International Journal of Statistical Distributions and Applications
2016; 2(1): 8-13
http://www.sciencepublishinggroup.com/j/ijsda
doi: 10.11648/j.ijsd.20160201.12
ISSN: 2472-3487 (Print); ISSN: 2472-3509 (Online)

Extended Intervened Geometric Distribution

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To cite this article:
C. Satheesh Kumar, S. Sreejakumari. Extended Intervened Geometric Distribution. *International Journal of Statistical Distributions and Applications.* Vol. 2, No. 1, 2016, pp. 8-13. doi: 10.11648/j.ijsd.20160201.12

Received: March 3, 2016; **Accepted**: March 29, 2016; **Published**: April 25, 2016

Abstract: Here we develop an extended version of the modified intervened geometric distribution of Kumar and Sreeja (*The Aligarh Journal of Statistics, 2014)* and investigate some of its important statistical properties. Parameters of the distribution are estimated by various methods of estimation such as the method of factorial moments, the method of mixed moments and the method of maximum likelihood. The distribution has been fitted to a real life data set for illustrating its practical relevance.

Keywords: Factorial Moments, Intervened Geometric Distribution, Method of Factorial Moments, Method of Mixed Moments, Method of Maximum Likelihood, Probability Generating Function, Probability Mass Function

1. Introduction

Intervened type distributions have found many applications in several areas such as epidemiological studies, life testing problems etc. In epidemiological studies health agencies takes various preventive actions. The information concerning the effect of such actions taken by the agencies can statistically analyzed by intervened type distributions. In life testing problem the failed items during the observational period are either replaced or repaired. This kind of actions changes the reliability of the system as only some of its components have longer life. The impact of such actions can be studied by intervened type distributions. The intervened type distributions such as intervened Poisson distribution (IPD), intervened geometric distribution (IGD) and modified intervened geometric distribution (MIGD) has been studied by several authors. For example see Shanmugan [1, 2], Huang and Fung [3], Scollink [4], Dhanavanthan [5, 6], Kumar and Shibu [7-15], Bartolucci et al [16], Kumar and Sreeja [17] etc.

Through this paper we consider a new class of intervened geometric distribution suitable for multiple intervention cases and named it as the extended intervened geometric distribution (EIGD), which contains the MIGD as its special case. The paper is organized as follows. In Section 2, we present a model leading to EIGD and obtain expression for its probability mass function, mean and variance. We also obtain a recurrence relation useful for the computation of probabilities of the EIGD. In Section 3, we consider the estimation of parameters of the EIGD by the method of maximum likelihood and the distribution has been fitted to a real life data set for highlighting the usefulness of the model.

We need the following series representation in the sequel

$$\sum_{i=0}^{\infty}\sum_{r=0}^{\infty} A(i,r) = \sum_{i=0}^{\infty}\sum_{r=0}^{i} A(i-r,r) \tag{1}$$

and

$$\sum_{i=0}^{\infty}\sum_{r=0}^{\infty} B(i,r) = \sum_{i=0}^{\infty}\sum_{r=0}^{\left[\frac{i}{m}\right]} B(i-mr,r), \tag{2}$$

where [a] represents the integer part of “a”, for any $a > 0$

2. Extended Intervened Geometric Distribution

Consider a discrete random variable X having intervened geometric distribution with the following probability mass function (pmf), in which $x = 1, 2, 3, \ldots$, $\theta \in (0, 1)$, $1 \neq \rho_1 > 0$ such that $\rho_1\theta \leq 1$.

$$h(x;\rho_1,\theta)=\frac{(1-\theta)(1-\rho_1\theta)}{1-\rho_1}(1-\rho_1{}^x)\theta^{x-1}$$

The probability generating function (pgf) of X is the following

$$P_X(s)=(1-\theta)(1-\rho_1\theta)s(1-s\theta)^{-1}(1-s\rho_1\theta)^{-1}$$

Let Y be a random variable having geometric distribution with the following pmf, in which $y = 1, 2, 3,..., \theta\in(0, 1)$, $1\neq\rho_2>0$ and such that $\rho_2\theta\le 1$.

$$f(y;\rho_2,\theta)=(1-\rho_2\theta)\,(\rho_2\theta)^y$$

The pgf $Q_Y(s)$ of Y is

$$Q_Y(s)=(1-\rho_2\theta)(1-s\rho_2\theta)^{-1}$$

Define Z = X+ mY, in which X and Y are assumed to be independent and m is a fixed but arbitrary positive integer. Then the pgf of Z is

$$G_Z(s)=P_X(s)\,Q_Y(s^m)$$

$$=\Lambda(\rho_1,\rho_2,\theta)s(1-s\theta)^{-1}(1-s\rho_1\theta)^{-1}(1-s^m\rho_2\theta)^{-1}, \tag{3}$$

where

$$\Lambda(\rho_1,\rho_2,\theta)=(1-\theta)(1-\rho_1\theta)(1-\rho_2\theta)$$

The distribution of a random variable whose pgf is (3) is called “the extended intervened geometric distribution” or in short “the EIGD”.

Result 2.1. Let Z follows EIGD with pgf given in equation number (3). Then the pmf g_z of Z is the following, for z =1,2,3,….

$$g_z=\Lambda(\rho_1,\rho_2,\theta)\sum_{j=0}^{z-1}\sum_{k=0}^{\left[\frac{j}{m}\right]}(\rho_1\theta)^{z-j-1}(\rho_2\theta)^k\,\theta^{j-mk}, \tag{4}$$

where $\Lambda(\rho_1,\rho_2,\theta)$ is as defined in (3), $\theta\in(0, 1)$, $0<\rho_j\neq 1$ such that $\rho_j\theta\le 1$, for j=1,2.

Proof: We have

$$G_Z(s)=\sum_{z=0}^{\infty}s^z g_z \tag{5}$$

$$=\Lambda(\rho_1,\rho_2,\theta)s(1-s\theta)^{-1}(1-s\rho_1\theta)^{-1}(1-s^m\rho_2\theta)^{-1}$$

$$=\Lambda(\rho_1,\rho_2,\theta)\,s\sum_{z=0}^{\infty}(s\rho_1\theta)^z\sum_{j=0}^{\infty}(s\theta)^j\sum_{k=0}^{\infty}(s^m\rho_2\theta)^k$$

$$=\Lambda(\rho_1,\rho_2,\theta)\sum_{z=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\rho_1{}^z\rho_2{}^k\theta^{z+j+k}s^{z+j+mk+1} \tag{6}$$

Now by applying the results given in equations (1) and (2) in equation (6) we get

$$G_z(s)=\Lambda(\rho_1,\rho_2,\theta)\sum_{z=0}^{\infty}\sum_{j=0}^{z}\sum_{k=0}^{\left[\frac{j}{m}\right]}\rho_1{}^{z-j}\rho_2{}^k\theta^{z-(m-1)k}s^{z+1} \tag{7}$$

On equating co efficient of s^z in the right hand side expressions of (5) and (7) we get (4) when $\rho_2\to 0$ and $\rho_1=\rho$ (4) reduces to the pmf of IGD defined by Bartalucci et.al [8]

Result 2.2 The first three factorial moments $\mu^1{}_{(1)},\mu^1{}_{(2)}$ and $\mu^1{}_{(3)}$ of the EIGD are

$$\mu^1{}_{(1)}=1+\delta\ +\delta_2+m\delta_3,$$

$$\mu^1{}_{(2)}=2m^2\delta_3{}^2+m(m+1)\delta_3+2\delta_2(1+\delta_2)+2\delta_1(1+\delta_1)+2m\delta_3(\delta_1+\delta_2)+2\delta_1\delta_2$$

and

$$\mu^1{}_{(3)}=6\left[m^3\delta_3{}^2\left(1+\delta_3\right)+\delta_2{}^2\left(1+\delta_2\right)+\delta_1{}^2\left(1+\delta_1\right)\right]+6m\delta_3(\delta_1{}^2+\delta_2{}^2+\delta_1\delta_2)+6\delta_1\delta_2(1+\delta_1+\delta_2)+m^2(m-1)\delta_3+3m\delta_3(\delta_1+\delta_2)(m+1+2m\delta_3), \tag{8}$$

in which $\delta_1=\theta(1-\theta)^{-1}$, $\delta_2=\rho_1\theta(1-\rho_1\theta)^{-1}$ and $\delta_3=\rho_2\theta(1-\rho_2\theta)^{-1}$.

Proof follows from the fact that

$$\mu^1{}_{(1)}=G_Z^{(1)}(s)/s=1,$$

$$\mu^1{}_{(2)}=G_Z^{(2)}(s)/s=1$$

and

$$\mu^1{}_{(3)}=G_Z^{(3)}(s)/s=1$$

in which

$$G_Z^{(r)}(s)=\frac{d^r}{ds^r}(G_Z(s))$$

with $G_Z(s)$ as the pgf of Z.

Result 2.3 The mean and variance of EIGD is

$$\text{Mean} = 1+\delta1 + \delta2 +m\ \delta3 \tag{9}$$

and

$$\text{Variance} = \delta1(1+\delta1)+\delta2(1+\delta2)+m^2\delta3(1+\delta3) \quad (10)$$

where δ_j for j=1,2,3 are as defined in Result 2.2.

Proof: On differentiating the pgf $G_Z(s)$ of "Z" given in equation (3) with respect to s and putting s=1, we get

$$\begin{aligned} G_Z^{(1)}(1) &= E(Z) = 1+\delta_1+\delta_2+m\delta_3 \\ G_Z^{(2)}(1) &= E(Z(Z-1)) \\ &= 2m^2\delta_3^{\ 2}+m(m+1)\delta_3+2m\delta_3(\delta_1+\delta_2)\,. \\ &\quad +2\delta_1(1+\delta_1)+2\delta_2(1+\delta_2)+2\delta_1\delta_2 \end{aligned}$$

in which

$$G_Z^{(r)}(1) = \frac{d^r}{ds^r}(G_Z(s))\,/\,s=1.$$

Now the mean and variance are obtained by using the following results

$$\text{Mean}= G_Z^{(1)}(1)$$

and

$$\text{Variance (Z)}= G_Z^{(2)}(1)+G_Z^{(1)}(1)-\left(G_Z^{(1)}(1)\right)^2$$

Result 2.4 For z ≥1, the following is a simple recurrence relation for probabilities g_z of the EIGD for z<m

$$zg_{z+1} = \sum_{i=0}^{z-1} (1+\rho_1^{\ i+1})\,\theta^{i+1}g_{z-i}$$

and for $z \geq m$

$$\begin{aligned} zg_{z+1} &= m\sum_{i=0}^{\left[\frac{z-m}{m}\right]} (\rho_2\theta)^{i+1}g_{z-mi-m+1} \\ &\quad +\sum_{i=0}^{z-1} (1+\rho_1^{\ i+1})\theta^{i+1}g_{z-i} \end{aligned} \quad (11)$$

Proof: From (3) we have

$$G_Z(s) = \sum_{z=0}^{\infty} s^z g_z \quad (12)$$

$$\begin{aligned} &= \Lambda(\rho_1,\rho_2,\theta) \\ &\quad s(1-s\theta)^{-1}(1-s\rho_1\theta)^{-1}(1-s^m\rho_2\theta)^{-1} \end{aligned} \quad (13)$$

On differentiating (12) and (13) with respect to s we get the following, in the light of (3),

$$\begin{aligned} \sum_{z=0}^{\infty} (z+1)g_{z+1}s^z &= \frac{G_z(s)}{s}+\theta\frac{G_z(s)}{1-s\theta} \\ &\quad +\rho_1\theta\frac{G_z(s)}{1-s\rho_1\theta}+m\rho_2\theta\frac{G_z(s)s^{m-1}}{1-s^m\rho_2\theta} \end{aligned} \quad (14)$$

By applying (1) and (2) in (14) to obtain

$$\begin{aligned} &\sum_{z=0}^{\infty}(z+1)s^z g_{z+1} = \\ &\quad \sum_{z=0}^{\infty}s^z g_{z+1}+\sum_{z=0}^{\infty}\sum_{i=0}^{z}(1+\rho_1^{\ i+1})\theta^{i+1}s^{z+1}g_{z-i+1} \\ &\quad +m\sum_{z=0}^{\infty}\sum_{i=0}^{\left[\frac{z}{m}\right]}(\rho_2\theta)^{i+1}s^{z+m}g_{z-mi+1}, \end{aligned}$$

which implies

$$\begin{aligned} \sum_{z=0}^{\infty}zs^z g_{z+1} &= \sum_{z=0}^{\infty}\sum_{i=0}^{z-1}(1+\rho_1^{\ i+1})\theta^{i+1}s^z g_{z-i} \\ &\quad +m\sum_{z=0}^{\infty}\sum_{i=0}^{\left[\frac{z-m}{m}\right]}(\rho_2\theta)^{i+1}s^z g_{z-mi-m+1} \end{aligned} \quad (15)$$

On equating coefficient of s^z on both sides of the expressions (15), we get (11).

3. Estimation

Here we discuss the estimation of the parameters of the EIGD by various methods of estimation such as the method of factorial moments, the method of mixed moments and the method of maximum likelihood. We assume that m is a fixed positive integer and the parameters ρ_1, ρ_2 and θ of the EIGD are estimated for possible values of m.

Method of factorial moments

In method of factorial moments, equate the first three factorial moments of the EIGD to the corresponding sample factorial moments say $m_1^{/}$, $m_2^{/}$, and $m_3^{/}$ and there by we obtain the following system of equations:

$$1+\delta\ +\delta_2+m\delta_3 = m_1^{/} \quad (16)$$

$$\begin{aligned} &2m^2\delta_3^{\ 2}+m(m+1)\delta_3 \\ &\quad +2\delta_2(1+\delta_2)+2\delta_1(1+\delta_1) \\ &\quad +2m\delta_3(\delta_1+\delta_2)+2\delta_1\delta_2 = m_2^{/} \end{aligned} \quad (17)$$

$$\begin{aligned} &6\left[m^3\delta_3^{\ 2}\left(1+\delta_3\ \right)+\delta_2^{\ 2}\left(1+\delta_2\ \right)+\delta_1^{\ 2}\left(1+\delta_1\ \right)\right] \\ &\quad +6m\delta_3(\delta_1^{\ 2}+\delta_2^{\ 2}+\delta_1\delta_2) \\ &\quad +6\delta_1\delta_2(1+\delta_1+\delta_2)+m^2(m-1)\delta_3 \\ &\quad +3m\delta_3(\delta_1+\delta_2)(m+1+2m\delta_3) = m_3^{/} \end{aligned} \quad (18)$$

where δ_1, δ_2 and δ_3 are given in (8). Now the parameters of EIGD are estimated by solving the non- linear equations (16), (17) and (18).

Method of mixed moments

In method of mixed moments, the parameters are estimated by using the first two sample factorial moments and the first observed frequency of the distribution. That is, the parameters are estimated by solving the following equation together with (16) and (17).

$$\Lambda(\rho_1, \rho_{2,} \theta) = \frac{p_1}{N} \quad (19)$$

where $\Lambda(\rho_1, \rho_{2,} \theta)$ is as defined in (3), p_1 is the observed frequency of the distribution corresponding to the observation x = 1 and N is the total frequency.

Method of maximum likelihood

Let a(z) be the observed frequency of z events and u be the highest value of z observed. Then the likelihood function of the sample is

$$L = \prod_{z=1}^{u} (g_z)^{a(z)}$$

which implies

$$\log L = \sum_{z=1}^{u} a(z) \log (g_z)$$

Let $\hat{\rho}_1, \hat{\rho}_2, \hat{\theta}$ denotes the maximum likelihood estimators of $\rho_{1,}$ ρ_2, θ respectively. Now $\hat{\rho}_1, \hat{\rho}_2, \hat{\theta}$ are obtained by solving the likelihood equations (20), (21) and (22) as given below.

$$\frac{\partial \log L}{\partial \theta} = 0$$

implies

$$\sum_{z=1}^{u} a(z)\Psi_1(\rho_1, \rho_2, \theta) + \sum_{z=1}^{u} a(z) \frac{\Psi_2(m, \rho_1, \rho_2, \theta)}{\Psi_3(m, \rho_1, \rho_2, \theta)} = \sum_{z=1}^{u} z a(z) \quad (20)$$

where

$$\Psi_1(\rho_1, \rho_2, \theta) = 1 + \frac{\theta}{1-\theta} + \frac{\rho_1\theta}{1-\rho_1\theta} + \frac{\rho_2\theta}{1-\rho_2\theta},$$

$$\Psi_2(m, \rho_1, \rho_2, \theta) = \sum_{j=0}^{z-1} \sum_{k=0}^{\left[\frac{j}{m}\right]} (m-1) k \left(\frac{1}{\rho_1}\right)^j \left(\frac{\rho_2}{\theta^{m-1}}\right)^k$$

and

$$E_3(m, \rho_1, \rho_2, \theta) = \sum_{j=0}^{z-1} \sum_{k=0}^{\left[\frac{j}{m}\right]} \left(\frac{1}{\rho_1}\right)^j \left(\frac{\rho_2}{\theta^{m-1}}\right)^k$$

$$\frac{\partial \log L}{\partial \rho_1} = 0$$

implies

$$\sum_{z=1}^{u} a(z)(1 + \frac{\rho_1\theta}{1-\rho_1\theta}) + \sum_{z=1}^{u} a(z) \frac{\Psi_4(m, \rho_1, \rho_2, \theta)}{\Psi_3(m, \rho_1, \rho_2, \theta)} = 0, \quad (21)$$

where

$$\Psi_4(m, \rho_1, \rho_2, \theta) = \sum_{j=0}^{z-1} \sum_{k=0}^{\left[\frac{j}{m}\right]} j \left(\frac{1}{\rho_1}\right)^j \left(\frac{\rho_2}{\theta^{m-1}}\right)^k$$ and

$\Psi_3(m, \rho_1, \rho_2, \theta)$ is as defined in (20). $\frac{\partial \log L}{\partial \rho_1} = 0$

implies

$$\sum_{z=1}^{u} a(z) \frac{\rho_2\theta}{1-\rho_2\theta} + \sum_{z=1}^{u} a(z) \frac{\Psi_5(m, \rho_1, \rho_2, \theta)}{\Psi_3(m, \rho_1, \rho_2, \theta)} = 0 \quad (22)$$

in which $\Psi_5(m, \rho_1, \rho_2, \theta) = \sum_{j=0}^{z-1} \sum_{k=0}^{\left[\frac{j}{m}\right]} k \left(\frac{1}{\rho_1}\right)^j \left(\frac{\rho_2}{\theta^{m-1}}\right)^k$ and

$\Psi_3(m, \rho_1, \rho_2, \theta)$ is as defined in (20).

We present the fitting of the intervened geometric distribution (IGD) and the extended intervened geometric distribution (EIGD) for particular values of m for the data set on the count of the number of European red mites on apple leaves taken from Jani and Shah [18]. We estimate the parameters by the method of factorial moments, the method of mixed moments and the method of maximum likelihood. We have computed the values of χ^2 statistics in the case of each model and the numerical results are summarized in Table 1,Table 2 and Table 3. From the tables it is obvious that the EIGD with m=3 gives a better fit compared to IGD as well as for the case m=1, m=2(MIGD) and m=4.

Table 1. *(Observed frequencies O_i and Expected frequencies E_i calculated by the method of factorial moments).*

x	Oi	Ei	Expected frequencies of EGD for different values of m			
		(IGD)	m=1	m=2	m=3	m=4
1	38	29	28	32	40	33
2	17	20	26	17	18	22
3	10	12	14	15	7	11
4	9	7	7	7	9	5
5^+	6	12	5	9	6	9
Total	80	80	80	80	80	80
Estimated values of parameters		$\hat{\rho}_1 = 0.17$ $\hat{\theta} = 0.596$	$\hat{\rho}_1 = 1.21$ $\hat{\rho}_2 = 1.8$ $\hat{\theta} = 0.216$	$\hat{\rho}_1 = 0.31$ $\hat{\rho}_2 = 0.54$ $\hat{\theta} = 0.416$	$\hat{\rho}_1 = 0.427$ $\hat{\rho}_2 = 0.491$ $\hat{\theta} = 0.326$	$\hat{\rho}_1 = 1.05$ $\hat{\rho}_2 = 0.187$ $\hat{\theta} = 0.326$
Chi- square value		7.148	8.601	4.363	1.441	6.185
p-value		0.128	0.072	0.359	0.837	0.186

Table 2. *(Observed frequencies O_i and Expected frequencies E_i calculated by the method of mixed moments).*

x	Oi	Ei	Expected frequencies of EGD for different values of m			
		(IGD)	m=1	m=2	m=3	m=4
1	38	27	28	26	40	29
2	17	22	23	17	16	19
3	10	14	14	15	6	10
4	9	8	7	9	10	5
5^+	6	9	8	13	8	17
Total	80	80	80	80	80	80
Estimated values of parameters		$\hat{\rho}_1 = 0.56$ $\hat{\theta} = 0.521$	$\hat{\rho}_1 = 0.35$ $\hat{\rho}_2 = 0.187$ $\hat{\theta} = 0.526$	$\hat{\rho}_1 = 0.25$ $\hat{\rho}_2 = 0.387$ $\hat{\theta} = 0.529$	$\hat{\rho}_1 = 0.246$ $\hat{\rho}_2 = 0.587$ $\hat{\theta} = 0.329$	$\hat{\rho}_1 = 0.57$ $\hat{\rho}_2 = 0.387$ $\hat{\theta} = 0.428$
Chi- square value		7.886	7.351	10.974	3.429	13.321
p-value		0.096	0.118	0.027	0.489	0.009

Table 3. *(Observed frequencies O_i and Expected frequencies E_i calculated by the method of maximum likelihood).*

x	Oi	Ei	Expected frequencies of EGD for different values of m			
		(IGD)	m=1	m=2	m=3	m=4
1	38	41	28	38	38	38
2	17	22	25	17	20	21
3	10	10	15	14	8	9
4	9	5	7	5	8	5
5^+	6	2	5	6	6	7
Total	80	80	80	80	80	80
Estimated values of parameters		$\hat{\rho}_1 = 0.275$ $\hat{\theta} = 0.426$	$\hat{\rho}_1 = 0.894$ $\hat{\rho}_2 = 1.207$ $\hat{\theta} = 0.284$	$\hat{\rho}_1 = 1.594$ $\hat{\rho}_2 = 1.407$ $\hat{\theta} = 0.164$	$\hat{\rho}_1 = 0.842$ $\hat{\rho}_2 = 0.448$ $\hat{\theta} = 0.284$	$\hat{\rho}_1 = 0.47$ $\hat{\rho}_2 = 0.207$ $\hat{\theta} = 0.375$
Chi- square value		12.556	8.57	4.343	1.075	4.216
p-value		0.014	0.073	0.362	0.898	0.378

Acknowledgements

Authors wish to express their sincere gratitude to the Editor-in Chief and anonymous referees for fruitful comments.

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