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The Discrete Poisson-Amarendra Distribution

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Abstract: In this paper, a Poisson mixture of the Amarendra distribution, introduced by Shanker (2016 c), is proposed, and called the, "Poisson-Amarendra distribution". The first four raw moments (about the origin) and central moments (about the mean) are obtained. The expression for coefficient of variation, skewness and kurtosis are also given. For the estimation of its parameter, the maximum likelihood estimation and the method of moments are discussed. Moreover, the distribution is fitted using maximum likelihood estimate to certain data sets to test its goodness of fit over Poisson, Poisson-Lindley and Poisson-Sujatha distributions. The corresponding fitting are found to be quite satisfactory in almost all data sets.

Keywords: Amarendra Distribution, Sujatha Distribution, Poisson-Lindley Distribution, Poisson-Sujatha Distribution, Compounding, Moments, Estimation of Parameter, Goodness of Fit

1. Introduction

The probability density function (p. d. f.) and the cumulative distribution function (c. d. f.) of Amarendra distribution, introduced by Shanker (2016 c) for modeling real lifetime data sets from engineering and biomedical science, are given by

$$f(x;\theta)=\frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\left(1+x+x^2+x^3\right)e^{-\theta x}\quad,\ x>0,\ \theta>0. \tag{1}$$

$$F(x,\theta)=1-\left[1+\frac{\theta^3x^3+\theta^2(\theta+3)x^2+\theta(\theta^2+2\theta+6)x}{\theta^3+\theta^2+2\theta+6}\right]e^{-\theta x},\ x>0,\ \theta>0. \tag{2}$$

It has been shown by Shanker (2016 c) that Amarendra distribution is a four component mixture of exponential (θ) distribution, a gamma $(2,\theta)$ istribution, a gamma $(3,\theta)$ distribution and a gamma $(4,\theta)$ distribution with their mixing proportions

$$\frac{\theta^3}{\theta^3+\theta^2+2\theta+6},\ \frac{\theta^2}{\theta^3+\theta^2+2\theta+6},\ \frac{2\theta}{\theta^3+\theta^2+2\theta+6},\ \text{and}\ \frac{6}{\theta^3+\theta^2+2\theta+6}$$

respectively. Shanker (2016 c) has discussed its various mathematical and statistical properties including its shape for different values of its parameter, moment generating function, moments, skewness, kurtosis, hazard rate function, mean residual life function, stochastic orderings, mean deviations, Bonferroni and Lorenz curves, some amongst others. Further, Shanker (2016 c) has also discussed the estimation of its parameter using maximum likelihood estimation and method of moments along with applications for modeling lifetime data and observed that it gives much closer fit than Akash, Shanker and Sujatha distributions introduced by Shanker (2015 a, 2015 b, 2016 a,), Lindley (1958) and exponential distributions. It would be worth mentioning that Shanker (2015 a, 2015 b, 2016 a) has proposed Akash, Shanker and Sujatha, distributions along with their various mathematical and statistical properties to model lifetime data arising from engineering and biomedical sciences and showed that these distributions provide much

closer fit than Lindley and exponential distributions.

In this paper, a Poisson mixture of Amarendra distribution, introduced by Shanker (2016 c), has been proposed, and called the "Poisson--Amarendra distribution (PAD)". Its important mathematical and statistical properties including, shape, raw moments, central moments, coefficients of variation, skewness and kurtosis are obtained and discussed. The estimation of PAD's parameter has been also discussed using both maximum likelihood estimation and method of moments. The distribution is then fitted using its maximum likelihood estimate on certain data sets in order to test its goodness of fit over Poisson distribution, Poisson-Lindley distribution (PLD), a Poisson-mixture of Lindley (1958) distribution, introduced by Sankaran (1970) and Poisson-Sujatha distribution (PSD) of Shanker (2016 b), a Poisson mixture of Sujatha distribution, introduced by Shanker (2016 a).

2. Poisson-Amarendra Distribution

Assuming the parameter λ of the Poisson distribution to follows Amarendra distribution (1), the Poisson mixture of Amarendra distribution can be obtained as

$$P(X=x)=\int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\cdot\frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-\theta\lambda}d\lambda \tag{3}$$

$$=\frac{\theta^4}{\left(\theta^3+\theta^2+2\theta+6\right)x!}\int_0^\infty \lambda^x\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-(\theta+1)\lambda}d\lambda$$

$$=\frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\cdot\frac{x^3+(\theta+7)x^2+\left(\theta^2+5\theta+15\right)x+\left(\theta^3+4\theta^2+7\theta+10\right)}{(\theta+1)^{x+4}};x=0,1,2,\ldots,\theta>0 \tag{4}$$

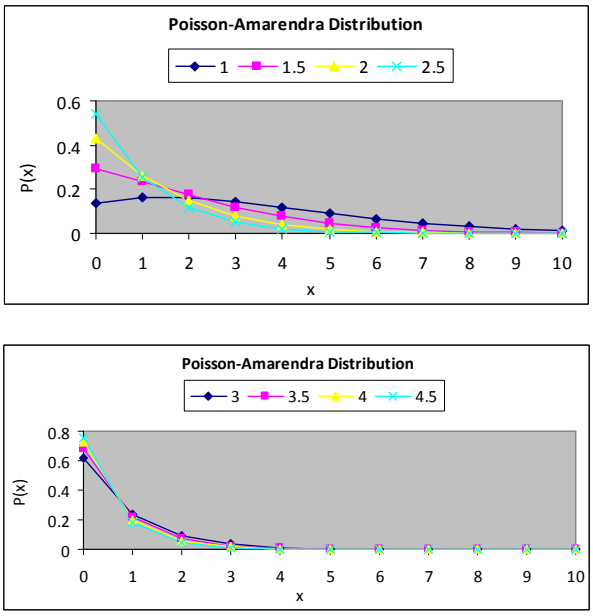


Fig. 1. *Graphs of the PAD' p. m. f. for varying values of its parameter* θ.

We shall refer to this distribution as the "Poisson-Amarendra distribution (PAD)". The graphs of the probability mass function (p. m. f) of PAD for varying values of its parameter θ are shown in the figure 1.

Recall that Sankaran (1970) obtained Poisson-Lindley distribution (PLD) having p. m. f. of the form

$$P(X=x)=\frac{\theta^2(x+\theta+2)}{(\theta+1)^{x+3}};x=0,1,2,\ldots,\theta>0 \tag{5}$$

by compounding Poisson distribution with Lindley distribution, introduced by Lindley (1958) having p. d. f of the form

$$f(x,\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{\theta x};x>0,\ \theta>0. \tag{6}$$

Shanker and Hagos (2015) has discussed the applications of PLD for modeling count data-sets from biological sciences and observed that it gives better fit than Poisson distribution.

Shanker (2016 b) obtained Poisson-Sujatha distribution (PSD) with its p. m. f. given by

$$P(X=x)=\frac{\theta^3}{\theta^2+\theta+2}\cdot\frac{x^2+(\theta+4)x+\left(\theta^2+3\theta+4\right)}{(\theta+1)^{x+3}};x=0,1,2,\ldots,\theta>0. \tag{7}$$

by compounding Poisson distribution with Sujatha distribution, introduced by Shanker (2016 a) having p. d. f. given by

$$f(x;\theta)=\frac{\theta^3}{\theta^2+\theta+2}\left(1+x+x^2\right)e^{-\theta x};x>0,\ \theta>0 \tag{8}$$

Sujatha distribution, introduced by Shanker (2016 a) is considered a better model than exponential and Lindley distributions when modeling lifetime data from biomedical science and engineering. Further, Shanker and Hagos (2016 a) provided a detailed study about the application of the Poisson-Sujatha distribution (PSD) when modeling biological science data, and observed a better fit than the Poisson-Lindley distribution (PLD) and Poisson-distribution.

Shanker and Hagos (2016 b, 2016 c) have obtained the size-biased Poisson-Sujatha distribution (SBPSD) and zero-truncated Poisson-Sujatha distribution (ZTPSD) and discussed their various mathematical and statistical properties, estimation of their parameter and applications. Further, Shanker and Hagos (2016 d) have also provided a detailed study regarding applications of the zero-truncated Poisson distribution (ZTPD), the zero-truncated Poisson-Lindley distribution (ZTPLD), and the zero-truncated Poisson-Sujatha distribution (ZTPSD) for modeling data sets (with zero counts excluded) from demography and biological sciences and concluded that in majority of data sets ZTPSD gives better fit than ZTPD and ZTPLD.

3. Moments and Related Measures

The r th factorial moment about origin of PAD (4) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right], \text{ where}$$
$$X^{(r)} = X(X-1)(X-2)...(X-r+1)$$

Using (3) the r th moment about origin of PAD (4) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)} \mid \lambda\right)\right]$$
$$= \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\int_0^\infty\left[\sum_{x=0}^\infty x^{(r)}\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-\theta\lambda}d\lambda$$
$$= \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\int_0^\infty\left[\lambda^r\sum_{x=r}^\infty\frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-\theta\lambda}d\lambda$$

Taking $x+r$ in place of x within bracket, we get

$$\mu_{(r)}' = \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\int_0^\infty\lambda^r\left[\sum_{x=0}^\infty\frac{e^{-\lambda}\lambda^x}{x!}\right]\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-\theta\lambda}d\lambda$$

The expression within the bracket is clearly unity and hence we have

$$\mu_{(r)}' = \frac{\theta^4}{\theta^3+\theta^2+2\theta+6}\int_0^\infty\lambda^r\left(1+\lambda+\lambda^2+\lambda^3\right)e^{-\theta\lambda}d\lambda$$

Using the gamma integral and some algebraic simplification, we get finally a general expression for the r th factorial raw moment (about the origin) of PAD (4) as

$$\mu_{(r)}' = \frac{r!\left[\theta^3+(r+1)\theta^2+(r+1)(r+2)\theta+(r+1)(r+2)(r+3)\right]}{\theta^r\left(\theta^3+\theta^2+2\theta+6\right)};\ r=1,2,3,.... \quad (9)$$

Substituting $r=1,2,3,$ and 4 in (9), the first four raw factorial moments can be obtained and using the relationship between factorial raw moments and the usual raw moments, the first four raw moments the PAD (4) are obtained as

$$\mu_1' = \frac{\theta^3+2\theta^2+6\theta+24}{\theta\left(\theta^3+\theta^2+2\theta+6\right)}$$

$$\mu_2' = \frac{\theta^4+4\theta^3+12\theta^2+48\theta+120}{\theta^2\left(\theta^3+\theta^2+2\theta+6\right)}$$

$$\mu_3' = \frac{\theta^5+8\theta^4+30\theta^3+120\theta^2+480\theta+720}{\theta^3\left(\theta^3+\theta^2+2\theta+6\right)}$$

$$\mu_4' = \frac{\theta^6+16\theta^5+84\theta^4+360\theta^3+1680\theta^2+5040\theta+5040}{\theta^4\left(\theta^3+\theta^2+2\theta+6\right)}$$

Using the relationship between the central moments (about the mean) and the raw moments (about the origin), the central moments of the PAD (4) are thus given by

$$\mu_2 = \frac{\theta^7+4\theta^6+14\theta^5+58\theta^4+144\theta^3+156\theta^2+240\theta+144}{\theta^2\left(\theta^3+\theta^2+2\theta+6\right)^2}$$

$$\mu_3 = \frac{\begin{pmatrix}\theta^{11}+7\theta^{10}+32\theta^9+146\theta^8+582\theta^7+1440\theta^6+2616\theta^5+4416\theta^4\\+4440\theta^3+4896\theta^2+4320\theta+1728\end{pmatrix}}{\theta^3\left(\theta^3+\theta^2+2\theta+6\right)^3}$$

$$\mu_4 = \frac{\left(\begin{array}{l}\theta^{15}+15\theta^{14}+99\theta^{13}+530\theta^{12}+2678\theta^{11}+10406\theta^{10}+29412\theta^{9}+71788\theta^{8}+149448\theta^{7}\\ +251496\theta^{6}+386880\theta^{5}+505296\theta^{4}+531648\theta^{3}+468288\theta^{2}+311040\theta+93312\end{array}\right)}{\theta^4\left(\theta^3+\theta^2+2\theta+6\right)^4}$$

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2) and index of dispersion (γ) of the PAD (4) are thus obtained as

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\theta^7+4\theta^6+14\theta^5+58\theta^4+144\theta^3+156\theta^2+240\theta+144}}{\theta^3+2\theta^2+6\theta+24}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\left(\begin{array}{l}\theta^{11}+7\theta^{10}+32\theta^{9}+146\theta^{8}+582\theta^{7}+1440\theta^{6}+2616\theta^{5}+4416\theta^{4}\\ +4440\theta^{3}+4896\theta^{2}+4320\theta+1728\end{array}\right)}{\left(\theta^7+4\theta^6+14\theta^5+58\theta^4+144\theta^3+156\theta^2+240\theta+144\right)^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^{2}} = \frac{\left(\begin{array}{l}\theta^{15}+15\theta^{14}+99\theta^{13}+530\theta^{12}+2678\theta^{11}+10406\theta^{10}+29412\theta^{9}+71788\theta^{8}\\ +149448\theta^{7}+251496\theta^{6}+386880\theta^{5}+505296\theta^{4}+531648\theta^{3}+468288\theta^{2}\\ +311040\theta+93312\end{array}\right)}{\left(\theta^7+4\theta^6+14\theta^5+58\theta^4+144\theta^3+156\theta^2+240\theta+144\right)^{2}}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\theta^7+4\theta^6+14\theta^5+58\theta^4+144\theta^3+156\theta^2+240\theta+144}{\theta\left(\theta^3+\theta^2+2\theta+6\right)\left(\theta^3+2\theta^2+6\theta+24\right)}$$

To study the nature and behavior of μ_1', μ_2, C.V, $\sqrt{\beta_1}, \beta_2$ and γ of the PLD, PSD and PAD, their numerical values are calculated for varying values of the parameter θ and the results are presented in tables 1, 2 and 3. It is clear that μ_1', μ_2, and γ are decreasing whereas C. V., $\sqrt{\beta_1}$, β_2 are increasing for increasing values of the parameter θ.

Table 1. *Numerical values of* μ_1', μ_2, C.V, $\sqrt{\beta_1}, \beta_2$ and γ *of PLD, for varying values the parameter* θ.

	Values of θ for Poisson-Lindley Distribution					
	1	**2**	**3**	**4**	**5**	**6**
μ_1'	1.5	0.666667	0.416667	0.3	0.233333	0.190476
μ_2	3.25	1.055556	0.576389	0.385	0.285556	0.225624
CV	1.20185	1.541104	1.822087	2.068279	2.290174	2.493742
$\sqrt{\beta_1}$	1.792108	2.083265	2.314307	2.517935	2.704839	2.87957
β_2	7.532544	8.941828	10.10611	11.17187	12.19654	13.203
γ	2.166667	1.583333	1.383333	1.283333	1.22381	1.184524

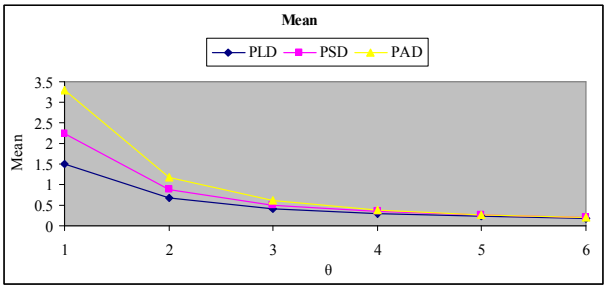
Table 2. *Numerical values of* μ_1', μ_2, C.V, $\sqrt{\beta_1}, \beta_2$ and γ *of PSD for varying values the parameter* θ.

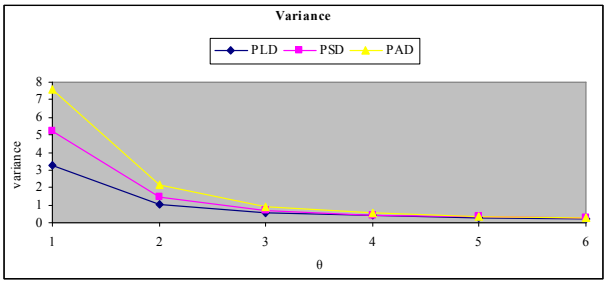
	Values of θ for Poisson-Sujatha Distribution					
	1	**2**	**3**	**4**	**5**	**6**
μ_1'	2.25	0.875	0.5	0.340909	0.25625	0.204545
μ_2	5.1875	1.484375	0.726190	0.451963	0.320586	0.246039
CV	1.012270	1.392399	1.704336	1.972026	2.209573	2.425006
$\sqrt{\beta_1}$	1.467931	1.898599	2.205880	2.443548	2.646990	2.831109
β_2	5.944113	7.864044	9.479441	10.777366	11.910905	12.969309
γ	2.305555	1.696428	1.452381	1.325757	1.251067	1.202862

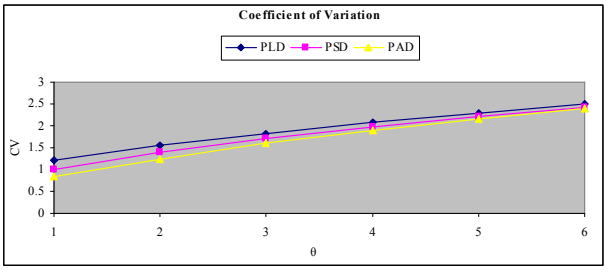
Table 3. *Numerical values of* μ_1', μ_2, C.V, $\sqrt{\beta_1}, \beta_2$ and γ *of PAD for varying values the parameter* θ.

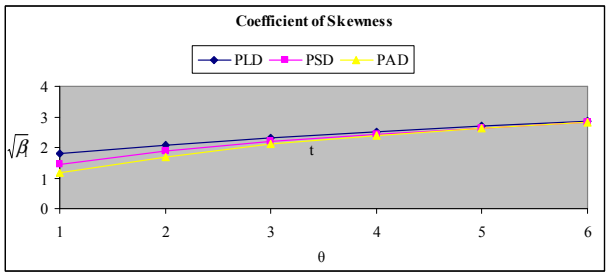
	Values of θ for Poisson-Amarendra Distribution					
	1	**2**	**3**	**4**	**5**	**6**
μ_1'	3.3	1.181818	0.604167	0.382979	0.275904	0.214815
μ_2	7.61	2.14876	0.933594	0.528859	0.353998	0.262497
CV	0.835946	1.240347	1.599272	1.898871	2.156467	2.385051
$\sqrt{\beta_1}$	1.172956	1.675185	2.106094	2.402408	2.626614	2.817693
β_2	4.856217	6.666575	8.883016	10.59733	11.89457	12.99572
γ	2.306061	1.818182	1.545259	1.38091	1.283048	1.221967

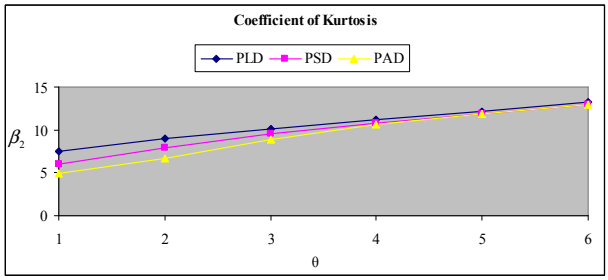
The corresponding sub-figures of figure 2 depict the nature and behavior of the mean, the variance, coefficient of variation, coefficient of skewness, coefficient of kurtosis, and index of dispersion of the PLD, PSD and PAD for varying values of the parameter θ. From these graphs it is obvious that the mean, variance and index of dispersion of PLD, PSD and PAD are decreasing whereas the corresponding coefficient of variation, coefficient of skewness, and coefficient of kurtosis are increasing for increasing values of the parameter θ.











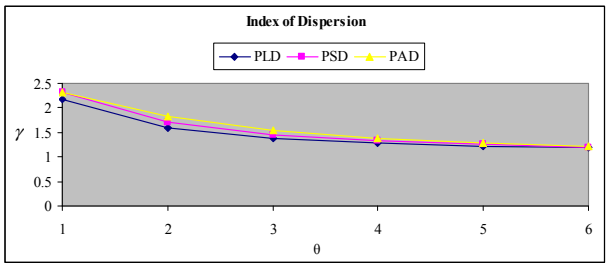


Fig. 2. *Graphs of the mean, variance, coefficient of variation, coefficient of skewness, coefficient of kurtosis and Index of dispersion of the PLD, PSD and PAD for varying values of the parameter* θ.

4. Statistical Properties

4.1. Increasing Hazard Rate (IHR) and Unimodality.

The PAD as in (4) has an increasing hazard rate and has a unimodal density function. Since

$$\frac{P(x+1;\theta)}{P(x;\theta)} = \frac{1}{\theta+1}\left[1+\frac{3x^2+(2\theta+17)x+(\theta^2+6\theta+16)}{x^3+(\theta+7)x^2+(\theta^2+5\theta+15)x+(\theta^3+4\theta^2+7\theta+10)}\right]$$

is decreasing function along x, $P(x;\theta)$ is log-concave. Therefore, the PAD has an increasing hazard rate (IHR) and its p. d. f. is unimodal. The interrelationship between log-concavity, unimodality and increasing hazard rate of discrete

distributions has been discussed in Grandell (1997).

4.2. Over-Dispersion

The PAD (4) is always over- dispersed $\left(\sigma^2 > \mu\right)$. We have

$$\sigma^2 = \frac{\theta^7 + 4\theta^6 + 14\theta^5 + 58\theta^4 + 144\theta^3 + 156\theta^2 + 240\theta + 144}{\theta^2\left(\theta^3 + \theta^2 + 2\theta + 6\right)^2}$$

$$= \frac{\theta^3 + 2\theta^2 + 6\theta + 24}{\theta\left(\theta^3 + \theta^2 + 2\theta + 6\right)}\left[\frac{\theta^7 + 4\theta^6 + 14\theta^5 + 58\theta^4 + 144\theta^3 + 156\theta^2 + 240\theta + 144}{\theta\left(\theta^3 + \theta^2 + 2\theta + 6\right)\left(\theta^3 + 2\theta^2 + 6\theta + 24\right)}\right]$$

$$= \mu\left[1 + \frac{\theta^6 + 4\theta^5 + 18\theta^4 + 96\theta^3 + 72\theta^2 + 96\theta + 144}{\theta\left(\theta^3 + \theta^2 + 2\theta + 6\right)\left(\theta^3 + 2\theta^2 + 6\theta + 24\right)}\right] > \mu.$$

This shows that PAD (4) is always over-dispersed.

5. Parameter Estimation

5.1. Maximum Likelihood Estimate (MLE) of the Parameter

Let $\left(x_1, x_2, ..., x_n\right)$ be a random sample of size n from the PAD (4) and let f_x be the observed frequency in the sample corresponding to $X = x$ $(x = 1, 2, 3, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the PAD (4) is given by

$$L = \left(\frac{\theta^4}{\theta^3 + \theta^2 + 2\theta + 6}\right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k} f_x (x+4)}} \prod_{x=1}^{k}\left[\begin{matrix} x^3 + (\theta+7)x^2 + \left(\theta^2 + 5\theta + 15\right)x \\ + \left(\theta^3 + 4\theta^2 + 7\theta + 10\right)\end{matrix}\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L = n\log\left(\frac{\theta^4}{\theta^3 + \theta^2 + 2\theta + 6}\right) - \sum_{x=1}^{k} f_x (x+4)\log(\theta+1) + \sum_{x=1}^{k} f_x \log\left[x^3 + (\theta+7)x^2 + \left(\theta^2 + 5\theta + 15\right)x + \left(\theta^3 + 4\theta^2 + 7\theta + 10\right)\right]$$

The first derivative of the log likelihood function is given by

$$\frac{d\log L}{d\theta} = \frac{n\left(\theta^3 + 2\theta^2 + 6\theta + 24\right)}{\theta\left(\theta^3 + \theta^2 + 2\theta + 6\right)} - \frac{n(\bar{x}+4)}{\theta+1} + \sum_{x=1}^{k} \frac{\left[x^2 + (2\theta+5)x + \left(3\theta^2 + 8\theta + 7\right)\right] f_x}{\left[x^3 + (\theta+7)x^2 + \left(\theta^2 + 5\theta + 15\right)x + \left(\theta^3 + 4\theta^2 + 7\theta + 10\right)\right]}$$

where $\bar{x}$ is the sample mean.

The maximum likelihood estimate (MLE), $\hat{\theta}$ of θ of PAD (4) is the solution of the equation $\frac{d\log L}{d\theta} = 0$ and is given by the solution of the following non-linear equation

$$\frac{n\left(\theta^3 + 2\theta^2 + 6\theta + 24\right)}{\theta\left(\theta^3 + \theta^2 + 2\theta + 6\right)} - \frac{n(\bar{x}+4)}{\theta+1} + \sum_{x=1}^{k} \frac{\left[x^2 + (2\theta+5)x + \left(3\theta^2 + 8\theta + 7\right)\right] f_x}{\left[x^3 + (\theta+7)x^2 + \left(\theta^2 + 5\theta + 15\right)x + \left(\theta^3 + 4\theta^2 + 7\theta + 10\right)\right]} = 0$$

This non-linear equation can be solved using any numerical iteration methods such as Newton - Raphson method, Bisection method, Regula - Falsi method etc. In this paper, Newton - Raphson method has been used to solve the above equation for estimating the parameter θ.

5.2. Method of Moment Estimate (MOME) of the Parameter

Let $\left(x_1, x_2, ..., x_n\right)$ be a random sample of size n from the PAD (4). Equating the population mean to the corresponding sample mean, the MOME $\tilde{\theta}$ of θ of PAD (4) is the solution of the following biquadratic equation

$$\bar{x}\theta^4 + (\bar{x}-1)\theta^3 + 2(\bar{x}-1)\theta^2 + 6(\bar{x}-1)\theta - 24 = 0$$

where $\bar{x}$ is the sample mean.

6. Applications and Goodness of Fit

In this section the PAD is fitted through four count data

sets in order to test the PAD's goodness of fit in applications and the fiting are then compared with that of Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Sujatha distribution (PSD). The corresponding maximum likelihood estimates (MLE) were used to fit those distributions. The first data set in table 4 is due to Beall (1940) regarding the distribution of Pyrausta nublilalis, while the data sets in table 5, 6 and 7 are relating to the observed number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11 year period of record for the months of June, July and August, January 1957 to December, 1967, available in Falls *et al* (1971). From the fitting of these distributions, it is obvious that PAD provides quite satisfactory fit in all data sets and hence PAD can be considered as an important distribution for modeling data sets over PD, PLD, and PSD.

Table4. *Distribution of Pyrausta nubilalis in 1937.*

No. of insects	Observed Frequency	Expected Frequency			
		PD	**PLD**	**PSD**	**PAD**
		26.4	31.5	31.5	31.6
0	33	19.8	14.2	14.2	14.0
1	12	**7.4**	**6.1**	**6.1**	6.1
2	6	**1.8**	**2.5**	**2.6**	**2.8**
3	3	**0.3**	**1.0**	**1.0**	**1.0**
4	1	**0.3**	**0.7**	**0.6**	**1.5**
5	1				
Total	56	56.0	56.0	56.0	56.0
ML estimate		$\hat{\theta} = 0.7500$	$\hat{\theta} = 1.8081$	$\hat{\theta} = 2.241454$	$\hat{\theta} = 2.640279$
χ^2		4.87	0.53	0.45	0.36
d. f.		1	1	1	2
p-value		0.0273	0.4666	0.5023	0.8353

Table 5. *Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of June, January 1957 to December 1967, Falls et al (1971).*

No. of Thunderstorms	Observed Frequency	Expected Frequency			
		PD	**PLD**	**PSD**	**PAD**
		155.6	185.3	184.8	185.4
0	187	117.0	83.5	83.6	82.7
1	77	43.9	35.9	36.3	36.3
2	40	**11.0**	15.0	15.2	15.4
3	17	**2.1**	**6.1**	**6.1**	**6.3**
4	6	**0.3**	**2.5**	**2.4**	**2.4**
5	2	**0.1**	**1.7**	**1.6**	**1.5**
6	1				
Total	330	330.0	330.0	330.0	330.0
ML estimate		$\hat{\theta} = 0.751515$	$\hat{\theta} = 1.804268$	$\hat{\theta} = 2.229891$	$\hat{\theta} = 2.625345$
χ^2		31.93	1.43	1.25	1.07
d. f.		2	3	3	3
p-value		0.0000	0.6985	0.7410	0.7843

Table 6. *Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of July, January 1957 to December 1967, Falls et al (1971).*

No. of Thunderstorms	Observed Frequency	Expected Frequency			
		PD	**PLD**	**PSD**	**PAD**
		142.3	177.7	176.5	176.7
0	177	124.4	88.0	88.4	87.6
1	80	54.3	41.5	42.2	42.3
2	47	**15.8**	18.9	19.2	19.5
3	26	**3.5**	**8.4**	**8.5**	**8.6**
4	9	**0.7**	**6.5**	**6.2**	**6.3**
5	2				
Total	341	341.0	341.0	341.0	341.0
ML estimate		$\hat{\theta} = 0.873900$	$\hat{\theta} = 1.583536$	$\hat{\theta} = 1.995806$	$\hat{\theta} = 2.390474$
χ^2		39.74	5.15	4.67	4.35
d. f.		2	3	3	3
p-value		0.0000	0.1611	0.1976	0.2261

Table 7. *Observed and expected number of days that experienced X thunderstorms events at Cape Kennedy, Florida for the 11-year period of record for the month of August, January 1957 to December 1967, Falls et al (1971).*

No. of Thunderstorms	Observed Frequency	Expected Frequency			
		PD	PLD	PSD	PAD
0	185	151.8	184.8	184.1	184.7
1	89	122.9	87.2	87.5	86.6
2	30	49.7	39.3	39.8	39.8
3	24	**13.4**	17.1	17.3	17.6
4	10	**2.7**	**7.3**	**7.3**	**7.4**
5	3	**0.5**	**5.3**	**5.0**	**4.9**
Total	341	341.0	341.0	341.0	341.0
ML estimate		$\hat{\theta} = 0.809384$	$\hat{\theta} = 1.693425$	$\hat{\theta} = 2.114545$	$\hat{\theta} = 2.511962$
χ^2		49.49	5.03	5.06	4.83
d. f.		2	3	3	3
p-value		0.0000	0.1696	0.1674	0.1847

7. Concluding Remarks

In the present paper the Poisson-Amarendra distribution (PAD) has been proposed by compounding Poisson distribution with Amarendra distribution, introduced by Shanker (2016 c). The r th factorial raw moment has been derived and the first four raw moments and the central moments have been given. The expression of the coefficient of variation, skewness, kurtosis and index of dispersion has also been obtained. The estimation of its parameter has been discussed using both maximum likelihood estimation and the method of moments. The PAD has been fitted using maximum likelihood estimate to certain data sets to test its goodness of fit over Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Sujatha distribution (PSD). It is found that Poisson-Amarendra distribution (PAD) provides better fit than PD, PLD, and PSD for all data sets.

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