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On Quasi Lindley Distribution and Its Applications to Model Lifetime Data

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Abstract: In this paper mathematical and statistical properties including moment generating function, mean deviations about mean and median, order statistics, Bonferroni and Lorenz curves, Renyi entropy and stress strength reliability of quasi Lindley distribution (QLD) introduced by Shanker and Mishra (2013 a) have been derived and discussed. The goodness of fit of QLD over exponential and Lindley distributions have been illustrated with five real lifetime data-sets and found that QLD provides better fit than exponential and Lindley distributions.

Keywords: Mean Deviations, Order Statistics, Bonferroni and Lorenz Curves, Renyi Entropy Measure, Stress-Strength Reliability, Goodness of Fit

1. Introduction

Lindley distribution, introduced in the context of Bayesian analysis as a counter example of fiducial statistics, having probability density function (p.d.f) and cumulative distribution function (c.d.f)

$$f(x;\theta)=\frac{\theta^2}{\theta+1}(1+x)e^{-\theta x} \quad ;x>0,\ \theta>0 \tag{1.1}$$

$$F(x;\theta)=1-\left[1+\frac{\theta x}{\theta+1}\right]e^{-\theta x} \ ;x>0,\theta>0 \tag{1.2}$$

has been introduced by Lindley (1958). A detailed study about its important mathematical and statistical properties, estimation of parameter and application showing the superiority of Lindley distribution over exponential distribution for the waiting times before service of the bank customers has been done by Ghitany *et al* (2008). The Lindley distribution has been generalized, extended, modified and mixed with other discrete distributions by different researchers including Zakerzadeh and Dolati (2009), Nadarajah et al (2011), Deniz and Ojeda (2011), Bakouch et al (2012), Shanker and Mishra (2013 a, 2013 b), Shanker and Amanuel (2013), Shanker *et al* (2013), Elbatal *et al* (2013), Ghitany *et al* (2013), Merovci (2013), Liyanage and Pararai (2014), Ashour and Eltehiwy (2014), Oluyede and Yang (2014), Singh *et al* (2014), Sharma *et al* (2015), Shanker *et al* (2015 a, 2015 b), Alkarni (2015), Pararai *et al* (2015) are some among others.

The probability density function (p.d.f.) and cumulative distribution function (c.d.f) of quasi Lindley distribution (QLD) of Shanker and Mishra (2013a) are given by

$$f(x;\alpha,\theta)=\frac{\theta}{\alpha+1}(\alpha+\theta x)e^{-\theta x} \quad ;x>0,\ \theta>0,\alpha>-1 \tag{1.3}$$

$$F(x;\alpha,\theta)=1-\left[\frac{1+\alpha+\theta x}{\alpha+1}\right]e^{-\theta x};x>0,\theta>0,\alpha>-1 \tag{1.4}$$

At $\alpha=\theta$, both (1.3) and (1.4) reduce to the corresponding expressions (1.1) and (1.2) of Lindley distribution. The first two moments about origin and the variance of QLD obtained by Shanker and Mishra (2013a) are

$$\mu_1'=\frac{1}{\theta}\left(\frac{\alpha+2}{\alpha+1}\right) \tag{1.5}$$

$$\mu_2' = \frac{2}{\theta^2}\left(\frac{\alpha+3}{\alpha+1}\right) \tag{1.6}$$

$$\mu_2 = \frac{\alpha^2+4\alpha+2}{\theta^2(\alpha+1)^2} \tag{1.7}$$

At $\alpha = \theta$, these moments reduce to the corresponding moments of Lindley distribution. Shanker and Mishra (2013 a) have derived and discussed some of its mathematical properties including its shape, moments, coefficient of variation, coefficient of skewness and kurtosis, hazard rate function, mean residual life function and stochastic orderings. They have also discussed the estimation of its parameters using maximum likelihood estimation and method of moments and its goodness of fit over Lindley and exponential distributions. It has been observed that many important mathematical and statistical properties of this distribution have not been derived and studied.

In the present paper some of the important mathematical and statistical properties including moment generating function, mean deviations about mean and median, order statistics, Bonferroni and Lorenz curves, Renyi entropy measure and stress strength reliability of QLD of Shanker and Mishra (2013 a) have been derived and discussed. Its goodness of fit over exponential and Lindley distributions have been illustrated with some real lifetime data-sets and found that QLD gives better fit than exponential and Lindley distributions.

2. Mathematical and Statistical Properties

2.1. Moment Generating Function

The moment generating function, $(M_X(t))$ of QLD (1.3) can be obtained as

$$M_X(t) = \frac{\theta}{\alpha+1}\int_0^\infty e^{-(\theta-t)}(\alpha+\theta x)\,dx = \frac{\theta}{\alpha+1}\left[\frac{\alpha}{\theta-t}+\frac{\theta}{(\theta-t)^2}\right]$$
$$= \frac{\theta}{\alpha+1}\left[\frac{\alpha}{\theta}\sum_{k=0}^{\infty}\left(\frac{t}{\theta}\right)^k + \frac{1}{\theta}\sum_{k=0}^{\infty}\binom{k+1}{k}\left(\frac{t}{\theta}\right)^k\right]$$
$$= \sum_{k=0}^{\infty}\left(\frac{\alpha+k+1}{\alpha+1}\right)\left(\frac{t}{\theta}\right)^k$$

It can be easily seen that the expression for μ_r', obtained as the coefficient of $\frac{t^r}{r!}$ in $M_X(t)$ is given as

$$\mu_r' = \frac{r!(\alpha+r+1)}{\theta^r(\alpha+1)}\;; r = 1,2,3,...$$

For $\alpha = \theta$, μ_r' reduces to the corresponding μ_r' of Lindley distribution. For $r = 1$ and $r = 2$ the first two moments about origin as given by (1.5) and (1.6) can easily be obtained.

2.2. Mean Deviations about Mean and Median

The amount of scatter in a population is measured to some extent by the totality of deviations usually from their mean and median and are known as the mean deviation about the mean and the mean deviation about the median, and are defined as

$$\delta_1(X) = \int_0^\infty |x-\mu| f(x)\,dx \quad \text{and} \quad \delta_2(X) = \int_0^\infty |x-M| f(x)\,dx\,,$$

respectively, where $\mu = E(X)$ and $M = \text{Median}(X)$. The derivation of measures $\delta_1(X)$ and $\delta_2(X)$ can be obtained using the following simplified relationships

$$\delta_1(X) = \int_0^\mu (\mu-x)f(x)\,dx + \int_\mu^\infty (x-\mu)f(x)\,dx$$
$$= \mu F(\mu) - \int_0^\mu x f(x)\,dx - \mu[1-F(\mu)] + \int_\mu^\infty x f(x)\,dx$$
$$= 2\mu F(\mu) - 2\mu + 2\int_\mu^\infty x f(x)\,dx$$
$$= 2\mu F(\mu) - 2\int_0^\mu x f(x)\,dx \tag{2.2.1}$$

and

$$\delta_2(X) = \int_0^M (M-x)f(x)\,dx + \int_M^\infty (x-M)f(x)\,dx$$
$$= M\,F(M) - \int_0^M x f(x)\,dx - M[1-F(M)] + \int_M^\infty x f(x)\,dx$$
$$= -\mu + 2\int_M^\infty x f(x)\,dx$$
$$= \mu - 2\int_0^M x f(x)\,dx \tag{2.2.2}$$

Using p.d.f. (1.3), and the expression for the mean of QLD (1.3), we have

$$\int_0^\mu x f(x)\,dx = \mu - \frac{\{\theta^2\mu^2 + (\alpha+2)\theta\mu + (\alpha+2)\}e^{-\theta\mu}}{\theta(\alpha+1)} \tag{2.2.3}$$

$$\int_0^M x f(x)\,dx = \mu - \frac{\{\theta^2 M^2 + (\alpha+2)\theta M + (\alpha+2)\}e^{-\theta M}}{\theta(\alpha+1)} \tag{2.2.4}$$

Using expressions from (2.2.1), (2.2.2), (2.2.3) and (2.2.4) and little algebraic simplification, $\delta_1(X)$ and $\delta_2(X)$ of QLD (1.3), are obtained as

$$\delta_1(X) = \frac{2(\theta\mu+\alpha+2)e^{-\theta\mu}}{\theta(\alpha+1)} \tag{2.2.5}$$

and

$$\delta_2(X)=\frac{2\left\{\theta^2M^2+(\alpha+2)\theta M+(\alpha+2)\right\}e^{-\theta M}}{\theta(\alpha+1)}-\mu \quad (2.2.6)$$

It can be easily verified that expressions (2.2.5) and (2.2.6) of QLD (1.3) reduce to the corresponding expressions of Lindley distribution at $\alpha=\theta$.

2.3. Distribution of Order Statistics

Let $\left(X_1, X_2, ..., X_n\right)$ be a random sample of size n from QLD (1.3). Let $X_{(1)}<X_{(2)}<...<X_{(n)}$ denote the corresponding order statistics. The p.d.f. and the c.d.f. of the k th order statistic, say $Y=X_{(k)}$ are given by

$$f_Y(y)=\frac{n!}{(k-1)!(n-k)!}F^{k-1}(y)\left\{1-F(y)\right\}^{n-k}f(y)$$

$$=\frac{n!}{(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l F^{k+l-1}(y)f(y)$$

and

$$F_Y(y)=\sum_{j=k}^{n}\binom{n}{j}F^j(y)\left\{1-F(y)\right\}^{n-j}$$

$$=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l F^{j+l}(y),$$

respectively, for $k=1,2,3,...,n$.

Thus, the p.d.f. and the c.d.f of the k th order statistics of QLD (1.3) are obtained as

$$f_Y(y)=\frac{n!\theta(\alpha+\theta x)e^{-\theta x}}{(\alpha+1)(k-1)!(n-k)!}\sum_{l=0}^{n-k}\binom{n-k}{l}(-1)^l\times\left[1-\frac{1+\alpha+\theta x}{\alpha+1}e^{-\theta x}\right]^{k+l-1}$$

and

$$F_Y(y)=\sum_{j=k}^{n}\sum_{l=0}^{n-j}\binom{n}{j}\binom{n-j}{l}(-1)^l\left[1-\frac{1+\alpha+\theta x}{\alpha+1}e^{-\theta x}\right]^{j+l}$$

It can be easily verified that the expressions for the p.d.f. and c.d.f. of the k th order statistics of QLD (1.3) reduce to the corresponding expressions for the p.d.f. and c.d.f. of the k th order statistics of Lindley distribution at $\alpha=\theta$.

2.4. Bonferroni and Lorenz Curves and Indices

The Bonferroni and Lorenz curves (Bonferroni, 1930) and Bonferroni and Gini indices have much applications in economics to study income and poverty. But now a days these indices have many applications in other fields of knowledge including reliability, demography, insurance, medicine and engineering. The Bonferroni and Lorenz curves are defined as

$$B(p)=\frac{1}{p\mu}\int_0^q x f(x)dx=\frac{1}{p\mu}\left[\int_0^\infty x f(x)dx-\int_q^\infty x f(x)dx\right]=\frac{1}{p\mu}\left[\mu-\int_q^\infty x f(x)dx\right] \quad (2.4.1)$$

and

$$L(p)=\frac{1}{\mu}\int_0^q x f(x)dx=\frac{1}{\mu}\left[\int_0^\infty x f(x)dx-\int_q^\infty x f(x)dx\right]=\frac{1}{\mu}\left[\mu-\int_q^\infty x f(x)dx\right] \quad (2.4.2)$$

respectively or equivalently

$$B(p)=\frac{1}{p\mu}\int_0^p F^{-1}(x)dx \quad (2.4.3)$$

and

$$L(p)=\frac{1}{\mu}\int_0^p F^{-1}(x)dx \quad (2.4.4)$$

respectively, where $\mu=E(X)$ and $q=F^{-1}(p)$.

The Bonferroni and Gini indices are thus defined as

$$B=1-\int_0^1 B(p)dp \quad (2.4.5)$$

and

$$G=1-2\int_0^1 L(p)dp \quad (2.4.6)$$

respectively.

Using p.d.f. (1.3), we have

$$\int_q^\infty x f(x)dx=\frac{\left\{\theta^2q^2+(\alpha+2)\theta q+(\alpha+2)\right\}e^{-\theta q}}{\theta(\alpha+1)} \quad (2.4.7)$$

Now using equation (2.4.7) in (2.4.1) and (2.4.2), we have

$$B(p)=\frac{1}{p}\left[1-\frac{\left\{\theta^2q^2+(\alpha+2)\theta q+(\alpha+2)\right\}e^{-\theta q}}{\alpha+2}\right] \quad (2.4.8)$$

And

$$L(p)=1-\frac{\left\{\theta^2q^2+(\alpha+2)\theta q+(\alpha+2)\right\}e^{-\theta q}}{\alpha+2} \quad (2.4.9)$$

Now using equations (2.4.8) and (2.4.9) in (2.4.5) and (2.4.6), the Bonferroni and Gini indices of QLD (1.3) are obtained as

$$B=1-\frac{\left\{\theta^2q^2+(\alpha+2)\theta q+(\alpha+2)\right\}e^{-\theta q}}{\alpha+2} \quad (2.4.10)$$

$$G=-1+\frac{\left\{\theta^2q^2+(\alpha+2)\theta q+(\alpha+2)\right\}e^{-\theta q}}{\alpha+2} \quad (2.4.11)$$

The Bonferroni and Gini indices of Lindley distribution are particular cases of the Bonferroni and Gini indices (2.4.10) and (2.4.11) of QLD (1.3) for $\alpha=\theta$.

2.5. Renyi Entropy Measure

The entropy of a random variable X is a measure of variation of uncertainty. A popular entropy measure is Renyi entropy (1961). If X is a continuous random variable having probability density function $f(.)$, then Renyi entropy is defined as

$$T_R(\gamma)=\frac{1}{1-\gamma}\log\left\{\int f^{\gamma}(x)\,dx\right\}$$

where $\gamma>0$ and $\gamma\neq1$.

Thus, the Renyi entropy for QLD (1.3) can be obtained as

$$\begin{aligned}
T_R(\gamma)&=\frac{1}{1-\gamma}\log\left[\int_0^\infty\left(\frac{\theta}{\alpha+1}\right)^{\gamma}(\alpha+\theta x)^{\gamma}e^{-\theta\gamma x}\,dx\right]\\
&=\frac{1}{1-\gamma}\log\left[\int_0^\infty\left(\frac{\theta}{\alpha+1}\right)^{\gamma}\alpha^{\gamma}\left(1+\frac{\theta}{\alpha}x\right)^{\gamma}e^{-\theta\gamma x}\,dx\right]\\
&=\frac{1}{1-\gamma}\log\left[\int_0^\infty\left(\frac{\alpha\theta}{\alpha+1}\right)^{\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\frac{\theta}{\alpha}x\right)^{j}e^{-\theta\gamma x}\,dx\right]\\
&=\frac{1}{1-\gamma}\log\left[\left(\frac{\alpha\theta}{\alpha+1}\right)^{\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\frac{\theta}{\alpha}\right)^{j}\int_0^\infty e^{-\theta\gamma x}x^{j+1-1}\,dx\right]\\
&=\frac{1}{1-\gamma}\log\left[\left(\frac{\alpha\theta}{\alpha+1}\right)^{\gamma}\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\frac{\theta}{\alpha}\right)^{j}\frac{\Gamma(j+1)}{(\theta\gamma)^{j+1}}\right]\\
&=\frac{1}{1-\gamma}\log\left[\sum_{j=0}^{\infty}\binom{\gamma}{j}\left(\frac{\alpha\theta}{\alpha+1}\right)^{\gamma}\left(\frac{\theta}{\alpha}\right)^{j}\frac{\Gamma(j+1)}{(\theta\gamma)^{j+1}}\right]
\end{aligned}$$

The Renyi entropy of Lindley distribution is a particular case of the Renyi entropy of QLD (1.3) at $\alpha=\theta$.

2.6. Stress-Strength Reliability

The stress- strength reliability of a component describes the life of the component having random strength X subject to a random stress Y. When the stress applied to the component exceeds the strength $(X<Y)$, the component fails instantly and the component will function satisfactorily till $X>Y$. Therefore, $R=P(Y<X)$ is a measure of the component reliability and is known as stress-strength parameter in reliability engineering. It has wide applications in almost all areas of knowledge especially in engineering such as structures, deterioration of rocket motors, static fatigue of ceramic components, aging of concrete pressure vessels etc.

Let X and Y be independent strength and stress random variables having QLD (1.3) with parameter (α_1,θ_1) and (α_2,θ_2) respectively. Then the stress-strength reliability R can be obtained as

$$\begin{aligned}
R=P(Y<X)&=\int_0^\infty P(Y<X\mid X=x)f_X(x)\,dx\\
&=\int_0^\infty f(x;\alpha_1,\theta_1)\,F(x;\alpha_2,\theta_2)\,dx\\
&=1-\frac{\theta_1\left[2\theta_1\theta_2+(\alpha_1\theta_2+\alpha_2\theta_1+\theta_1)(\theta_1+\theta_2)+\alpha_1(1+\alpha_2)(\theta_1+\theta_2)^2\right]}{(\alpha_1+1)(\alpha_2+1)(\theta_1+\theta_2)^3}
\end{aligned}$$

The expression of stress-strength reliability of QLD reduces to the corresponding expression for stress-strength reliability of Lindley distribution at $\alpha_1=\theta_1$ and $\alpha_2=\theta_2$.

3. Estimation of Parameters

3.1. Method of Moment Estimate (MOME) of Parameters

Since the QLD (1.3) has two parameters to be estimated, the first two moments about origin are required to estimate its parameters. Using the first two moments about origin of QLD (1.3), we have

$$\frac{\mu_2'}{(\mu_1')^2}=K\,(\text{Say})=\frac{2(\alpha+3)(\alpha+1)}{(\alpha+2)^2} \quad (3.1.1)$$

Equation (3.1.1) gives a quadratic equation in α as

$$(2-k)\alpha^2+4(2-k)\alpha+2(3-2k)=0 \quad (3.1.2)$$

Replacing μ_1' and μ_2' by their respective sample moments in (3.1.1), an estimate of k can be obtained and substituting the value of k in equation (3.1.2), moment estimate $\tilde{\alpha}$ of α can be obtained. Substituting the moment estimate of α in the expression for the mean of QLD (1.3), moment estimate $\tilde{\theta}$ of θ can be obtained as

$$\tilde{\theta}=\left(\frac{\alpha+2}{\alpha+1}\right)\frac{1}{\bar{x}} \quad (3.1.3)$$

3.2. Maximum Likelihood Estimate (MLE) of Parameters

Let $(x_1, x_2, x_3, \ldots, x_n)$ be a random sample of size n from QLD (1.3). Let f_x be the observed frequency in the sample corresponding to $X = x\,(x = 1,2,3,\ldots k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function, L of QLD (1.3) is given by

$$L = \left(\frac{\theta}{\alpha+1}\right)^2 \prod_{x=1}^{n} \left(\alpha+\theta x\right)^{f_x} e^{-n\theta\bar{x}} \qquad (3.2.1)$$

The natural log likelihood function is thus obtained as

$$\ln L = n\ln\theta - n\ln(\alpha+1) + \sum_{x=1}^{k} f_x \ln(\alpha+\theta x) - n\theta\bar{x} \quad (3.2.2)$$

where $\bar{x}$ is the sample mean.

The two log likelihood equations are obtained as

$$\frac{\partial \ln L}{\partial \theta} = \frac{n}{\theta} + \sum_{x=1}^{k} \frac{x f_x}{\alpha+\theta x} - n\bar{x} = 0 \qquad (3.2.3)$$

$$\frac{\partial \ln L}{\partial \alpha} = -\frac{n}{\alpha+1} + \sum_{x=1}^{k} \frac{f_x}{\alpha+\theta x} = 0 \qquad (3.2.4)$$

The equations (3.2.3) and (3.2.4) do not seem to be solved directly. However, Fisher's scoring method can be applied to solve these equations iteratively. We have

$$\frac{\partial^2 \ln L}{\partial \theta^2} = -\frac{n}{\theta^2} - \sum_{x=1}^{k} \frac{x^2 f_x}{(\alpha+\theta x)^2} \qquad (3.2.5)$$

$$\frac{\partial^2 \ln L}{\partial \theta \partial \alpha} = -\sum_{x=1}^{k} \frac{x f_x}{(\alpha+\theta x)^2} \qquad (3.2.6)$$

$$\frac{\partial^2 \ln L}{\partial \alpha^2} = -\frac{n}{(\alpha+1)^2} - \sum_{x=1}^{k} \frac{f_x}{(\alpha+\theta x)^2} \qquad (3.2.7)$$

The maximum likelihood estimates $\hat{\theta}$ and $\hat{\alpha}$ of parameters θ and α are the solution of the following equations

$$\begin{bmatrix} \frac{\partial^2 \ln L}{\partial \theta^2} & \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} \\ \frac{\partial^2 \ln L}{\partial \theta \partial \alpha} & \frac{\partial^2 \ln L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta}-\theta_0 \\ \hat{\alpha}-\alpha_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \ln L}{\partial \theta} \\ \frac{\partial \ln L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

where θ_0 and α_0 are initial values of θ and α, preferably method of moment estimates of the parameters. These equations are solved iteratively till sufficiently close estimates of $\hat{\theta}$ and $\hat{\alpha}$ are obtained.

4. Goodness of Fit to Real Lifetime Data-Sets

The quasi Lindley distribution (QLD) has been fitted to a number of lifetime data- sets. In this section, we present the fitting of QLD to five real lifetime data-sets and compare its goodness of fit with exponential and Lindley distributions. The following five lifetime data-sets have been considered for comparing the goodness of fit of QLD with Lindley and exponential distributions.

Data set 1: This data set represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark (1975, P. 105).

1.1, 1.4, 1.3, 1.7, 1.9, 1.8, 1.6, 2.2, 1.7, 2.7, 4.1, 1.8,1.5, 1.2, 1.4 3 ,1.7, 2.3, 1.6 2

Data Set 2: This data set is the strength data of glass of the aircraft window reported by Fuller *et al* (1994):

18.83, 20.8, 21.657, 23.03, 23.23, 24.05, 24.321, 25.5 25.52, 25.8, 26.69, 26.77, 26.78, 27.05, 27.67, 29.9, 31.11, 33.2, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381

Data set 3: This data set represents the waiting times (in minutes) before service of 100 Bank customers and examined and analyzed by Ghitany *et al* (2008) for fitting the Lindley (1958) distribution.

0.8, 0.8, 1.3, 1.5, 1.8, 1.9, 1.9, 2.1, 2.6, 2.7, 2.9, 3.1,3.2, 3.3, 3.5, 3.6, 4.0, 4.1, 4.2, 4.2, 4.3, 4.3, 4.4, 4.4, 4.6, 4.7, 4.7, 4.8, 4.9 ,4.9, 5.0, 5.3, 5.5, 5.7, 5.7, 6.1,,6.2, 6.2, 6.2, 6.3, 6.7 ,6.9 ,7.1, 7.1, 7.1, 7.1, 7.4, 7.6, 7.7, 8.0, 8.2, 8.6, 8.6, 8.6, 8.8, 8.8, 8.9, 8.9, 9.5, 9.6,9.7, 9.8, 10.7, 10.9, 11.0, 11.0, 11.1, 11.2, 11.2, 11.5, 11.9, 12.4,12.5, 12.9, 13.0, 13.1, 13.3 , 13.6 , 13.7, 13.9, 14.1, 15.4, 15.4, 17.3,17.3, 18.1, 18.2, 18.4, 18.9, 19.0, 19.9, 20.6, 21.3, 21.4, 21.9, 23.0,27.0, 31.6, 33.1, 38.5

Data Set 4: The data set represents the strength of 1.5cm glass fibers measured at the National Physical Laboratory, England. Unfortunately, the units of measurements are not given in the paper, and they are taken from Smith and Naylor (1987)

0.55, 0.93, 1.25, 1.36, 1.49, 1.52, 1.58, 1.61, 1.64, 1.68, 1.73, 1.81, 2.00, 0.74, 1.04, 1.27, 1.39, 1.49, 1.53, 1.59, 1.61, 1.66, 1.68, 1.76,,1.82, 2.01, 0.77, 1.11, 1.28, 1.42, 1.50, 1.54, 1.60, 1.62, 1.66, 1.69,1.76, 1.84, 2.24, 0.81, 1.13, 1.29, 1.48,1.50, 1.55, 1.61, 1.62, 1.66,1.70, 1.77, 1.84, 0.84, 1.24, 1.30, 1.48, 1.51, 1.55, 1.61, 1.63, 1.67

1.70 1.78 1.89

Data Set 5: The data set is from Lawless (1982, p-228). The data given arose in tests on endurance of deep groove ball bearings. The data are the number of million revolutions before failure for each of the 23 ball bearings in the life tests and they are:

17.88 28.92 33.00 41.52 42.12 45.60 48.80 51.84 51.96 54.12 55.56 67.80

68.44 68.64 68.88 84.12 93.12 98.64 105.12 105.84 127.92 128.04 173.40

In order to compare QLD, exponential and Lindley distributions, $-2\ln L$, AIC (Akaike Information Criterion), AICC (Akaike Information Criterion Corrected), BIC (Bayesian Information Criterion),K-S Statistics (Kolmogorov-Smirnov Statistics) for five real lifetime data - sets have been computed and presented in table 1. The formulae for computing AIC, AICC, BIC, and K-S Statistics are as follows:

$$\text{AIC} = -2\ln L + 2k,\ \ \text{AICC} = \text{AIC} + \frac{2k(k+1)}{(n-k-1)},$$

$$\text{BIC} = -2\ln L + k\ln n \ \text{ and } \ D = \operatorname*{Sup}_x \left| F_n(x) - F_0(x) \right|,$$

where k = the number of parameters, n = the sample size and $F_n(x)$ is the empirical distribution function. The best distribution is the distribution corresponding to the lower values of $-2\ln L$, AIC, AICC, BIC, and K-S statistics.

Table 1. *MLE's, − 2ln L, AIC, AICC, BIC, K-S Statistics of the fitted distributions of data sets 1-5.*

	Model	Estimate of Parameters		− 2ln L	AIC	AICC	BIC	K-S Statistics
		$\hat{\theta}$	$\hat{\alpha}$					
Data 1	Lindley	0.816118		60.50	62.50	62.72	63.49	0.34
	Exponential	0.526316		65.67	67.67	67.90	68.67	0.39
	QLD	1.545110	− 0.483393	40.71	44.71	45.41	46.70	0.20
Data 2	Lindley	0.062988		253.99	255.99	256.13	257.42	0.33
	Exponential	0.032455		274.53	276.53	276.67	277.96	0.43
	QLD	0.103985	− 0.546267	231.82	235.82	236.25	238.69	0.30
Data 3	Lindley	0.186571		638.07	640.07	640.12	642.68	0.06
	Exponential	0.101245		658.04	660.04	660.08	662.65	0.16
	QLD	0.196209	0.066138	635.75	639.75	639.87	639.75	0.05
Data 4	Lindley	0.996116		162.56	164.56	164.62	166.70	0.37
	Exponential	0.663647		177.66	179.66	179.73	181.80	0.40
	QLD	2.146473	-0.552445	91.56	95.56	95.63	97.36	0.36
Data 5	Lindley	0.027321		231.47	233.47	233.66	234.61	0.15
	Exponential	0.013845		242.87	244.87	245.06	246.01	0.26
	QLD	0.035434	-0.358716	223.52	227.52	228.12	229.79	0.10

It is obvious from the fitting of QLD, Lindley and exponential distributions in the table 1 that QLD gives much closer fit than Lindley and exponential distributions in all data- sets, and therefore QLD can be preferred over Lindley and exponential distributions for modeling lifetime data-sets from biomedical science, engineering and other fields of knowledge.

5. Concluding Remarks

In the this paper some important mathematical and statistical properties including moment generating function, mean deviations about mean and median, order statistics, Bonferroni and Lorenz curves, Renyi entropy measure and stress strength reliability of quasi Lindley distribution (QLD) of Shanker and Mishra (2013 a) have been derived and discussed. The distribution has been fitted to some real lifetime data-sets to test its goodness of fit over exponential and Lindley distributions. It is clear from the fitting of QLD that it gives better fitting than exponential and Lindley distributions and hence QLD can be recommended over exponential and Lindley distributions for modeling real lifetime data-sets from biomedical science and engineering.

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