusing on the interplay between nonlinear viscous damping and source terms, by the potential well framework [16, 17]. To the best of our knowledge, the system of nonlinear Timoshenko equation have not been well studied.

We also mention that for one equation boundary value problem with nonlinear damping and source terms at first time investigated by Lions, J.-L [22]. The problem for the linear and semilinear system of equations of hyperbolic-elliptic type, including property of changing time direction in general form PDE investigated at first time by M. A. Nurmammadov [23-25] which is system equations contains partition part all classical and semilinear systems, degenerating elliptic, degenerating hyperbolic, mixed and composite type differential equations. In this case he considered new boundary value problems.

In this paper, we consider the problem (1)-(6). We give an equivalent inequality between $\|u_x\|^2 + k\|u+v_x\|^2$ and the standard norm on the function space $H_0^1\times H_0^1$, then we obtain local existence of solution of problem (1)-(6) following very carefully the techniques used in [19]. Following the equivalent inequality, we introduce the stable set, then we get global existence of solution for problem (1)-(6) by potential well method combined Galerkin procedure, and decay estimates of the energy functions are established by means of Nakao's inequality [18].

2. Preliminaries

Throughout this paper, we denote $L^p(0,1)$ and $H_0^1(0,1)$ by L^p and H_0^1, respectively. $\|\cdot\|$ and $\|\cdot\|_p$ denote the usual L^2 norm and L^p norm, respectively. And let us define $(\varphi,\psi)=\int_0^1\varphi(x)\psi(x)dx$ as the usual L^2 inner product. The standard duality between $(H^1)'$ and H^1 will be denote also by $(\cdot,\cdot)$. For $\varphi\in H_0^1$, it is well-known that the norm $\|\varphi_x\|$ is the equivalent of the H_0^1 norm $\|\varphi\|_{H_0^1}=(\|\varphi\|^2+\|\varphi_x\|^2)^{\frac{1}{2}}$ (see [22,25]). Let V denotes the following Hilbert space $V=H_0^1\times H_0^1$, and endowed with the following norms $\|\Phi\|_V^2 = \|u_x\|^2 + \|v_x\|^2$, for $\Phi=(u,v)\in V$. Throughout this paper, $C, C_1, C_2, \cdots$ are positive generic constants, which may be different in various occurrences. In addition, we denote C_* is the Poincare constants, that is

$$\|u\|_s \le C_*\|u_x\|, \quad for \quad u\in H_0^1, \quad 2\le s\le +\infty. \tag{13}$$

Concerning the nonlinear functions $f_1(u,v)$ and $f_2(u,v)$, we assume that

$$f_1(u,v) = a|u+v|^{2(\rho+1)}(u+v) + b|u|^{\rho}u|v|^{\rho+2}, \tag{14}$$

$$f_2(u,v) = a|u+v|^{2(\rho+1)}(u+v) + b|v|^{\rho}v|u|^{\rho+2}, \tag{15}$$

where $a,b>0$ and $\rho\ge 0$ are constants. It is easy to see that

$$uf_1(u,v) + vf_2(u,v) = 2(\rho+2)F(u,v), \quad \forall(u,v)\in R^2, \tag{16}$$

where $F(u,v)=\frac{1}{2(\rho+2)}[a|u+v|^{2(\rho+2)} + b|uv|^{\rho+2}]$. Moreover, a quick computation will show that there exist two positive constants C_0 and C_1 such that the following inequality holds (see [19, 20])

$$C_0(|u|^{2(\rho+2)} + |v|^{2(\rho+2)}) \le 2(\rho+2)F(u,v) \le C_1(|u|^{2(\rho+2)} + |v|^{2(\rho+2)}). \tag{17}$$

Now, we give the following lemmas which will be used later. By a simple computation, we have the following result:

Lemma 2.1 For $(u,v)\in H_0^1$, there exist positive constants $\alpha_1>0, \alpha_2>0$ such that the inequality holds

$$\alpha_1(\|u_x\|^2 + k\|u+v_x\|^2) \le \|u_x\|^2 + \|v_x\|^2 \le \alpha_2(\|u_x\|^2 + k\|u+v_x\|^2) \tag{18}$$

where $\alpha_2 = max\{1+2C_*), \frac{2}{k}\}, \alpha_1^{-1} = max\{(1+2kC_*), 2k\}$.

Lemma 2.2 [18] Let $\varphi(t)$ be nonincreasing and nonnegative function defined on $[0,\infty), T>1$, satisfying

$$\varphi^{1+\alpha}(t) \le w_0(\varphi(t)-\varphi(t+1)), \qquad t\in[0,\infty),$$

where w_0 is a positive constant, α is a nonnegative constant. Then we have for each $t\in[0,\infty)$,

$$\varphi(t) \le \varphi(0)e^{-w_1[t-1]^+}, \qquad \alpha=0,$$
$$\varphi(t) \le (\varphi^{-\alpha}(0) + w_0^{-1}\alpha[t-1]^+)^{-\frac{1}{2}}, \qquad \alpha>0,$$

where $[t-1]^+ = max\{t-1,0\}$ and $w_1 = ln\frac{w_0}{w_0-1}$.

Now, we state the local existence of the problem (1)-(6) and the proof follows very carefully the techniques used in [19].

Theorem 2.3 (Local existence) Assume that the assumptions (14)-(17) hold. Suppose further that $p,q>1, \rho>0$, then for any initial data $u_0, v_0\in H_0^1$ and $u_1, v_1\in L^2$, there exists a local weak solution (u,v) of problem (1)-(6) defined in $[0,T_0]$ for some $T_0>0$ and satisfies the energy identity.

$$E(t)+\int_0^t (\| u_s(s)\|_{p+1}^{p+1}+\| v_s(s)\|_{q+1}^{q+1})ds = E(0), \tag{19}$$

where $E(t)$ is defined by

$$E(t)=\frac{1}{2}(\|u_t\|^2+\|v_t\|^2)+\frac{1}{2}\|u_x\|^2+\frac{k}{2}\|u+v_x\|^2-\int_0^1 F(u,v)dx, \tag{20}$$

and

$$E(0)=\frac{1}{2}(\|u_1\|^2+\|v_1\|^2)+\frac{1}{2}\|u_{0x}\|^2+\frac{k}{2}\|u_0+v_{0x}\|^2-\int_0^1 F(u_0,v_0)dx. \tag{21}$$

3. Global Existence and Decay of Solution

In this section, we discuss the global existence and decay of the solution for problem (1)-(6). In order to do so, let us introduce the functions

$$J(t)=J(u(t),v(t))=\frac{1}{2}(\|u_x\|^2+k\|u+v_x\|^2)-\int_0^1 F(u,v)dx, \tag{22}$$

$$I(t)=I(u(t),v(t))=\|u_x\|^2+k\|u+v_x\|^2-2(\rho+2)\int_0^1 F(u,v)dx. \tag{23}$$

It follows from Theorem 2.3 that

$$\tfrac{d}{dt}E(t)=-\|u_t\|_{p+1}^{p+1}-\|v_t\|_{q+1}^{q+1}, \tag{24}$$

then the energy function $E(t)$ is a nonincreasing function.

Now, we define

$$W=\{(u,v)\in H_0^1\times H_0^1,\quad I(u,v)>0\}\cup\{(0,0)\}. \tag{25}$$

Lemma 3.1 There exist $\eta>0$ such that for any $(u,v)\in H_0^1\times H_0^1$, the following inequality holds,

$$\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{\rho+2}^{\rho+2}\le \eta(\|u_x\|^2+\|u_x\|^2)^{\rho+2}\le \eta\alpha_2^{\rho+2}(\|u_x\|^2+k\|u+v_x\|^2)^{\rho+2}. \tag{26}$$

Proof A combination of the following inequality [20]

$$\|u+v\|_{2(\rho+2)}^{2(\rho+2)}+2\|uv\|_{2(\rho+2)}^{2(\rho+2)}\le \eta(\|u_x\|^2+\|u_x\|^2)^{\rho+2},$$

with Lemma 2.1 yields (26).

Lemma 3.2 Let $(u_0,v_0)\in W$ and $(u_1,v_1)\in L^2\times L^2$, such that

$$\beta=\eta\alpha_2^{\rho+2}(\tfrac{2(\rho+2)}{\rho+1}E(0))^{\rho+1}<1, \tag{27}$$

then $(u,v)\in W$ for each $t>0$.

Proof Since $I(0)>0$, then it follows from the continuity of $u(t)$ that $I(t)>0$ for some interval near $t=0$. Let T_m be the maximum time (possible $T_m=T$) when $I(t)>0$ on $[0,T_m]$. This implies that for all $t\in[0,T_m]$.

$$J(t)=\tfrac{\rho+1}{2(\rho+2)}(\|u_x\|^2+k\|u+v_x\|^2)+\tfrac{1}{2(\rho+2)}I(t)\ge \tfrac{\rho+1}{2(\rho+2)}(\|u_x\|^2+k\|u+v_x\|^2). \tag{28}$$

Hence, we get

$$\|u_x\|^2+k\|u+v_x\|^2\le \tfrac{2(\rho+2)}{\rho+1}J(t)\le \tfrac{2(\rho+2)}{\rho+1}E(t)\le \tfrac{2(\rho+2)}{\rho+1}E(0). \tag{29}$$

By recalling (2.4), (2.5) and (3.5), we have

$$\begin{aligned}2(\rho+2)\int_0^1 F(u,v)dx &\le \eta\alpha_2^{\rho+2}(\|u_x\|^2+k\|u+v_x\|^2)^{\rho+2}\\ &\le \eta\alpha_2^{\rho+2}(\tfrac{2(\rho+2)}{\rho+1}E(0))^{\rho+1}(\|u_x\|^2+k\|u+v_x\|^2)\\ &<\|u_x\|^2+k\|u+v_x\|^2,\ on\ t\in[0,T_m].\end{aligned} \tag{30}$$

Therefore, we conclude that $I(t)>0$ for all $t\in[0,T_m]$. By repeating the procedure, T_m is extended to T. The proof of Lemma 3.2 is completed.

Also, the following inequality is easily obtained

$$\|u_x\|^2+k\|u+v_x\|^2\le \frac{1}{1-\eta\alpha_2^{\rho+2}(\frac{2(\rho+2)}{\rho+1}E(0))^{\rho+1}}I(t). \tag{31}$$

Theorem 3.3 Suppose that (14)-(17) hold. If $(u_0,v_0)\in W$ satisfying (27), then the solution of problem (1)- (6) is global.

Proof It is sufficient to show that $\|u_t\|^2+\|v_t\|^2+\|u_x\|^2+k\|u+v_x\|^2$ is bounded for bounded t. To achieve this, we use the fact that $E(t)$ is a nonincreasing function and $I(t)>0$ by Lemma 3.2, to obtain

$$E(0)>E(t)\ge \tfrac{1}{2}(\|u_t\|^2+\|v_t\|^2)+\tfrac{\rho+1}{2(\rho+2)}(\|u_x\|^2+k\|u+v_x\|^2)$$

Therefore,

$$\|u_t\|^2+\|v_t\|^2+\|u_x\|^2+k\|u+v_x\|^2<CE(0),$$

where $C = max\{2, \frac{2(\rho+2)}{\rho+1}\}$. Then by Theorem 2.3, we have the global existence result.

Theorem 3.4 Let the assumptions of Theorem 3.3 hold, thus we have the following decay estimates

$$E(t) \le E(0)e^{-w_1[t-1]^+}, \; if \;\; p=q=1,$$
$$E(t) \le [E(0)^{-\alpha} + w_2[t-1]^+]^{\frac{1}{\alpha}}, \; if \;\; p,q>1,$$

where w_1, w_2, α are positive constants.

Proof By integrating (24) over $[t, t+1](t>0)$, we have

$$E(t) = E(t+1) + \int_t^{t+1} (\|u_s(s)\|_{p+1}^{p+1} + \|v_s(s)\|_{q+1}^{q+1}) ds = E(t+1) + D_1^{p+1}(t) + D_2^{q+1}(t). \tag{32}$$

By virtue of (32) and Hölder inequality, we observe that

$$\int_t^{t+1}\int_0^1 |u_s(s)|^2 \, dxds \le CD_1^2(t), \tag{33}$$

$$\int_t^{t+1}\int_0^1 |v_s(s)|^2 \, dxds \le CD_2^2(t). \tag{34}$$

Hence, from (3.12) and (3.13), there exist $t_1 \in [t, t+\frac{1}{4}]$ and $t_2 \in [t+\frac{3}{4}, t+1]$ such that

$$\|u_t(t_i)\| \le CD_1(t), \qquad \|v_t(t_i)\| \le CD_2(t), i=1,2. \tag{35}$$

Multiplying equation (1) by u, equation (2) by v, then adding them and integrating it over $[0,1]\times[t_1,t_2]$, we get

$$\int_{t_1}^{t_2} I(t)dt \le -\int_{t_1}^{t_2}\int_0^1 (uu_{tt} + vv_{tt})dxdt - \int_{t_1}^{t_2}\int_0^1 (|u_t|^{p-1} u_t u + |v_t|^{q-1} v_t v)dxdt. \tag{36}$$

Integrating by parts and using Cauchy-Schwartz inequality in the first term of the right hand side of (36), we obtain

$$\int_{t_1}^{t_2} I(t)dt \le \sum_{i=1}^{m} \|u_t(t_i)\| \|u(t_i)\| + \sum_{i=1}^{m} \|v_t(t_i)\| \|v(t_i)\|$$
$$+\int_{t_1}^{t_2} (\|u_t(t)\|^2 + \|v_t(t)\|^2)dt - \int_{t_1}^{t_2}\int_0^1 (|u_t|^{p-1} u_t u + |v_t|^{q-1} v_t v)dxdt. \tag{37}$$

Now we estimate the last term in the right hand side of (37). By using Hölder inequality and embedding theorem (e.g., [20-21]) and (29), we find

$$\int_{t_1}^{t_2}\int_0^1 |u_t|^{p-1} u_t u dxdt \le \int_{t_1}^{t_2} \|u_t\|_{p+1}^p \|u\|_{p+1} dt$$
$$\le C_*\int_{t_1}^{t_2} \|u_t\|_{p+1}^p \|u_x\| dt$$
$$\le C_*(\tfrac{2(\rho+2)}{\rho+1})^{\frac{1}{2}} \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s)\int_{t_1}^{t_2} \|u_t\|_{p+1}^p dt$$
$$\le C_*(\tfrac{2(\rho+2)}{\rho+1})^{\frac{1}{2}} D_1^p(t) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s). \tag{38}$$

From (29), (30) and Poincare inequality, we have

$$\|u_t(t_i)\|\|u(t_i)\| \le CD_1(t) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s), i=1,2. \tag{39}$$

Similarly, we get

$$\int_{t_1}^{t_2}\int_0^1 |v_t|^{q-1} v_t v dxdt \le \int_{t_1}^{t_2} \|v_t\|_{q+1}^q \|v\|_{p+1} dt \le C_*(\tfrac{2(\rho+2)}{\rho+1})^{\frac{1}{2}} D_2^q(t) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s). \tag{40}$$

$$\|v_t(t_i)\|\|v(t_i)\| \le CD_2(t) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s), \qquad i=1,2. \tag{41}$$

Then, by (37)- (41) and (33), (34), we have

$$\int_{t_1}^{t_2} I(t)dt \le C\{(D_1(t)+D_2(t)) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s)$$
$$+D_1^2(t) + D_2^2(t) + C_*(\tfrac{2(\rho+2)}{\rho+1})^{\frac{1}{2}}(D_1^p(t)+D_2^q(t)) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s)\} \tag{42}$$

On the other hand, from (31), we obtain

$$E(t) = \tfrac{1}{2}(\|u_t\|^2 + \|v_t\|^2) + \tfrac{\rho+1}{2(\rho+2)}(\|u_x\|^2 + k\|u+v_x\|^2) + \tfrac{1}{2(\rho+2)} I(t)$$
$$\le \tfrac{1}{2}(\|u_t\|^2 + \|v_t\|^2) + CI(t), \tag{43}$$

where $C = \frac{\rho+1}{2(\rho+2)[1-\eta\alpha_2^{\rho+2}(\frac{2(\rho+2)}{\rho+1}E(0))^{\rho+1}]} + \frac{1}{2(\rho+2)}$. Integrating (43) over $[t_1,t_2]$, by (33), (34) and (42), we have

$$\int_{t_1}^{t_2} E(t)dt \le \frac{1}{2}\int_{t_1}^{t_2} (\|u_t\|^2 + \|v_t\|^2)dt + C\int_{t_1}^{t_2} I(t)dt$$
$$\le \tfrac{1}{2}C(D_1^2(t)+D_2^2(t)) + C\{(D_1(t)+D_2(t)) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s) + (D_1^p(t)$$
$$+D_2^q(t)) \sup_{t_1\le s\le t_2} E^{\frac{1}{2}}(s)\}. \tag{44}$$

By integrating (24) over $[t, t_2]$, we obtain

$$E(t) = E(t_2) + \int_t^{t_2} (\|u_s(s)\|_{p+1}^{p+1} + \|v_s(s)\|_{q+1}^{q+1})ds \ge E(t_2). \tag{45}$$

Therefore, since $t_2 - t_1 > \frac{1}{2}$, integrating (45) over $[t_1,t_2]$, we conclude that

$$\int_{t_1}^{t_2} E(t)dt \le \int_{t_1}^{t_2} E(t_2)dt \le \tfrac{1}{2}E(t_2),$$

that is,

$$E(t_2) \le 2\int_{t_1}^{t_2} E(t)dt\,. \tag{46}$$

Since $t < t_1, t_2 < t+1$, and $E(t)$ is nonincreasing, then by (32), (44)

$$E(t) = E(t+1) + D_1^{p+1}(t) + D_2^{q+1}(t) \;\le E(t) + D_1^{p+1}(t) + D_2^{q+1}(t)$$

$$\leq 2\int_{t_1}^{t_2} E(t)dt + D_1^{p+1}(t) + D_2^{q+1}(t)$$
$$\leq C(D_1^2(t) + D_2^2(t) + D_1^{p+1}(t) + D_2^{q+1}(t))$$
$$+C(D_1(t) + D_2(t) + D_1^p(t) + D_2^q(t))E^{\frac{1}{2}}(t). \quad (47)$$

It follows from Young's inequality that

$$E(t) \leq C(D_1^2(t) + D_2^2(t) + D_1^{p+1}(t) + D_2^{q+1}(t) + D_1^{2p}(t) + D_2^{2q}(t)). \quad (48)$$

If $p = q = 1$, then

$$E(t) \leq C(D_1^2(t) + D_2^2(t) = C(E(t) - E(t+1)).$$

If $p,q > 1$, then from (48), (32) and $E(t) < E(0)$, we arrive to

$$E(t) \leq CD_1^2(t)(1 + D_1^{p-1}(t) + D_1^{2(p-1)}(t)) + CD_2^2(t)(1 + D_2^{q-1}(t) + D_2^{2(q-1)}(t))$$
$$\leq C(D_1^2(t) + D_2^2(t))(1 + D_1^{p-1}(t) + D_1^{2(p-1)} + D_2^{q-1} + D_2^{2(q-1)}(t)))$$
$$\leq C(D_1^2(t) + D_2^2(t))(1 + E^{\frac{p-1}{p+1}}(0) + E^{\frac{2(p-1)}{p+1}}(0) + E^{\frac{q-1}{q+1}}(0) + E^{\frac{2(q-1)}{q+1}}(0))$$
$$\leq C(D_1^2(t) + D_2^2(t).$$

Let $\alpha = \max\{\frac{p-1}{2}, \frac{q-1}{2}\}$ and $\beta = \max\{p+1, q+1\}$, then we obtain

$$E^{1+\alpha}(t) \leq C(D_1^2(t) + D_2^2(t))^{1+\alpha} \leq C(D_1^\beta(t) + D_2^\beta(t))$$
$$= C(D_1^{p+1}(t)D_1^{\beta-p-1}(t) + D_2^{q+1}(t)D_2^{\beta-q-1}(t)$$
$$\leq C(D_1^{p+1}(t)E^{\frac{\beta-p-1}{p+1}}(0) + D_2^{q+1}(t)E^{\frac{\beta-q-1}{p+1}}(0)$$
$$\leq C(D_1^{p+1}(t) + D_2^{q+1}(t) = C(E(t) - E(t+1)). \quad (49)$$

Thus, from (49) and Lemma 2.2, we complete the proof.

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