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Analytical Study of Electromagnetic Wave Diffraction Through a Circular Aperture with Fringes on a Perfect Conducting Screen

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Abstract: This work focused on the study and analysis of the behavior of fringes formed by EM wave diffraction through small circular aperture of finite diameter on a cylindrical perfect conducting screen of a finite thickness, L. The nature of the fringes intensity formed on the perfect conducting screen was analyzed with reference to the three windows within EM spectrum such as ultraviolet, visible and infrared. The intensity of the fringes distribution was found to depend on the wavelength of the radiation associated to each window. The work also showed the different diffraction patterns that formed normal distribution pattern for different wavelengths within electromagnetic wave spectrum with a central fringes being more intense in each case than the others that appeared like background fringes. The graphs of the ratio of spectral energy to the conductivity of the screen plotted for the three windows considered in this work as a function of thickness of screen respectively.

Keywords: Electromagnetic Wave, Analysis, Fringes, Perfectly Conducting Screen, Circular Aperture, Diffraction, Distribution, Intensity

1. Introduction

The electromagnetic fields passing through a circular aperture have been investigated for a longtime, [1, 9] and the incident light model of a Gaussian beam in transverse distribution has been generally studied, [6; 7] while the plane wave has also been taken into consideration for simpler incident light models. [8; 9] Recently, more accurate solutions to the electromagnetic equations for aperture diffraction have received much attention. Generally, the scalar representation from paraxial approximation cannot satisfy the Maxwell equations and cannot be used in a large divergent beam or a small beam spot size comparable to the wavelength. To overcome this drawback, the vector representation based on the nonparaxial framework has been developed to resolve the Maxwell equations, such as the vector angular spectrum method, [10; 11] the Hertz vector diffraction theory, [3, 7, 9] the vectorial Rayleigh diffraction, and so on. [6; 12] The vector angular spectrum method has received more and more attention recently because it is a useful and convenient tool to resolve the propagation of a light field [10; 13, 20]and the far-field diffraction of nonparaxial Gaussian beams at a circular aperture. [11] At the same time, the Hertz vector diffraction theory has also been taken as an important method to study circular aperture diffraction.[3; 7, 9]. In 2005, Guha and Gillen [8] derived integral expressions for the electric and magnetic field components of light within and beyond an aperture plane for an incident plane wave by using the nonparaxial vector diffraction theory. In 2009, Gillen *et al.* [7] investigated a focused TEM00 Gaussian light field passing through a circular aperture by using the Hertz vector diffraction theory. In addition, in 2003, Liu and Duan [6] investigated the nonparaxial propagation of vectorial Gaussian beams diffracted at a circular aperture by using the vectorial Rayleigh diffraction integral. And in 2004, Gillen and Guha [12] also used the complete Rayleigh–Somerfield method to discuss the model and propagation of near-field diffraction patterns. Moreover, as is well known, the diffraction of light at an aperture with a radius comparable to or smaller than the wavelength of light is a challenging researching topic and must be described by the vector theory, in which the light field is restricted to about a wavelength size scale, and it is composed of an evanescent field and a propagating wave in the near-field. So far, the analytical non-paraxial vector expression for the diffraction through the circular aperture with subwavelength size in the near-field and the properties of the evolving evanescent field and the propagating wave, as well as their relationship, has not been achieved completely. The diffraction through a circular sub-wavelength-size aperture in the near-field is important for aperture near field scanning optical microscopy using a near-field optical probe with a sub-wavelength-sized opening, which has already become a widely used tool for optical imaging with several 10-nm spatial resolutions. Therefore, it would be interesting and worthwhile to study the diffraction through a circular sub-wavelength-size aperture. In this paper, analytical nonparaxial vectorial electric field expressions in the form of convergent series and the power transmission functions and their prosperities of the diffraction through a circular subwavelength-size aperture are investigated based on the vector plane angular spectrum. The rest of the present paper the diffracted through a circular aperture for Gaussian beams in terms of ultraviolet, visible and infrared spectra of electromagnetic wave and. analyzed diffracted intensity distribution prosperities in as exhibited by each region

2. Theoretical Approach

Considering the wave equation in one dimensional component

$$\nabla^2 E = \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} \tag{1}$$

Whose solution is generalized in terms of real part $E(x,y,z,t) = \mathrm{Re}[\exp(i\omega t)]$ Of the solution of wave equation, we obtain a desired scalar wave equation of the form

$$(\nabla^2 + k^2)E = 0 \tag{2}$$

If we assume a negligible influence of aperture on the diffraction of the field for a distance of the order of a few wavelength, then

$$E = \frac{1}{i\lambda}\iint E_o \exp(ikr_{io})\cos ds \tag{3}$$

Which can be simply be of the form

$$E = A\exp ikr \tag{4}$$

For spherical wave of amplitude A diverging from a point source

As the fringe formed is in the direction of propagated wave along the z axis with circular aperture of radius r, located at the plane z=0, the wave just at the aperture according to(Duan and LU 2003) can be expressed thus

$$E_x(x_o, y_o, 0) = t(x_o, y_o)\exp\left(\frac{-x_o, y_o}{w^2}\right) \tag{5a}$$

$$E_y(x_o, y_o, 0) = 0 \tag{5b}$$

Where w is the beam waist and the window function of the circular aperture $t(x_o, y_o) = 1$
or as

$$x_o^2 + y_o^2 \le R^2 \tag{6}$$

and

$$t(x_o, y_o) = 0 \tag{7}$$

In any case according Wang et al, 2012 the angular spectral representation of any electric field can be expanded into an angular spectrum of plane waves in the region of propagation z at any point, $z \ge 0$ the result of this expansion would culminate to first order Bessel function

$$J_l(x) = \sum_{m=1}^{\infty}(-1)^{m+1}\frac{x^{2m+l}}{m!(m+l)!2^{2m+l}} \tag{8}$$

This function can be used to describe the intensity of the irradiance distribution of the fringe pattern,

$$I_x = I_0\left[\frac{2J_l(x)}{x}\right]^2 \tag{9}$$

$$r = \frac{d\pi\sin\theta}{\lambda} \tag{10}$$

The energy impacted on the screen by the fringe which is assumed to be spherical on reaching the cylindrical screen can be obtained using Lorentz force,

$$\vec{F} = q\vec{E} + qvx\vec{B} \tag{11}$$

Since the fringe being formed on screen excite electrons that move along the thickness in it with constant mean radius, the current density due to the excited electrons will cause the Lorentz force to be of the form

$$\vec{F} = dq\vec{E} + dq\frac{dl}{dt}x\vec{B} \tag{12}$$

To obtain the work done by the fringe by exciting the electrons in the screen, we have

$$dF.dl = dq\vec{E}.dl + dq\frac{dl}{dt}x\vec{B}.dl \tag{13}$$

But the second term in the equation 13 is zero, it means that

$$d\vec{F}dl = dq\vec{E}dl$$

Represent the work done and as such can be can be written as

$$dw = JEAdl \text{ i}$$

$$\int_0^l dw = \int_0^l JEAdl = \frac{1}{2}JEAl^2 \tag{14}$$

By approximation of the fringe distribution on the cylindrical screen to circular in nature, equation 14 becomes

$$wl = 2\pi rlJEl^2 \tag{15}$$

as J tends infinity, the conductivity of the perfect conduct screen also tend to infinity and as such

$$\frac{J}{\sigma} = E$$

this leads to

$$\alpha = \frac{w}{\sigma} = \pi^2 r\vec{E}^2 l^2 \tag{16}$$

From equation 8,14 becomes

$$\alpha = \frac{\pi^2 E^2 l^2 d\sin\theta}{\lambda} \tag{17}$$

That specifies the ratio of the spectral energy intensity to the conductivity of the screen.

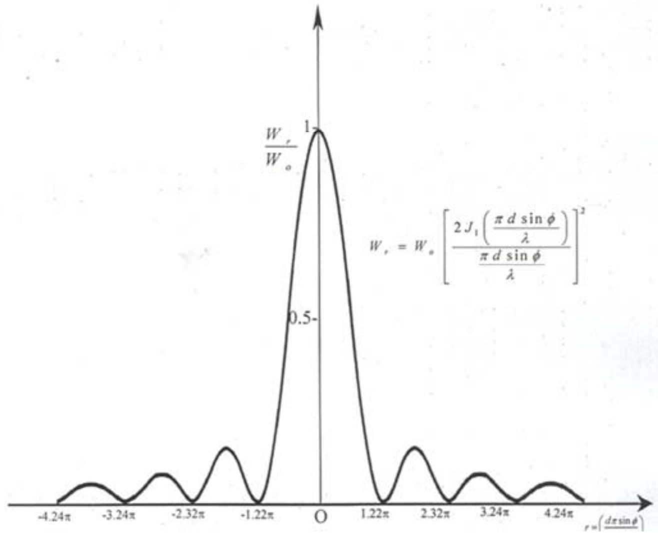

Figure 1. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 250\mu m$ *and* $\lambda = 250nm$.

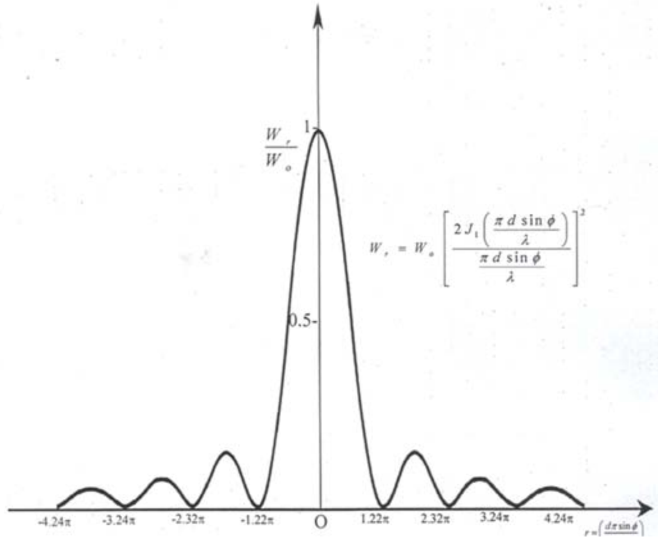

Figure 2. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 250\mu m$ *and* $\lambda = 450nm$.

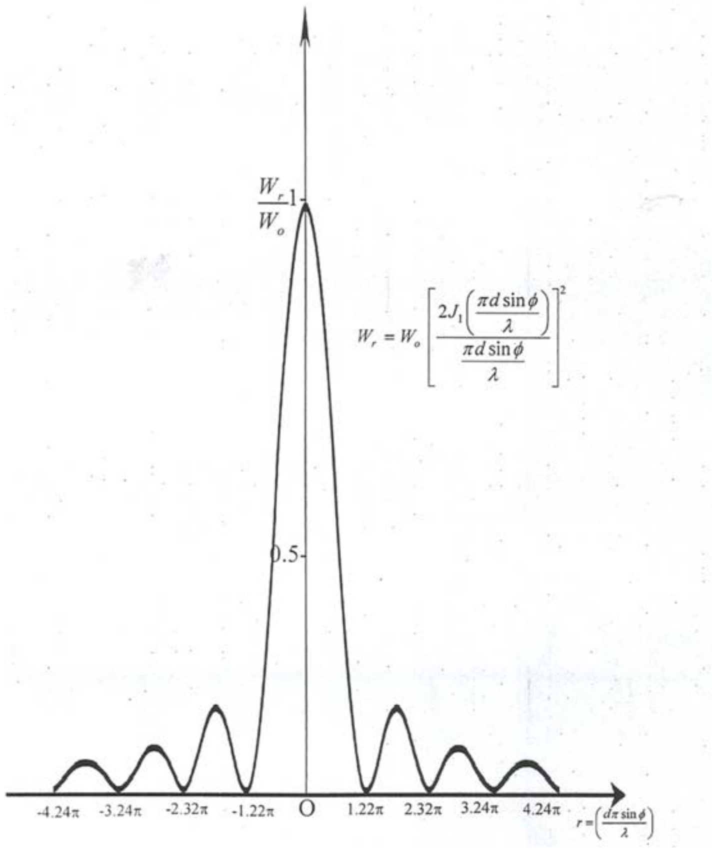

Figure 3. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at d=500μm and λ= 450μm.*

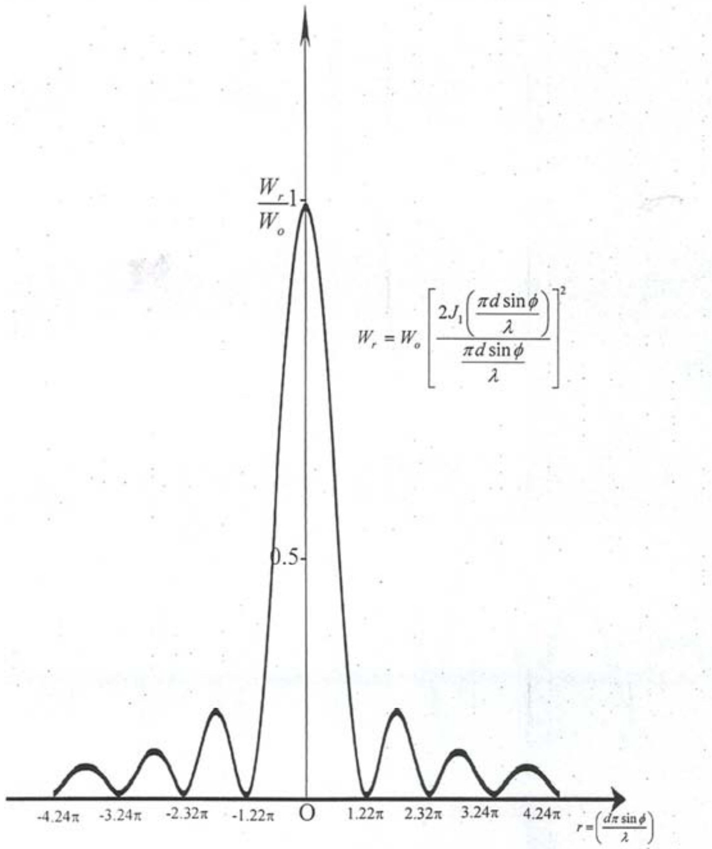

Figure 4. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at $d = 700\mu m$ and $\lambda = 350nm$.*

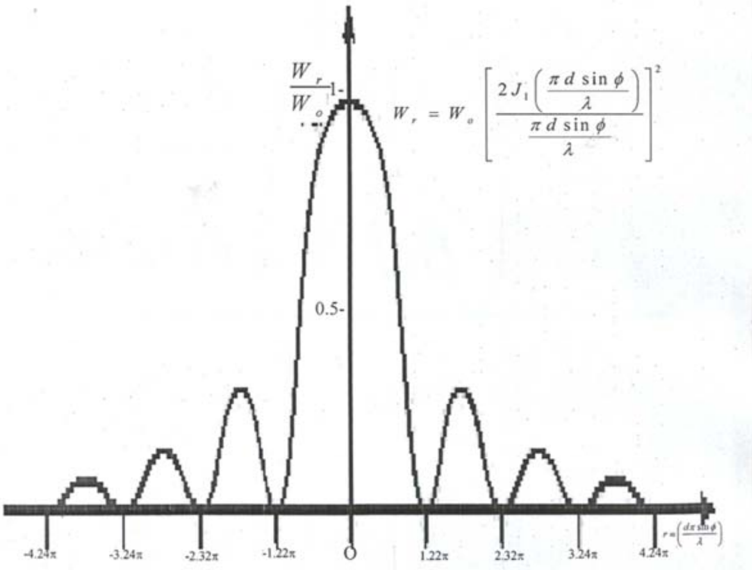

Figure 5. *Graph of relative energy intensity (W_r/W_o) against radius of the fringes at* $d = 250\mu m$ *and* $\lambda = 550nm$

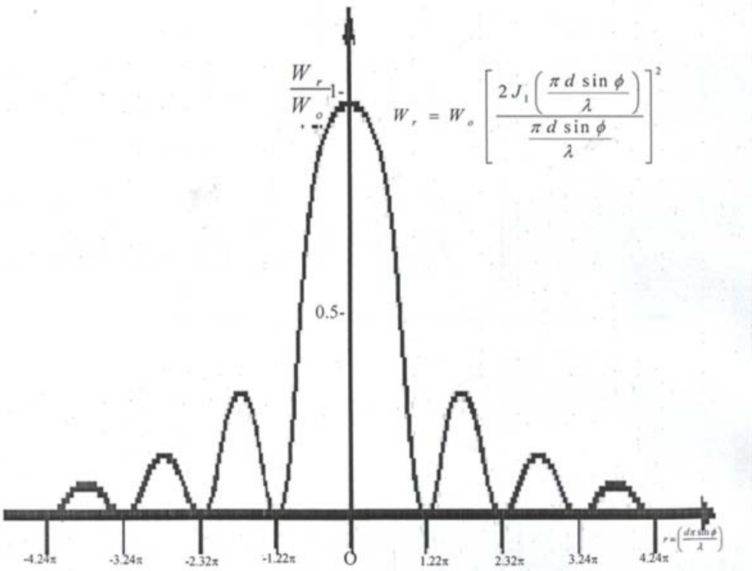

Figure 6. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 250\mu m$ *and* $\lambda = 750nm$.

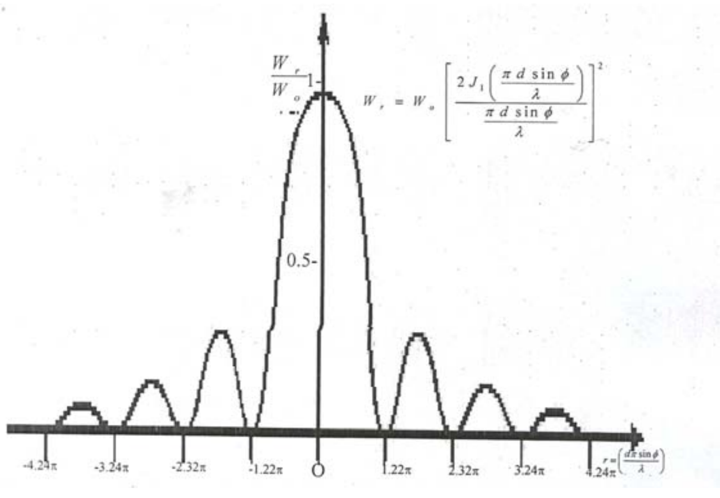

Figure 7. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 500\mu m$ *and* $\lambda = 650nm$.

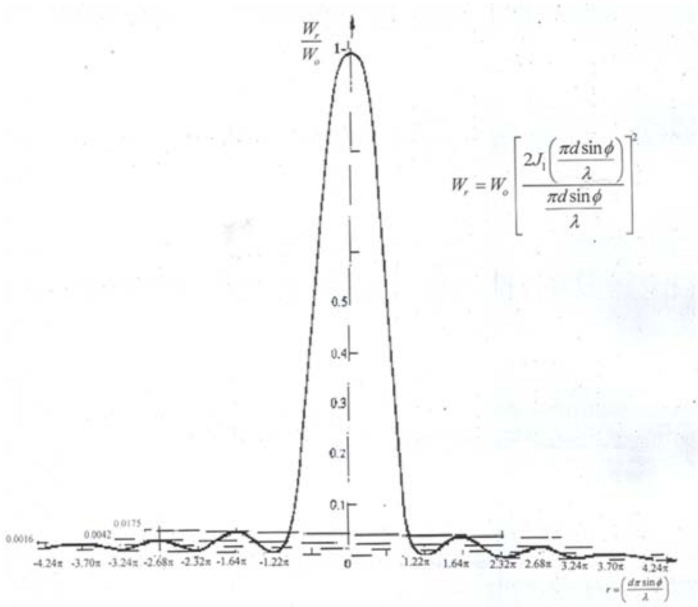

Figure 8. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 700\mu m$ *and* $\lambda = 650nm$.

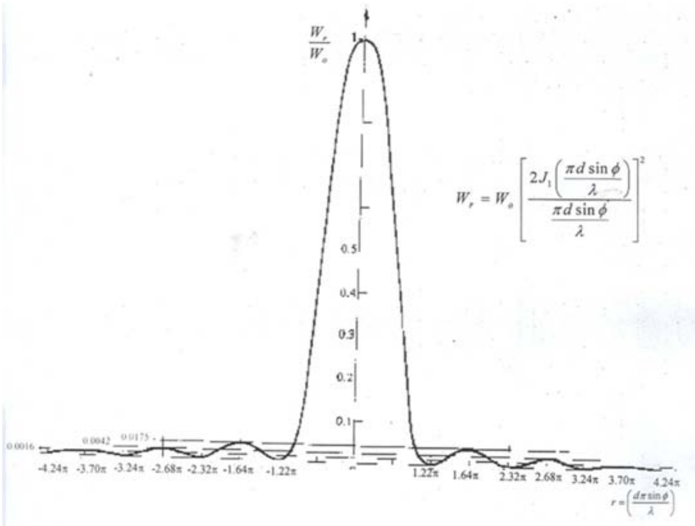

Figure 9. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 250\mu m$ *and* $\lambda = 850nm$.

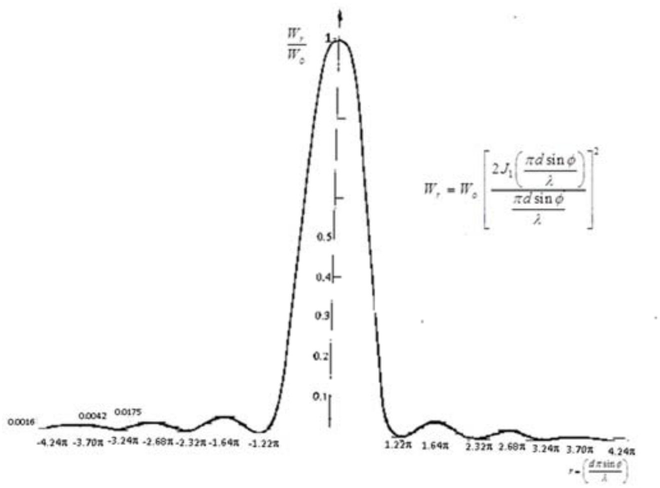

Figure 10. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at* $d = 250\mu m$ *and* $\lambda = 1200nm$.

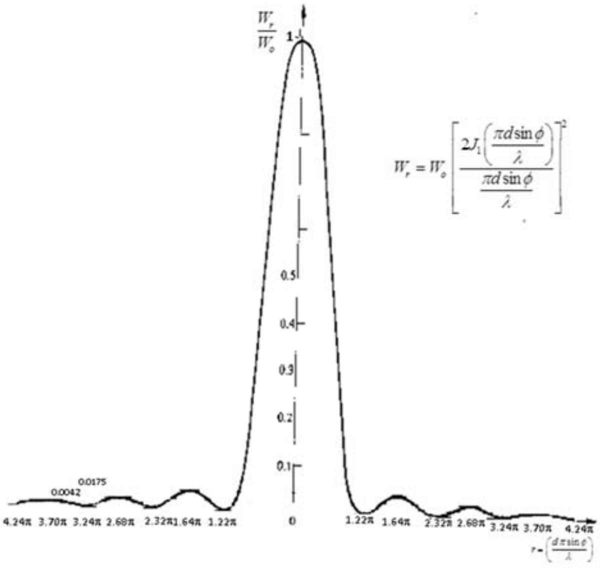

Figure 11. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at d= 500μm and λ=950μm*

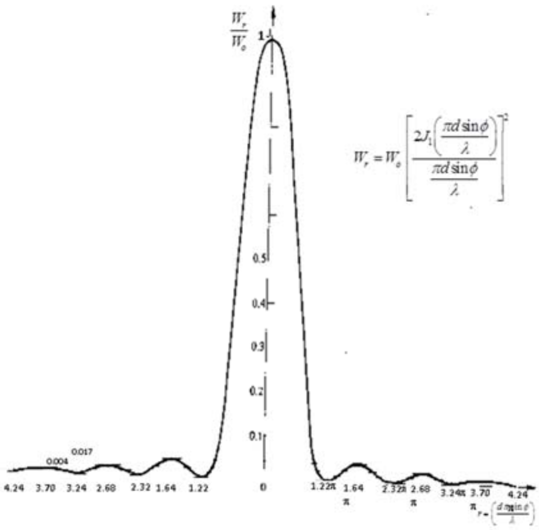

Figure 12. *Graph of relative fringe intensity (W_r/W_o) against waist of the fringes at $d = 700\mu m$ and $\lambda = 950nm$.*

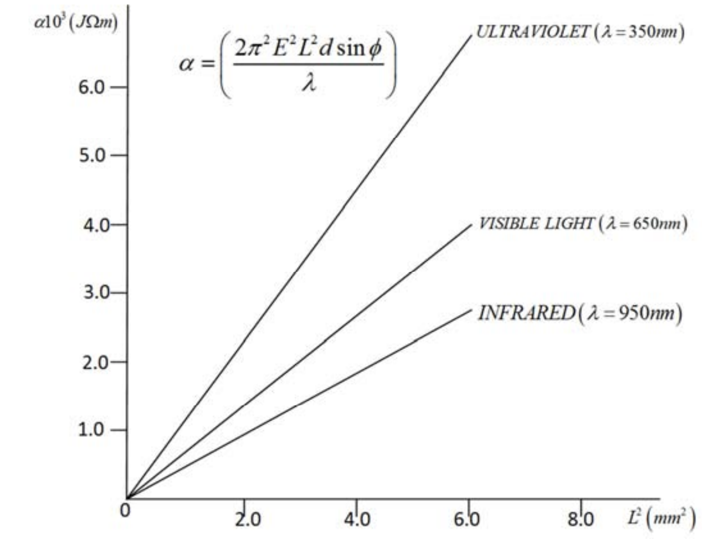

Figure 13. *Graph of the ratio of spectral energy to the conductivity (α) of the screen against thickness (L^2) at constant $d = 250\mu m$.*

3. Result/Discussion

The distribution of fringe intensity formed on a perfect cylindrical conducting screen as obtained from the Bessel's function of the first kind of order of unity were shown on the graphs in figures 1 to figures 12 in accordance with the wavelength of the electromagnetic wave spectrum considered in this work viz; ultraviolet, visible and infrared region respectively.

Figure 1 to figure 4 depict the relative fringe intensity within the ultraviolet region for different dimension of diffracting aperture while figure 5 to figure 7 are the graphs for visible region for different diffracting apartures. Contained in figures 9 to 12 were that of relative fringe intensity within infrared region while figure 13 is the ratio of the spectral energy to the conductivity of the screen used.

From the graphs of the relative intensity of the fringes the regions considered, it was observed that that within the ultraviolet appeared very sharp with that within the position of the central bright fringe well distributed for all the diffracting apertures used in the computation. On the other hand, the ones within the visible region appeared were not sharp although well distributed like that of the former ones while that within the infrared region were not equally as bright and sharp as that of ultraviolet. The ratio of the spectral energy to the conductivity of the screen for a given diffracting aperture was found to be constant for the three considered regions.

4. Conclusion

The study of the relative fringe intensity as formed on the perfect conducting cylindrical screen by diffracting electromagnetic radiation through a given dimension of diffracting circular aperture has been carried out with our formalism, Bessel's function of first kind order unity. Considering the nature and the behaviour of relative fringe intensity obtained for the three region of the E. M wave spectrum, it would be observed that the brightness of the visible region was not as sharp as the other two regions a situation that might be attributed to effect of the energy impacted on screen by the fringes in which that within the visible region might have likely caused disturbance of electrons on the screen which could have occasioned the unsharpness of the fringes in the visible region.

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