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Supply Chain Network Design Optimization Model for Multi-period Multi-product Under Uncertainty

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Abstract: This research is a development of a stochastic mixed integer linear programming (SMILP) model considering stochastic customer demand, to tackle the multi-product SCND problems. It also considers multi-period, multi-echelons, products inventories, considering locations capacities and associated cost elements. The model represents both location and allocation decisions of the supply chain which maximize the total expected profit. The effect of demand mean on the total expected profit and the effect of the number of scenarios on the CPU time are studied. The results have shown the effect of customers' demands for each product in each period on the quantities of material delivered from each supplier to each factory, the quantities of products delivered from each factory and factory store to each distributor, the inventory of each product in each factory and distributor, the quantities of each type of product delivered from each distributor to each customer in each period. The model has been verified through a detailed example.

Keywords: Supply Chain Network Design (SCND), Stochastic Mixed Integer Linear Programming (SMILP), Location, Allocation, Modeling, Multi-products, Multi-echelon and Multi-periods

1. Introduction

Supply chain (SC) generally include two main inter-related processes of (1) production planning and inventory control that deals with production, storage, and the relation between them, and (2) logistics and distribution that determine how to transport products to customers (Ballou, R. H., [3]).

Haq, A. N. et al., proposed an integrated production-inventory-distribution model incorporating many realistic conditions to determine optimal production and distribution as well as inventory level, where a mixed-integer-linear programming (MILP) was developed to minimize the total cost of the system [5].

Maqsood, I., et al. proposed an interval-parameter fuzzy two-stage stochastic programming (IFTSP) method for the planning of water-resources-management systems under uncertainty [6].

Santoso, T., et al. proposed a stochastic programming model and solution algorithm for solving supply chain network design problems of a realistic scale. their solution methodology integrates a recently proposed sampling strategy, the sample average approximation (SAA) scheme, with an accelerated benders decomposition algorithm to quickly compute high-quality solutions to large-scale stochastic supply chain design problems with a huge (potentially infinite) number of scenarios [8].

El-Sayed, M., et al. developed a multi-period multi-echelon forward–reverse logistics network design under risk model maximizing the total expected profit. The proposed network structure considers first customer zones in which the demands are stochastic and second customer zones in which the demand is assumed to be deterministic [4].

Wang, F., et al. studied a supply chain network design problem with environmental concerns. They proposed a multi-objective optimization model that captures the trade-off between the total cost and the environment influence [10].

Pishvaee, M. S., & Razmi, J. proposed an interactive fuzzy solution approach to solve a multi-objective fuzzy mathematical programming model of an environmental

SCND with objectives of minimizing total cost and minimization of the total environmental impact [7].

Badri, H. et al. developed a new multi-commodity SCND model with different time resolutions for strategic and tactical decisions. In addition, a mathematical technique based on the Lagrangian relaxation method was developed to solve the problem [2].

Wu, J., & Li, J. studied dynamic factory location and supply chain planning through minimizing the costs of factory location, path selection and transportation of coal under demand uncertainty [11].

Xia R. & Matsukawa H. investigated a supplier-retailer supply chain that experiences disruptions in supplier during the planning horizon. While determining what suppliers, parts, processes, and transportation modes to select at each stage in the supply chain, options disruption must be considered [13].

Adabi, F., & Omrani, H. formulated a mixed integer programming model considering two objective functions where the first one maximizes the efficiency of the supply chain and the second one minimizes the cost of facility layout as well as the production of different products [1].

Serdar E. T. & Al-Ashhab M. S. mathematically modeled an SCN in a mixed integer linear programming (MILP) form considering deterministic demand maximizing the total profit [9].

This research is a development of a stochastic mixed integer linear programming (SMILP) model considering stochastic customer demand, to tackle the multi-product SCND problems. It also considers multi-period, multi-echelons, products inventories, considering locations capacities and associated cost elements. The model represents both location and allocation decisions of the supply chain which maximize the total expected profit. The nature of the logistic decisions encompasses procurement of raw materials from suppliers, production of finished product at factories, distribution of finished product to customers via distributors, and the storage of end product at factories and distributors. The proposed scheme of supply chain consists of three echelons (three suppliers, three factories, and three distributors) to serve four customers as shown in Figure 1.

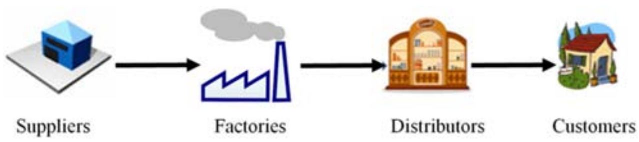

Figure 1. *Supply chain network scheme.*

2. Model Assumptions and Limitations

The problem is formulated using multi-stage stochastic mixed integer linear programming (SMILP) and it is solved using Xpress-SP software which uses Mosel language [12].

The following assumptions are considered:

a. Customers' demands are stochastic and known for all product in all periods.
b. The model is multi-product, where actions and flow of materials take place for multi-product.
c. Product weight affected material, transportation, holding
d. Costs parameters (fixed costs, material costs, manufacturing costs, non-utilized capacity costs, shortage costs, transportation costs, and inventory holding costs) are known for each location, each product at each period.
e. The shortage cost depends on the shortage quantity for each product and time.
f. The manufacturing cost depends on the manufacturing hours for each product and manufacturing cost per hours

The stochastic demand of the customers is normally distributed with mean μ and standard deviation σ, it is discretized into N points.

3. Model Formulation

The model involves the following sets, parameters, and variables:

Sets:

S, F, D, and D: potential number of suppliers, factories, distributors, and first customers,

T: number of periods, indexed by t.

P: number of products, indexed by p.

Parameters:

F_i: fixed cost of opening location *i*,

$DEMAND_{cpt}$: demand of customer *c* from product *p* in period *t*,

μ_{ct}: demand mean of customer *c* in period *t*,

σ_{ct}: demand standard deviation of customer *c* in period *t*,

P_{pct}: unit price of product *p* at customer *c* in period *t*,

W_p: weight of product *p*,

MH_p: manufacturing hours for product *p*,

D_{sf}: distance between supplier *s* and factory *f*,

D_{fd}: distance between factory *f* and distributor *d*,

D_{dc}: distance between distributor *d* and customer *c*,

CAP_{st}: capacity of supplier *s* in period *t* (kg),

$CAPM_{ft}$: capacity of factory *f* Raw Material Store in period *t*,

$CAPH_{ft}$: capacity in manufacturing hours of factory *f* in period *t*,

$CAPFS_{ft}$: storing capacity of factory *f* in period *t*,

CAP_{dt}: capacity of distributor *d* in period *t* (kg),

MatCost: material cost per unit supplied by supplier *s* in period *t*,

MC_{ft}: manufacturing cost per hour for factory *f* in period *t*,

MH_p: manufacturing hours for product *p*,

NUCCf: non-utilized manufacturing capacity cost per hour of factory *f*,

$SCPU_p$: shortage cost per unit per period of product *p*,

HF_p: holding cost per unit per period at factory *f* store (kg) of product *p*,

HD_p: holding cost per unit per period at distributor *d* store (kg) of product *p*,

B_s: batch size for supplier *s*,

$B_{fp\&}$ B_{dp}: batch size for factory *f* for product *p* and distributor *d* for product *p*,

TCperkm: transportation cost per unit per kilometer,

M: big number,

S: small number.

Decision Variables:

L_i: binary variable equal to 1 if a location *i* is opened and equal to 0 otherwise,

Li_{ij}: binary variable equal to 1 if a transportation link is activated between location *i* location *j*,

Q_{ijpt}: flow of batches from location i to location j of product p in period t,

I_{fpt}: flow of batches from factory f to its store of product p in period t,

I_{fdpt}: flow of batches from the store of factory f to distributor d of product p in period,

R_{fpt}: residual inventory of the period t at the store of factory f for product p,

R_{dpt}: residual inventory of the period *t* at distributor *d* for product *p*.

3.1. Objective Function

The objective of the model is to maximize the total expected profit of the supply chain network.

Total expected profit = Total expected income – Total expected cost

3.1.1. Total Expected Income

$$\text{Total income} = \sum_{d\in D}\sum_{c\in C}\sum_{p\in P}\sum_{t\in T} Q_{dcpt}\ B_{dp}\ P_{pct} \tag{1}$$

3.1.2. Total Expected Cost

Total expected cost = fixed costs + material costs + manufacturing costs + non-utilized capacity costs + shortage costs + transportation costs + inventory holding costs.

i. Fixed Costs

$$\sum_{s\in S} F_s L_s + \sum_{f\in F} F_f L_f + \sum_{d\in D} F_d L_d \tag{2}$$

ii. Material Cost

$$\sum_{s\in S}\sum_{f\in F}\sum_{t\in T} Q_{sft}\ B_s\ \text{MatCost}_{st} \tag{3}$$

iii. Manufacturing Costs

$$\sum_{f\in F}\sum_{d\in D}\sum_{p\in P}\sum_{t\in T} Q_{fdpt}\ B_{fp}\ MH_p\ MC_{ft} + \sum_{f\in F}\sum_{d\in D}\sum_{p\in P}\sum_{t\in T} I_{fpt}\ B_{fp}\ MH_p\ MC_{ft} \tag{4}$$

iv. Non-Utilized Capacity Cost (for factories)

$$\sum_{f\in F}\Big(\sum_{p\in P}\sum_{t\in T}((CAPH_{ft})\ L_f - \sum_{d\in D}(Q_{fdpt}\ B_{fp} MH_p) - \sum_{d\in D}(I_{ffpt}\ B_{fp} MH_p))NUCC_f\Big) \tag{5}$$

v. Shortage Cost (for distributor)

$$\sum_{p\in P}\Big(\sum_{c\in C}\Big(\sum_{t\in T}\Big(\sum_{1}^{t} DEMAND_{cpt} - \sum_{1}^{t}\sum_{d\in D} Q_{dcpt}\ B_{dp}\Big)\Big)\Big)\ SCPU_p \tag{6}$$

vi. Transportation Costs

$$\sum_{t\in T}\sum_{s\in S}\sum_{f\in F} Q_{sft}\ B_s\ T_s\ DS_{sf} + \sum_{p\in P}\Big(\sum_{t\in T}\sum_{f\in F}\sum_{d\in D} Q_{fdpt}\ B_{fp}\ W_p T_f\ D_{fd} + \sum_{t\in T}\sum_{f\in F}\sum_{d\in D} I_{fdpt}\ B_{fp} W_p T_f\ D_{fd}(1+s) + \sum_{d\in D}\sum_{c\in C}\sum_{t\in T} Q_{dcpt}\ B_{dp}\ W_p T_d\ D_{dc}\Big) \tag{7}$$

vii. Inventory Holding Costs

$$\sum_{p\in P}\Big(\sum_{f\in F}\sum_{t\in T} R_{fpt}\ W_p\ HF_f + \sum_{d\in D}\sum_{t\in T} R_{dpt}\ W_p\ HD_d\Big) \tag{8}$$

3.2. Constraints

3.2.1. Balance Constraints

$$\sum_{s\in S} Q_{sft} B_s = \sum_{d\in D} \sum_{p\in P} Q_{fdpt} B_{fp} W_p + I_{fpt} B_{fp} W_p, \forall t\in T, \forall f\in F \quad (9)$$

$$I_{fpt} B_{fp} + R_{fp(t-1)} B_{fp} = R_{fpt} B_{fp} + \sum_{d\in D} I_{fdpt} B_{fp}, \forall t\in T, \forall f\in F, \forall p\in P \quad (10)$$

$$\sum_{p\in P} \sum_{f\in F} (Q_{fdpt} + I_{fdpt}) B_{fp} + R_{dp(t-1)} B_{dp} = R_{dpt} B_{dp} + \sum_{p\in P} \sum_{c\in C} Q_{dcpt} B_{dp}, \forall t\in 2\rightarrow T, \forall d\in D \quad (11)$$

$$\sum_{d\in D} Q_{dcpt} B_{dp} \leq \text{DEMAND}_{cpt} + \sum_{1\rightarrow t} \text{DEMAND}_{cp(t-1)} - \sum_{d\in D} Q_{dcp(t-1)} B_{dp}, \forall t\in T, \forall c\in C, \forall p\in P \quad (12)$$

3.2.2. Capacity Constraints

$$\sum_{f\in F} \mathrm{Q_{sft}}\ \mathrm{B_s} \leq \mathrm{CAP_{st}}\ \mathrm{L_s}, \forall \mathrm{t}\in \mathrm{T}, \forall \mathrm{s}\in \mathrm{S} \quad (13)$$

$$\sum_{s\in S} \mathrm{Q_{sft}}\ \mathrm{B_s} \leq \mathrm{CAPM_{ft}}\ \mathrm{L_f}, \forall \mathrm{t}\in \mathrm{T}, \forall \mathrm{f}\in \mathrm{F} \quad (14)$$

$$(\sum_{\mathrm{d}\in \mathrm{D}} \mathrm{Q_{fdpt}}\ \mathrm{B_{fp}} + \sum_{\mathrm{d}\in \mathrm{D}} \mathrm{I_{ffpt}}\ \mathrm{B_{fp}})\ \mathrm{MH_p} \leq \mathrm{CAPH_{ft}}\ \mathrm{L_f}, \forall \mathrm{t}\in \mathrm{T}, \forall \mathrm{f}\in \mathrm{F}, \forall \mathrm{p}\in \mathrm{P} \quad (15)$$

$$\sum_{p\in P} \mathrm{R_{fpt}} \mathrm{B_{fp}} \mathrm{W_p} \leq \mathrm{CAPFS_{ft}}\ \mathrm{L_f}, \forall \mathrm{t}\in \mathrm{T}, \forall \mathrm{f}\in \mathrm{F} \quad (16)$$

$$\sum_{\mathrm{f}\in \mathrm{F}} (\mathrm{Q_{fdpt}} + \mathrm{I_{fdpt}})\ \mathrm{B_{fp}} \mathrm{W_p} + \sum_{t\in \mathrm{T}} \mathrm{R_{dpt\text{-}1}} \mathrm{B_{fp}} \mathrm{W_p} \leq \mathrm{CAP_{dt}}\ \mathrm{L_d}, \forall \mathrm{t}\in \mathrm{T}, \forall \mathrm{d}\in \mathrm{D}, \forall \mathrm{p}\in \mathrm{P} \quad (17)$$

4. Effect of Demand Mean and Number of Scenarios

The relationship between demand mean and total expected profit has been studied at different values of scenarios of 1, 8, 27 and 64. Figures 2-6 show that the general shape of the relation between demand means and total expected profit is almost the same for a different number of scenarios. In general, the increase in demand mean increases the total expected profit as shown in Figure 6. The total expected profit is linearly proportional to the total demand. At transient ranges, it decreases slightly due to the shortage cost as it is not profitable to open an extra location. At certain demand mean, it is profitable to open another location to fulfill the extra demand. The same behavior continues with the increase in demand mean until the total demand exceeds the maximum permissible capacity of the network and it is not possible to open extra locations.

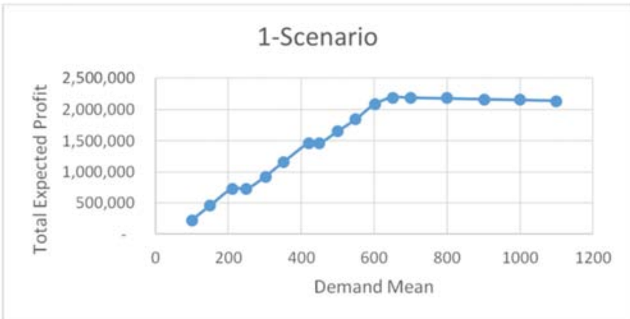

Figure 2. *The relationship between demand mean and the total expected profit for 1 scenario.*

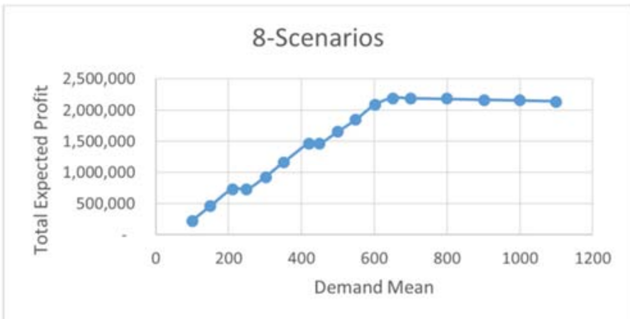

Figure 3. *The Relationship between demand mean and the total expected profit for 8 scenarios.*

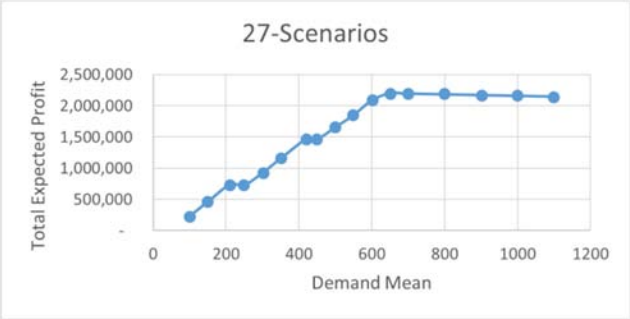

Figure 4. *The Relationship between demand mean and the total expected profit for 27 scenarios.*

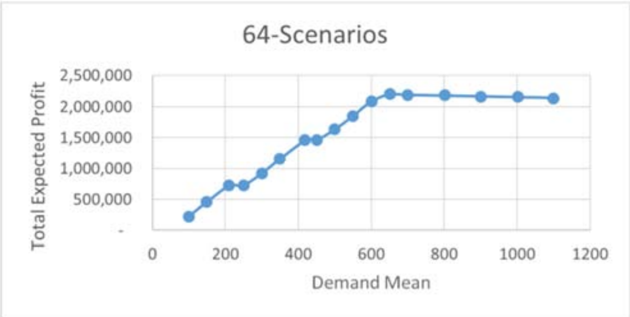

Figure 5. *The Relationship between demand mean and the total expected profit for 64 scenarios.*

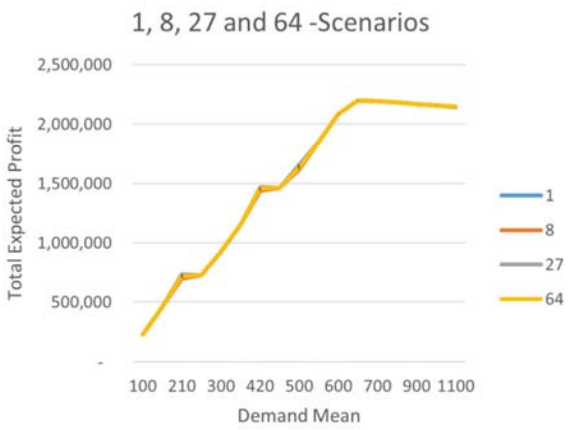

Figure 6. *The Relationship between demand mean and the total expected profit for all scenarios.*

The effect of changing number of scenarios on the total expected profit has been studied for different values of demand means. The changing percentage is calculated by dividing the resulted total expected profit of the given number of scenarios by the resulted total expected profit of the 1-scenario case. Figure 7 depicts the effect of the number of scenarios on the total expected profit. As it is noticed in Figure 8 the most changes happened for demand means of 200, 400 and 500 respectively which are located in the transient ranges since the increase of the total expected profit in the high demand scenarios is not equal to the decrease of it in the low demand scenarios.

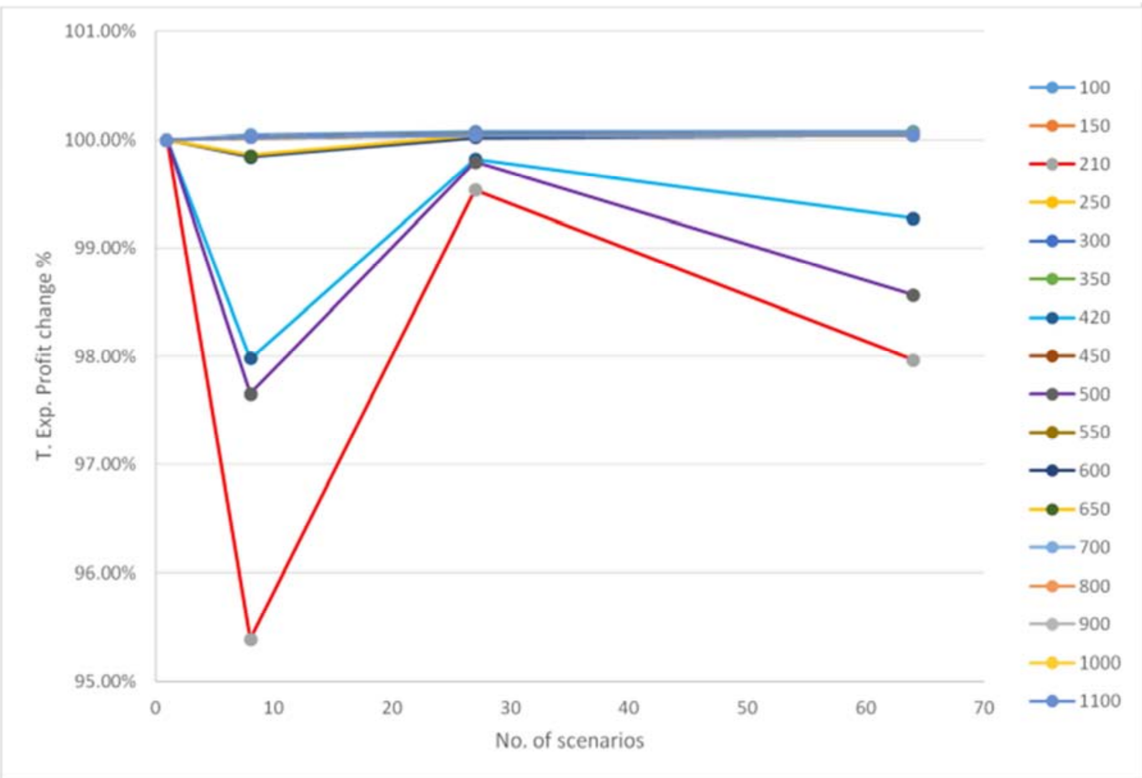

Figure 7. *Effect of no. of scenarios on the total expected profit.*

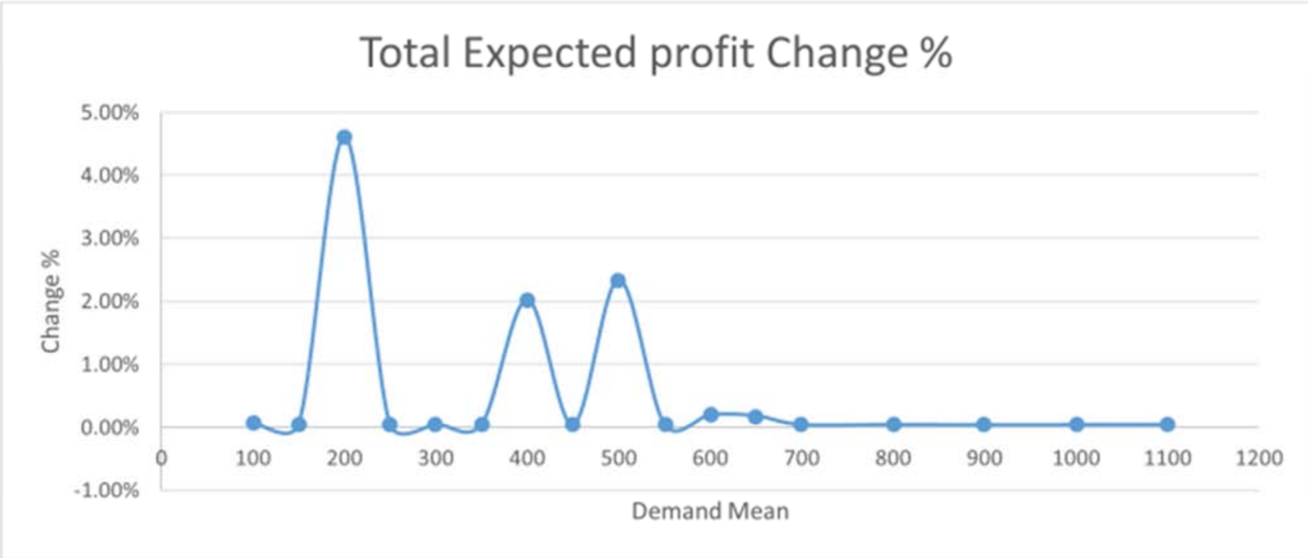

Figure 8. *Maximum change percentage in total expected profit vs. demand mean.*

5. Computational Results

In this section, we describe numerical experiment using the proposed model for solving a supply chain network design problem.

5.1. Model Inputs

The model has been verified through the following example where the input parameters are considered as shown in Table 1.

Table 1. *Verification model parameters.*

Parameter	Value	Parameter	Value
Number of suppliers	3	Material Cost per unit weight	10
Number of factories	3	Manufacturing Cost per hour	10
Number of Distributors	3	Manufacturing hours for product (1)	1
Number of Customers	4	Manufacturing hours for product (2)	2
Number of products	3	Manufacturing hours for product (3)	3
Fixed costs for supplier & distributor	20000	Transportation cost per kilometer per unit	0.001
Fixed costs for factory	50000	Factory holding cost	2
Weight of Product (1) Kg	1	Distributor holding cost	2
Weight of Product (2) Kg	2	Capacity of each supplier in each period	6000
Weight of Product (3) Kg	3	Supplier batch size	10
Price of Product (1)	100	Factory Batch size for product p	5
Price of Product (2)	150	Distributor Batch size for product p	1
Price of Product (3)	200	Capacity of each Factory Raw Material Store in each period	5000
Customers' Demands standard deviation	10	Factory capacity in hours	5000
Customers' Demands mean of all products for all customers in all periods	100-600	Capacity of each Factory Store in each period	2000
Non-utilized cost per hour per period	10	Capacity of each Distributor Store in each period	6000
Shortage cost of product (1) per period	5	Shortage cost of product (3) per period	15
Shortage cost of product (2) per period	10		

In this case, demand means and standard deviations for all customers in all periods for all products are assumed to be the same to simplify discussion, demand mean is assumed to be 200 units and demand standard deviation is assumed to be 10 units

5.2. Model Outputs

In this section, the model outputs are presented. One of the outputs is the probabilities of scenarios generated by the model and they are as shown in Table 2.

Table 2. *Scenarios probabilities.*

Scenario	Probability	Scenario	Probability	Scenario	Probability	Scenario	Probability	Scenario	Probability
1	4.0094E-12	26	2.89954E-09	51	1.94308E-08	76	2.89954E-09	101	4.0102E-12
2	2.89954E-09	27	2.0969E-06	52	1.40521E-05	77	2.0969E-06	102	2.90012E-09
3	1.94308E-08	28	1.40521E-05	53	9.41675E-05	78	1.40521E-05	103	1.94347E-08
4	2.89954E-09	29	2.0969E-06	54	1.40521E-05	79	2.0969E-06	104	2.90012E-09
5	4.0102E-12	30	2.90012E-09	55	1.94347E-08	80	2.90012E-09	105	4.01101E-12
6	2.89954E-09	31	2.0969E-06	56	1.40521E-05	81	2.0969E-06	106	2.90012E-09
7	2.0969E-06	32	0.00151645	57	0.0101622	82	0.00151645	107	2.09733E-06
8	1.40521E-05	33	0.0101622	58	0.0681006	83	0.0101622	108	1.40549E-05
9	2.0969E-06	34	0.00151645	59	0.0101622	84	0.00151645	109	2.09733E-06
10	2.90012E-09	35	2.09733E-06	60	1.40549E-05	85	2.09733E-06	110	2.90071E-09
11	1.94308E-08	36	1.40521E-05	61	9.41675E-05	86	1.40521E-05	111	1.94347E-08
12	1.40521E-05	37	0.0101622	62	0.0681006	87	0.0101622	112	1.40549E-05
13	9.41675E-05	38	0.0681006	63	0.456365	88	0.0681006	113	9.41865E-05
14	1.40521E-05	39	0.0101622	64	0.0681006	89	0.0101622	114	1.40549E-05
15	1.94347E-08	40	1.40549E-05	65	9.41865E-05	90	1.40549E-05	115	1.94386E-08
16	2.89954E-09	41	2.0969E-06	66	1.40521E-05	91	2.0969E-06	116	2.90012E-09
17	2.0969E-06	42	0.00151645	67	0.0101622	92	0.00151645	117	2.09733E-06
18	1.40521E-05	43	0.0101622	68	0.0681006	93	0.0101622	118	1.40549E-05
19	2.0969E-06	44	0.00151645	69	0.0101622	94	0.00151645	119	2.09733E-06
20	2.90012E-09	45	2.09733E-06	70	1.40549E-05	95	2.09733E-06	120	2.90071E-09
21	4.0102E-12	46	2.90012E-09	71	1.94347E-08	96	2.90012E-09	121	4.01101E-12
22	2.90012E-09	47	2.09733E-06	72	1.40549E-05	97	2.09733E-06	122	2.90071E-09
23	1.94347E-08	48	1.40549E-05	73	9.41865E-05	98	1.40549E-05	123	1.94386E-08
24	2.90012E-09	49	2.09733E-06	74	1.40549E-05	99	2.09733E-06	124	2.90071E-09
25	4.01101E-12	50	2.90071E-09	75	1.94386E-08	100	2.90071E-09	125	4.01182E-12

The model also generates the demand for each scenario according to the given distribution which is assumed to be normal in this research. Figure 9 depicts the generated scenario tree.

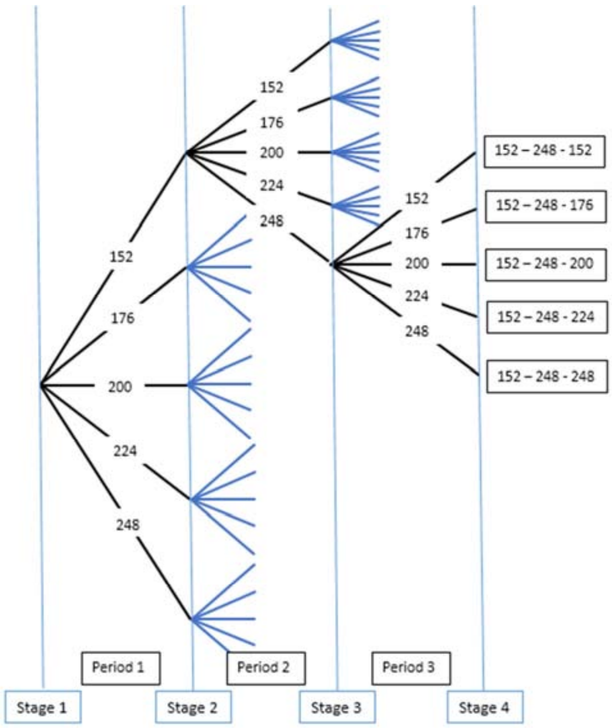

Figure 9. *Scenario tree and demands.*

The resulted optimal supply chain network obtained from the model is shown in Figure 10 where it is decided to open the second supplier, factory, and distributor to serve the four customers.

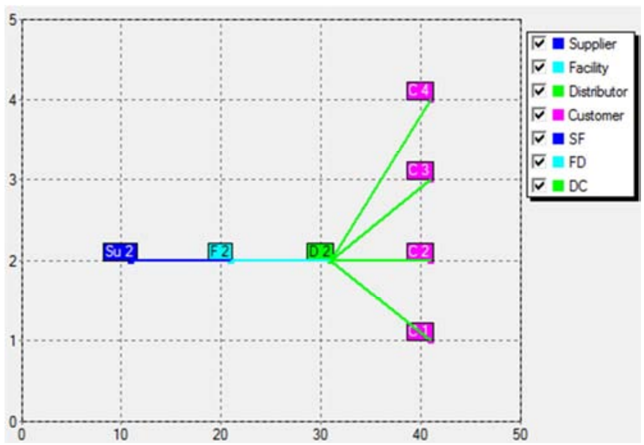

Figure 10. *The Resulted Optimal Network Design.*

The resulted total expected profit is 691870. Table 3 presents the number of material batches transferred from the supplier to the factory in all scenarios. Table 4 presents the number of batches transferred from the factory to the distributor from product 1 in all scenarios while the numbers of batches transferred from product 2 in all scenarios are presented Table 5, and Table 6 presents the number of batches transferred from product 3 in all scenarios.

Table 3. *Number of material batches transferred from supplier to factory in all scenarios.*

Scen.	Qsft			Scen.	Qsft			Scen.	Qsft			Scen.	Qsft			Scen.	Qsft		
	222	**223**	**224**		**222**	**223**	**224**		**222**	**223**	**224**		**222**	**223**	**224**		**222**	**223**	**224**
1	380	347	3	26	440	383	204	51	497	384	328	76	500	438	98	101	500	459	45
2	380	347	199	27	440	383	386	52	497	384	385	77	500	438	385	102	500	459	149
3	380	347	482	28	440	383	443	53	497	384	442	78	500	438	442	103	500	459	480
4	380	347	194	29	440	383	500	54	497	384	499	79	500	438	500	104	500	459	336
5	380	347	0	30	440	383	322	55	497	384	500	80	500	438	191	105	500	459	0
6	380	443	221	31	440	440	329	56	497	443	326	81	500	496	327	106	500	500	138
7	380	443	386	32	440	440	386	57	497	443	383	82	500	496	384	107	500	500	440
8	380	443	443	33	440	440	443	58	497	443	440	83	500	496	443	108	500	500	497
9	380	443	500	34	440	440	500	59	497	443	500	84	500	496	500	109	500	500	500
10	380	443	370	35	440	440	500	60	497	443	500	85	500	496	499	110	500	500	164
11	380	500	327	36	440	500	326	61	497	500	326	86	500	499	381	111	500	500	440
12	380	500	386	37	440	500	383	62	497	500	383	87	500	499	441	112	500	500	497
13	380	500	443	38	440	500	440	63	497	500	443	88	500	499	498	113	500	500	500
14	380	500	500	39	440	500	500	64	497	500	500	89	500	499	500	114	500	500	500
15	380	500	500	40	440	500	500	65	497	500	500	90	500	499	500	115	500	500	500
16	380	500	111	41	440	500	383	66	497	500	383	91	500	500	440	116	500	500	198
17	380	500	443	42	440	500	440	67	497	500	443	92	500	500	497	117	500	500	500
18	380	500	500	43	440	500	500	68	497	500	500	93	500	500	500	118	500	500	500
19	380	500	500	44	440	500	500	69	497	500	500	94	500	500	500	119	500	500	500
20	380	500	372	45	440	500	500	70	497	500	500	95	500	500	500	120	500	500	320
21	380	500	0	46	440	500	142	71	497	500	443	96	500	500	119	121	500	500	31
22	380	500	253	47	440	500	500	72	497	500	500	97	500	500	500	122	500	500	407
23	380	500	500	48	440	500	500	73	497	500	500	98	500	500	500	123	500	500	500
24	380	500	254	49	440	500	500	74	497	500	500	99	500	500	500	124	500	500	406
25	380	500	0	50	440	500	336	75	497	500	500	100	500	500	323	125	500	500	0

Table 4. *Number of batches of product 1 transferred from factory to distributor in all scenarios.*

Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..		
	..12	**..13**	**..14**		**..12**	**..13**	**..14**		**..12**	**..13**	**..14**		**..12**	**..13**	**..14**		**..12**	**..13**	**..14**
1	123	120	0	26	141	122	90	51	178	104	53	76	179	121	29	101	196	122	0
2	123	120	35	27	141	122	138	52	178	104	72	77	179	121	141	102	196	122	36
3	123	120	160	28	141	122	157	53	178	104	91	78	179	121	160	103	196	122	160
4	123	120	44	29	141	122	176	54	178	104	110	79	179	121	178	104	196	122	45
5	123	120	0	30	141	122	76	55	178	104	123	80	179	121	50	105	196	122	0
6	123	140	2	31	141	141	119	56	178	123	121	81	179	141	121	106	196	142	30
7	123	140	112	32	141	141	138	57	178	123	140	82	179	141	140	107	196	142	142
8	123	140	131	33	141	141	157	58	178	123	159	83	179	141	160	108	196	142	161
9	123	140	150	34	141	141	176	59	178	123	179	84	179	141	179	109	196	142	180
10	123	140	22	35	141	141	195	60	178	123	198	85	179	141	197	110	196	142	49
11	123	159	120	36	141	160	120	61	178	143	120	86	179	159	122	111	196	160	124
12	123	159	140	37	141	160	139	62	178	143	139	87	179	159	142	112	196	160	143
13	123	159	159	38	141	160	158	63	178	143	159	88	179	159	161	113	196	160	162
14	123	159	178	39	141	160	178	64	178	143	178	89	179	159	180	114	196	160	181
15	123	159	197	40	141	160	197	65	178	143	197	90	179	159	197	115	196	160	200
16	123	176	31	41	141	178	121	66	178	160	122	91	179	178	123	116	196	180	30
17	123	176	142	42	141	178	140	67	178	160	142	92	179	178	142	117	196	180	142
18	123	176	161	43	141	178	160	68	178	160	161	93	179	178	161	118	196	180	161
19	123	176	181	44	141	178	179	69	178	160	180	94	179	178	180	119	196	180	180
20	123	176	48	45	141	178	198	70	178	160	199	95	179	178	199	120	196	180	50
21	123	148	0	46	141	145	74	71	178	179	123	96	179	198	31	121	196	199	0
22	123	148	35	47	141	145	193	72	178	179	142	97	179	198	141	122	196	199	34
23	123	148	208	48	141	145	212	73	178	179	161	98	179	198	160	123	196	199	160
24	123	148	0	49	141	145	231	74	178	179	180	99	179	198	179	124	196	199	0
25	123	148	0	50	141	145	94	75	178	179	199	100	179	198	47	125	196	199	0

Table 5. *Number of batches of product 2 transferred from factory to distributor.*

Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..		
	..22	..23	..24		..22	..23	..24		..22	..23	..24		..22	..23	..24		..22	..23	..24
1	122	110	0	26	141	122	69	51	162	121	120	76	178	122	1	101	198	122	0
2	122	110	0	27	141	122	107	52	162	121	139	77	178	122	112	102	198	122	29
3	122	110	171	28	141	122	126	53	162	121	158	78	178	122	131	103	198	122	160
4	122	110	49	29	141	122	145	54	162	121	177	79	178	122	150	104	198	122	45
5	122	110	0	30	141	122	65	55	162	121	197	80	178	122	28	105	198	122	0
6	122	141	46	31	141	141	106	56	162	140	120	81	178	142	121	106	198	141	0
7	122	141	126	32	141	141	125	57	162	140	139	82	178	142	140	107	198	141	141
8	122	141	145	33	141	141	144	58	162	140	158	83	178	142	159	108	198	141	160
9	122	141	164	34	141	141	163	59	162	140	178	84	178	142	178	109	198	141	179
10	122	141	98	35	141	141	182	60	162	140	197	85	178	142	198	110	198	141	135
11	122	161	120	36	141	162	119	61	162	160	119	86	178	160	122	111	198	159	123
12	122	161	139	37	141	162	138	62	162	160	138	87	178	160	142	112	198	159	142
13	122	161	158	38	141	162	157	63	162	160	158	88	178	160	161	113	198	159	161
14	122	161	177	39	141	162	177	64	162	160	177	89	178	160	179	114	198	159	180
15	122	161	196	40	141	162	196	65	162	160	196	90	178	160	199	115	198	159	199
16	122	175	34	41	141	177	123	66	162	174	124	91	178	180	122	116	198	179	30
17	122	175	144	42	141	177	142	67	162	174	144	92	178	180	141	117	198	179	141
18	122	175	163	43	141	177	162	68	162	174	163	93	178	180	160	118	198	179	160
19	122	175	183	44	141	177	181	69	162	174	182	94	178	180	179	119	198	179	179
20	122	175	126	45	141	177	200	70	162	174	201	95	178	180	198	120	198	179	97
21	122	171	0	46	141	198	30	71	162	196	122	96	178	197	30	121	198	156	31
22	122	171	129	47	141	198	141	72	162	196	141	97	178	197	143	122	198	156	78
23	122	171	186	48	141	198	160	73	162	196	160	98	178	197	162	123	198	156	201
24	122	171	95	49	141	198	179	74	162	196	179	99	178	197	181	124	198	156	127
25	122	171	0	50	141	198	49	75	162	196	198	100	178	197	52	125	198	156	0

Table 6. *Number of batches of product 3 transferred from factory to distributor.*

Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..			Scen.	Qfdpt22..		
	..32	..33	..34		..32	..33	..34		..32	..33	..34		..32	..33	..34		..32	..33	..34
1	121	118	2	26	141	111	60	51	164	118	121	76	155	150	55	101	136	184	30
2	121	118	121	27	141	111	140	52	164	118	140	77	155	150	135	102	136	184	68
3	121	118	154	28	141	111	159	53	164	118	159	78	155	150	154	103	136	184	160
4	121	118	82	29	141	111	178	54	164	118	178	79	155	150	174	104	136	184	179
5	121	118	0	30	141	111	146	55	164	118	161	80	155	150	92	105	136	184	0
6	121	136	116	31	141	141	109	56	164	143	97	81	155	188	97	106	136	192	82
7	121	136	136	32	141	141	128	57	164	143	116	82	155	188	116	107	136	192	152
8	121	136	155	33	141	141	147	58	164	143	135	83	155	188	136	108	136	192	171
9	121	136	174	34	141	141	166	59	164	143	155	84	155	188	155	109	136	192	154
10	121	136	174	35	141	141	147	60	164	143	136	85	155	188	135	110	136	192	3
11	121	173	98	36	141	172	98	61	164	179	98	86	155	173	132	111	136	174	170
12	121	173	118	37	141	172	117	62	164	179	117	87	155	173	152	112	136	174	189
13	121	173	137	38	141	172	136	63	164	179	137	88	155	173	171	113	136	174	172
14	121	173	156	39	141	172	156	64	164	179	156	89	155	173	154	114	136	174	153
15	121	173	137	40	141	172	137	65	164	179	137	90	155	173	135	115	136	174	134
16	121	158	41	41	141	156	133	66	164	164	132	91	155	154	171	116	136	154	102
17	121	158	152	42	141	156	152	67	164	164	152	92	155	154	190	117	136	154	192
18	121	158	171	43	141	156	172	68	164	164	171	93	155	154	173	118	136	154	173
19	121	158	151	44	141	156	153	69	164	164	152	94	155	154	154	119	136	154	154
20	121	158	148	45	141	156	134	70	164	164	133	95	155	154	135	120	136	154	132
21	121	170	0	46	141	153	50	71	164	143	173	96	155	136	49	121	136	163	0
22	121	170	71	47	141	153	175	72	164	143	192	97	155	136	191	122	136	163	208
23	121	170	140	48	141	153	156	73	164	143	173	98	155	136	172	123	136	163	146
24	121	170	106	49	141	153	137	74	164	143	154	99	155	136	153	124	136	163	186
25	121	170	0	50	141	153	160	75	164	143	135	100	155	136	165	125	136	163	0

There are a huge amount of data representing the number of batches transferred from the distributor to customers so, the data of only first 25 scenarios are represented by the paper and the full data are represented in appendix A or through this link https://drive.google.com/open?id=0B448W9rNzRcPRk9iWHlPN0dQa1U. Table 7 represents the number of batches (units) transferred from the distributor to the first and second customers for the three products for only 25 scenarios and Table 8 represents the number of batches (units) transferred from the distributor to the third and fourth customers for the three products for only 25 scenarios.

Table 7. *Number of batches (units) transferred from distributors to the first and second customers for the three products for only 25 scenarios.*

Scen.	Qdcpt21..			Qdcpt21..			Qdcpt21..			Qdcpt22..			Qdcpt22..			Qdcpt22..		
	..12	..13	..14	..22	..23	..24	..32	..33	..34	..12	..13	..14	..22	..23	..24	..32	..33	..34
1	152	152	0	151	152	0	149	154	0	152	151	0	152	152	0	152	152	40
2	152	152	0	151	152	0	149	154	169	152	151	175	152	152	0	152	152	176
3	152	152	200	151	152	200	149	154	200	152	151	200	152	152	200	152	152	200
4	152	152	0	151	152	0	149	154	0	152	151	220	152	152	219	152	152	221
5	152	152	0	151	152	0	149	154	0	152	151	0	152	152	0	152	152	0
6	152	176	0	151	176	152	149	178	149	152	176	1	152	176	152	152	176	152
7	152	176	176	151	176	176	149	178	177	152	176	176	152	176	176	152	176	176
8	152	176	199	151	176	200	149	178	200	152	176	200	152	176	200	152	176	200
9	152	176	223	151	176	224	149	178	225	152	176	224	152	176	224	152	176	224
10	152	176	0	151	176	100	149	178	168	152	176	5	152	176	217	152	176	248
11	152	200	152	151	200	153	149	201	152	152	200	152	152	200	152	152	199	152
12	152	200	176	151	200	176	149	201	176	152	200	176	152	200	176	152	199	177
13	152	200	200	151	200	198	149	201	200	152	200	200	152	200	200	152	199	200
14	152	200	224	151	200	222	149	201	224	152	200	224	152	200	224	152	199	224
15	152	200	248	151	200	248	149	201	248	152	200	248	152	200	248	152	199	248
16	152	224	0	151	224	0	149	227	0	152	224	0	152	206	170	152	224	0
17	152	224	176	151	224	174	149	227	176	152	224	176	152	206	194	152	224	176
18	152	224	200	151	224	197	149	227	200	152	224	200	152	206	218	152	224	200
19	152	224	224	151	224	225	149	227	224	152	224	224	152	206	242	152	224	224
20	152	224	0	151	224	248	149	227	248	152	224	240	152	206	134	152	224	248
21	152	248	0	151	248	0	149	169	0	152	248	0	152	248	0	152	248	0
22	152	248	0	151	248	174	149	169	0	152	248	175	152	248	176	152	248	0
23	152	248	200	151	248	200	149	169	282	152	248	200	152	248	200	152	248	200
24	152	248	0	151	248	0	149	169	306	152	248	0	152	248	0	152	248	224
25	152	248	0	151	248	0	149	169	0	152	248	0	152	248	0	152	248	0

Table 8. *Number of batches (units) transferred from distributors to the third and fourth customers for the three products for only 25 scenarios.*

Scen.	Qdcpt 23..			Qdcpt 23..			Qdcpt 23..			Qdcpt 24..			Qdcpt 24..			Qdcpt 24..		
	..12	..13	..14	..22	..23	..24	..32	..33	..34	..12	..13	..14	..22	..23	..24	..32	..33	..34
1	152	152	0	152	152	0	152	152	0	152	152	0	152	97	0	152	152	0
2	152	152	0	152	152	0	152	152	174	152	152	0	152	97	0	152	152	116
3	152	152	200	152	152	200	152	152	200	152	152	200	152	97	255	152	152	200
4	152	152	0	152	152	26	152	152	0	152	152	0	152	97	0	152	152	219
5	152	152	0	152	152	0	152	152	0	152	152	0	152	97	0	152	152	30
6	152	176	152	152	175	1	152	176	152	152	176	0	152	176	0	152	175	152
7	152	176	175	152	175	177	152	176	176	152	176	176	152	176	176	152	175	176
8	152	176	199	152	175	200	152	176	200	152	176	200	152	176	200	152	175	200
9	152	176	223	152	175	224	152	176	224	152	176	223	152	176	223	152	175	222
10	152	176	0	152	175	248	152	176	248	152	176	248	152	176	0	152	175	231
11	152	199	148	152	200	152	152	200	152	152	199	152	152	200	151	152	200	149
12	152	199	176	152	200	176	152	200	176	152	199	176	152	200	175	152	200	176
13	152	199	199	152	200	200	152	200	200	152	199	200	152	200	200	152	200	200
14	152	199	223	152	200	224	152	200	224	152	199	223	152	200	223	152	200	223
15	152	199	247	152	200	248	152	200	248	152	199	246	152	200	244	152	200	56
16	152	224	0	152	224	0	152	171	205	152	215	155	152	224	0	152	218	0
17	152	224	176	152	224	176	152	171	229	152	215	182	152	224	176	152	218	179
18	152	224	200	152	224	200	152	171	253	152	215	205	152	224	200	152	218	202
19	152	224	224	152	224	224	152	171	277	152	215	233	152	224	224	152	218	30
20	152	224	0	152	224	0	152	171	244	152	215	0	152	224	248	152	218	0
21	152	4	0	152	114	0	152	248	0	152	247	0	152	248	0	152	235	0
22	152	4	0	152	114	295	152	248	176	152	247	0	152	248	0	152	235	179
23	152	4	444	152	114	334	152	248	200	152	247	196	152	248	196	152	235	18
24	152	4	0	152	114	358	152	248	0	152	247	0	152	248	117	152	235	0
25	152	4	0	152	114	0	152	248	0	152	247	0	152	248	0	152	235	0

The number of batches transferred from factory to its store (I_{fpt}), the number of batches transferred from this store to the distributor (I_{fdpt}), and the residual batches in both factory (R_{fpt}) and distributor (R_{dpt}) stores for the three products for the first 25 scenarios are shown in Table 9.

Table 9. *Number of batches transferred from factory to its store (Ifpt), Transferred from this store to the distributor (Ifdpt) and Residual batches in both factory (Rfpt) and distributor (Rdpt) stores for the three products for only 25 scenarios.*

Scen.	Ifpt 2		Ifpt 2		Ifpt 2		Ifdpt 22..		Ifdpt 22..		Ifdpt 22..		Rfpt 2..		Rfpt 2..		Rfpt 2..		Rdpt 2..		Rdpt 2..		Rdpt 2...	
	12	13	22	23	32	33	13	14	23	24	33	34	12	13	22	23	32	33	12	13	22	23	32	33
1	0	0	0	0	10	0	0	0	0	0	4	6	0	0	0	0	10	6	7	0	3	0	0	0
2	0	0	0	0	10	0	0	0	0	0	4	6	0	0	0	0	10	6	7	0	3	0	0	0
3	0	0	0	0	10	0	0	0	0	0	4	6	0	0	0	0	10	6	7	0	3	0	0	0
4	0	0	0	0	10	0	0	0	0	0	4	6	0	0	0	0	10	6	7	0	3	0	0	0
5	0	0	0	0	10	0	0	0	0	0	4	6	0	0	0	0	10	6	7	0	3	0	0	0
6	0	28	0	14	10	0	0	28	0	14	5	5	0	28	0	14	10	5	7	3	3	5	0	0
7	0	28	0	14	10	0	0	28	0	14	5	5	0	28	0	14	10	5	7	3	3	5	0	0
8	0	28	0	14	10	0	0	28	0	14	5	5	0	28	0	14	10	5	7	3	3	5	0	0
9	0	28	0	14	10	0	0	28	0	14	5	5	0	28	0	14	10	5	7	3	3	5	0	0
10	0	28	0	14	10	0	0	28	0	14	5	5	0	28	0	14	10	5	7	3	3	5	0	0
11	0	0	0	0	10	0	0	0	0	0	0	10	0	0	0	0	10	10	7	4	3	8	0	65
12	0	0	0	0	10	0	0	0	0	0	0	10	0	0	0	0	10	10	7	4	3	8	0	65
13	0	0	0	0	10	0	0	0	0	0	0	10	0	0	0	0	10	10	7	4	3	8	0	65
14	0	0	0	0	10	0	0	0	0	0	0	10	0	0	0	0	10	10	7	4	3	8	0	65
15	0	0	0	0	10	0	0	0	0	0	0	10	0	0	0	0	10	10	7	4	3	8	0	65
16	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
17	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
18	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
19	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
20	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
21	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
22	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
23	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
24	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
25	0	0	0	0	10	0	0	0	0	0	10	0	0	0	0	0	10	0	7	0	3	0	0	0
26	1	1	0	33	11	0	0	2	0	33	10	1	1	2	0	33	11	1	1	3	1	4	1	0

5.3. Model Results Analysis

Table 10 represents the mean demand and its required material weight in kilograms and the required manufacturing hours. It can be noticed that the total expected required material equals 1200 kg which is smaller than the supplying capacity of one supplier so it is reasonable to open only one supplier. The total expected required manufacturing hours equals 1200 hour which is smaller than the manufacturing capacity of one factory so also, it is reasonable to open only one factory and one distributor to transfer them to customers.

Table 10. *Mean demand fulfillment requirements.*

Product	Mean demand per period	Unit weight (Kg.)	Exp. Req. weight (Kg.)	Man. Hours	Exp. Req. Hours
1	200	1	200	1	200
2	200	2	400	2	400
3	200	3	600	3	600
			1200		1200

Considering transportation cost, the model optimally decided to open the second raw of facilities as shown in Figure 10 to reduce the total transportation cost to the four customers.

Verification of the network flow balancing is done for only the middle scenario which has the most probability. Demands of all products for all customers are 200, 200 and 200 in the three periods. Table 11 depicts the quantities of batches and weights of raw material transferred from supplier 2 to factory 2, and the quantities of batches and weights of products transferred from factory 2 to all customers through distributor 2. Balancing is noticed in Table 11 for the transferred weights since the transferred amount from any echelon to other are the same of 14,400 kilograms.

Table 11. *No. of batches and weights transferred from the supplier to customers.*

From-To	S2-F2	F2-D2			D2C1			D2C2			D2C3			D2C4		
Period	**RM**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**	**P1**	**P2**	**P3**
1	497	178	162	164	200	200	200	200	200	200	200	200	200	200	200	200
2	500	143	160	179	200	200	200	200	200	200	200	200	200	200	200	200
3	443	159	158	137	200	200	200	200	200	200	200	200	200	200	200	200
Weight (Kg.)	14400	2400	4800	7200	600	1200	1800	600	1200	1800	600	1200	1800	600	1200	1800
Total Weights	14400	14400			14400											

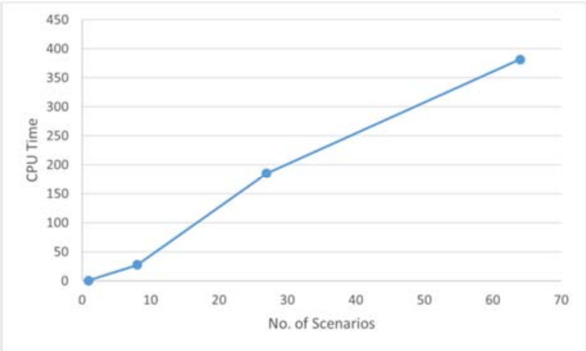

Figure 11. *CPU seconds versus No. of Scenarios up to 64 scenarios.*

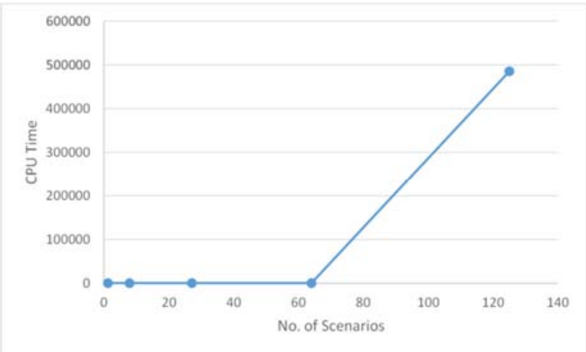

Figure 12. *CPU seconds versus No. of Scenarios up to 125 scenarios.*

The effect of changing the number of scenarios on the processing time is studied and it was as shown in Figures 11 and 12 from which it is noticed that increasing the number of scenarios dramatically increases CPU time. Figure 12 shows that the processing time of 125-scenarios is too big comparing to other values. The CPU time of this example is 485,925 seconds, 135 hours or 5.6 days.

6. Conclusion

The proposed model is successful in designing supply chain networks while considering multi-product, multi-period stochastic demand with three echelons (suppliers, facilities, and distributors). It can only be used for single item problems. The model is flexible to solve larger problems; however, it requires more powerful hardware since the CPU time increases exponentially with the increase of the number of scenarios which increases by increasing the number of periods.

The application of the proposed model showed that the total expected profit is directly affected by demand mean for a given capacity of the network.

The proposed design model is capable of supply chain networks while considering inventory at the factory and distribution centers.

The proposed design model takes into account different types of costs like the non-utilized capacity cost for factories, transportation cost between all nodes, and the holding cost of inventory in both factories and distributors and shortage cost to enhance customers' satisfaction.

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