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A Study of Fundamental Law of Thermal Radiation and Thermal Equilibrium Process

Chen Dayou

Shaanxi Society of Optics, Photoelectron and Quantum Electronics, Xi'an, Chian

Email address:
Chendayou668@163.com

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Abstract: The fundamental law of thermal equilibrium radiation includes two elements: the law of energy distribution of matter vibrators in the radiation field and the law of energy exchange between vibrators and the radiation field. This paper discovers the law of how vibrators stimulate and absorb radiation, by a study of the black-body radiation law and the characteristics of vibrators' absorption of radiation. As for the fundamental law of thermal equilibrium radiation, its complete expression should be: the energy distribution of vibrators in the thermal equilibrium radiation field follows the energy distribution law by L. Boltzmann; the probability of vibrators' stimulating radiation is directly proportional to their state of energy levels and that of their absorbing radiation is directly proportional to their energy distribution probability. The author, on the basis of the fundamental law of thermal radiation, proposes conditions for thermal equilibrium radiation and analyses the micro momentum theory and characteristics in the process of thermal equilibrium.

Keywords: Thermal Equilibrium Radiation, State of Energy Levels, Probability of Energy Levels, Thermal Equilibrium Process, Fundamental Law of Thermal Radiation

1. Introduction

The theoretic study and lab experiment of thermal radiation is of special significance in the history of development of physics. Especially the study of black-body radiation law results in the founding of quantum physics. Historically, there have been two main methods. One is to derive the black-body radiation formula by calculating the average energy of vibrators on cavity, as was done by M. Planck; the other is adopted by L. Rayleigh and J. H. Jeans to calculate the average energy of standing waves in cavity.

According to modern radiation theory, the thermal radiation is the electromagnetic radiation of some special waves and the photon (wave packet) is of wave article duality. An empty cavity is a radiation field. If the radiating body reaches a balanced state with the cavity temperature, the Planck formula may be derived by calculating the average energy of the standing waves in cavity. From the comparative study of Planck's and Rayleigh's work, we find that the black-body radiation law has more profound content. If we do further research on this basis, we can reveal the fundamental law of thermal radiation.

The fundamental law of thermal radiation includes two aspects of the vibrators' energy distribution law and the energy exchange law in the radiation field, the former being described by Boltzmann's energy distribution law while the latter awaits further study. Our study is focused on revealing the energy exchange law between the body and the radiation field. The energy exchange also covers two levels of vibrators stimulating and absorbing radiation. Let's first of all analyse the law of vibrators' stimulating radiation.

2. Law of Vibrators' Stimulating Radiation

The Planck's formula is of immediate significance for revealing the law of vibrators' stimulating radiation. We can even claim that it implicitly contains the law.

Planck holds the view that the energy spectrum density in the radiation field $u(v, T)$ is determined by the state density of the radiation field and the average energy of vibrators. On the issue of how to calculate the average energy of vibrators, Planck advanced the revolutionary concept of quantum of energy and thus founded his quantum theory. According to this theory the vibrator's energy is discontinuous and has its minimal unit. For vibrators whose frequency is v, the minimal unit of energy $\varepsilon=h\nu$ and vibrators can only locate themselves in ε, 2ε, 3ε, …. one of a series of discontinuous states of energy. According to Boltzmann's law of energy distribution, the cavity frequency is ν and the normalized distribution law of $n\varepsilon$ energy vibrators is

$$B(\varepsilon_n, T) = De^{-\varepsilon_n/kT} \qquad (n=1,2,\cdots) \tag{1}$$

In the formula D stands for the normalization coefficient

$$D = \frac{1}{\sum_{n=0}^{n=\infty} e^{-\varepsilon_n/kT}\Delta\varepsilon} \tag{2}$$

Now that the energy levels are separate, we cannot but directly calculate the average energy of vibrators. Noticing $\varepsilon=h\nu$, the average energy is

$$\begin{aligned}\overline{\varepsilon} &= \frac{\sum_{n=0}^{n=\infty}\varepsilon_n e^{-\varepsilon_n/kT}\Delta\varepsilon}{\sum_{n=0}^{n=\infty} e^{-\varepsilon_n/kT}\Delta\varepsilon} = \frac{\sum_{n=0}^{n=\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{n=\infty} e^{-nh\nu/kT}} \\ &= \frac{-\partial}{\partial(1/kT)}\mathrm{In}\left(\sum_{n=0}^{n=\infty} e^{nh\nu/kT}\right) = \frac{h\nu}{e^{h\nu/kT}-1}\end{aligned} \tag{3}$$

The density in the $\nu\rightarrow\nu+d\nu$ area is represented by $g(\nu)d\nu$.[1] When the result of Rayleigh and Jeans calculation is used

$$g(\nu)\mathrm{d}\nu = \frac{8\pi\nu^2\mathrm{d}\nu}{c^3} \tag{4}$$

So the Planck formula

$$\begin{aligned}u(\nu,T) &= g(\nu)\overline{\varepsilon}\mathrm{d}\nu \\ &= \frac{8\pi\nu^2}{c^3}\frac{h\nu\mathrm{d}\nu}{e^{h\nu/kt}-1}\end{aligned} \tag{5}$$

Before Planck, Rayleigh calculated the state density of electromagnetic standing waves in the cavity and offers the formula of black-body energy spectrum, on the basis of the existence of electromagnetic medium in vacuum.

$$u(\nu,T) = \frac{8\pi\nu^2}{c^3}kT \tag{6}$$

This formula conforms to the experiment in long waves, but differs a lot in short waves. The Rayleigh formula is a natural result of the classic electromagnetic theory and the energy equal distribution law. Then what's wrong with the Rayleigh-Jeans formula? In our view, first of all the state density can not serve as the basis of calculating the number of standing waves. Otherwise, it will result in "the ultraviolet disaster" that the higher the frequency is, the greater the number of standing waves becomes, and the radiation energy tend to be infinite. As a matter of fact, the number of standing waves in the cavity is determined by two independent factors. It is related not only to the state density, but also to the vibrators in the cavity whose frequency is ν. This conclusion is easy to understand. The experiments reveal that the strongest cosmos ray received so far is no higher than 10^{26}in frequency.[2][3]The vibrator whose frequency is higher than the limit cannot possibly exist in any known body. But the state density given in the Rayleigh-Jeans formula is astonishingly high. Besides, while calculating the energy of standing waves, classic theory is used to give the average energy of vibrators.

To find the law of vibrators' stimulating radiation, we repeat our calculation of the number of standing waves in the $\nu\rightarrow\nu+d\nu$ area. For the cavity radiation field with the T temperature, $p_i(\nu, T)$ stands for the probability of radiation stimulation by vibrators with ν frequency and $ih\nu$ energy. It is also the probability of the number of transmitted photons. We hold the view that, although vibrators whose frequency is ν and whose energy levels are respectively $h\nu$, $2h\nu$, $3h\nu$, … etc. hop and shift between energy levels of the same basic frequency, they all stimulate photons with ν frequency. But the probabilities of different energy level vibrators stimulating radiation are different. It is easier for vibrators of high energy state to transfer and stimulate radiation than those of low energy state. And the probabilities of vibrators' stimulating radiation are surely directly proportional to their energy level state. That is to say, for vibrators whose energy levels are respectively $h\nu$, $2h\nu$, $3h\nu$, … etc., the probability relationship of the number of stimulated radiation in unit time is

$$h\nu : 2h\nu : 3h\nu = 1:2:3 \tag{7}$$

The radiation stimulation probability of vibrators with ν frequency and i energy level is determined by the two factors of energy level state and energy distribution probability. That is to say

$$p_i(\nu,T) = ie^{-ih\nu/kT} \tag{8}$$

So the average probability of radiation stimulation of vibrators with ν frequency is

$$f(\nu,T) = \frac{\sum_{i=0}^{i=\infty} ie^{-ih\nu/kT}}{\sum_{i=0}^{i=\infty} e^{-ih\nu/kT}} = \frac{1}{e^{h\nu/kT}-1} \tag{9}$$

If $P(\nu, T)$ stands for the number of standing waves in the areas neighboring unit volume frequency ν, obviously the number of standing wave in cavity is determined by the product of the density of state multiplied by average stimulating probability.

$$P(\nu,T) = g(\nu)f(\nu,T) = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/kT}-1} \tag{10}$$

Formula (10) is multiplied by the energy of wave packets and we have Planck Formula.

The Planck formula completely agrees with the experiment. This means that the number of standing waves shown in Formula (10) conforms to the experiment. And furtherly, the relationship of the radiation probability revealed by Formula (7) is directly proportional to the vibrators state of energy levels, which conforms to the reality. That's why we claim that Planck formula not only reveals the law of black-body radiation, but also contains the relationship of the probability of vibrators' stimulating radiation being directly proportional to their state of energy levels. So, as for the fundamental law of thermal stimulation, our conclusion is: the probability of stimulating radiation of vibrators in thermal equilibrium radiation field is directly proportional to their state of energy levels. The radiation stimulating probability of every specific vibrator is directly proportional to its state of energy level. For large numbers of vibrators, their average radiation stimulating probabilities are directly proportional to their average energy.

3. Law of Vibrators' Absorbing Radiation

We've already studied the law of vibrators' stimulating radiation. Now on the basis of Boltzmann's energy distribution law and the law of vibrators' stimulating radiation, we'll analyze the law of vibrators' absorbing radiation. The energy and frequency distribution of vibrators is shown in Table 1[4]. In Table 1, the verticals are the Boltzmann distribution of vibrators' energy and horizontals are the Planck distribution of vibrators' frequencies. N_i and n_i are respectively corresponding to the number of vibrators. The horizontal lines in the middle of the table stand for energy levels.

Table 1. *Energy Level and Frequency Distribution of Vibrators.*

Boltamann distribution									
energy	**Distri-bution**	n_0	n_1	n_2	**...**	n_i	**...**	n_m	
nε+dε	$B(\varepsilon_n)$	*	* * * *	* * *	* *	*	*		N*n*
......	......	*	* * * *	* * *	* *	*			...
iε+dε	$B(\varepsilon_i)$	*	* * * *	* * *	* *	*	*	*	N_i
......	......	*	* * * *	* * *	* *	*			...
2ε+dε	$B(\varepsilon_2)$	*	* * * *	* * *	* *	*	*		N_2
1ε+dε	$B(\varepsilon_1)$	*	* * * *	* * *	* *	*		*	N_1
0ε+dε	$B(\varepsilon_0)$	*	* * * *	* * *	* *	*	*		N_0
		v_0+dv	v_1+dv	v_2+dv	...	v_i+dv	...	v_m+dv	Froquency
		$P(v_0)$	$P(v_1)$	$P(v_2)$	...	$P(v_i)$	...	$P(v_m)$	Planck distribution

In Table 1 $B(\varepsilon, T)$ stands for normalized Boltzmann energy distribution coefficient. And $P(v, T)$ represent the Planck frequency distribution coefficient.

A. Einstein divided the energy exchange of thermal radiation into 3 different processes: spontaneous radiation, stimulated radiation and stimulated absorption radiation. Spontaneous radiation refers to the radiation of spontaneous transition with no relation between vibrators and the radiation field. At this moment the photons stimulated by vibrators maintain randomness in a state of polarization, in phase position or in direction of propagation. Stimulated radiation refers to photons sent out when vibrators are stimulated by the sent-in photons in the radiation field. It keeps consistent with the frequency, phase position, polarization state and propagation direction of sent-in photons. [5] The radiation field in the cavity is the result of the two kinds of radiations.

The process of absorption is just opposite. Vibrators can only resonantly absorb photons of same frequency. Since the density of state in cavity is independent and there is no degeneracy between photons, vibrators can only absorb same-frequency photons one by one. This is the essential characteristic of vibrators' absorbing photons. [6]

The energy levels of harmonic vibrators are characteristic of equal intervals. If vibrators with v_i frequency and n energy level transit to the state of m energy level, they stride over $(n\text{-}m)$ energy levels and stimulate $(n\text{-}m)$ photons of v_i frequency, instead of one photon of $(n\text{-}m)v_i$ products. This is different from the hop of atom energy levels.[7] In the table k_i $(i=1,2,3,\ldots)$ indicates the number of energy levels of vibrators of different frequencies in the same energy division. The low-frequency vibrator k is great in numerical value and dense in energy levels. The high-frequency vibrator k is small in numerical value and the intervals are big between energy levels.

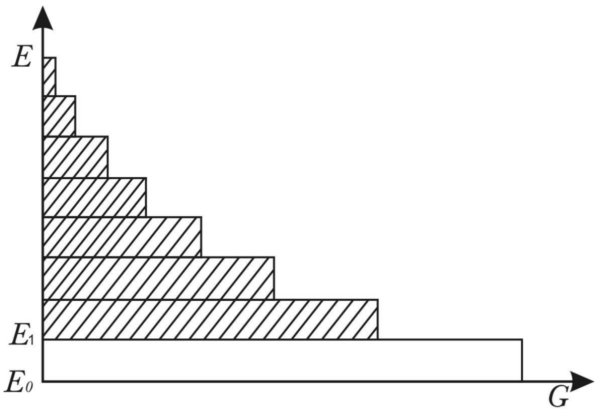
To find out the law of how vibrators absorb radiation, let's consider such a state of extremity. Suppose the matter vibrators in the cavity send out, in a period of time, all the photons of corresponding frequencies which they carry and all return to the ground state. Then we let the cavity absorb photons in the former environment so as to enable energy to redistribute from the ground state and ultimately reach the former state of equilibrium. Even so, according to Boltzmann's law of energy distribution, vibrators will choose the former state of distribution.

According to quantum mechanics, vibrators of ground state have the minimum energy E_0 $(E_0=hv/2)$. At this moment they do not stimulate photons. In accordance with the supposition that vibrators send out all photons of various frequencies they carry, the probability on ground state at this moment is $1 (G_0=1)$. If the probability of vibrators absorbing at least one photon is G_1, then

$$\begin{aligned} G_1 &= 1 - De^{-0hv/kT}\Delta\varepsilon = 1 - D\Delta\varepsilon \\ &= 1-(1-e^{-hv/kT}) \\ &= e^{-hv/kT} \end{aligned} \tag{11}$$

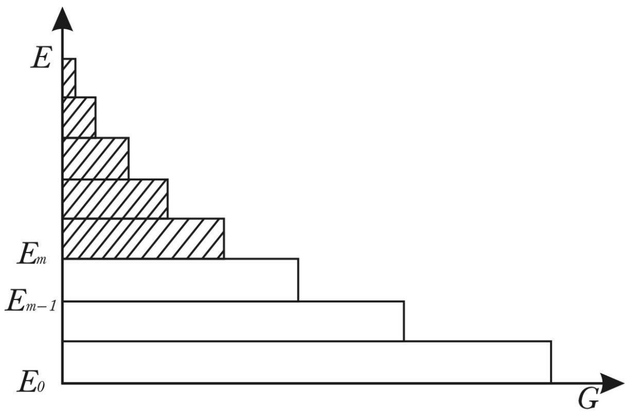
In the formula, D stands for normalized factor. It is the same as Formula (2).

Formula (11) means the probability that vibrators absorb at least one photon, i.e. the probability of vibrators not being in the ground state. The idea is self-evident. To absorb one photon is absorption, so is to absorb *i* number of photons. According to the characteristics of vibrators' absorbing photons, to absorb *i* number of photons is always a re-absorption after absorbing one. Therefore, all vibrators whose energy levels are above ground state are within this probability. So the probability of ground-state vibrators' absorbing radiation is given by Formula (11). The probability of how vibrators absorb photons is shown in Fig. 1.



(a)

The probability of the oscillator absorbing at least one photon, i.e. the distribution probability of the oscillator in the non-ground state, as is shown in the shadow of Fig. (a)



(b)

The photon-absorption probability of the oscillator of Em-1 energy level, i.e. the distribution probability of the oscillator whose energy level is great than Em-1.

Fig. 1. *Sketch of How Vibrators Absorb Photons.*

Likewise, the probability of oscillators on the E_1 state absorbing radiation is

$$\begin{aligned} G_2 &= 1 - D(1 + e^{-hv/kT})\Delta\varepsilon \\ &= 1 - (1 + e^{-hv/kT})(1 - e^{-hv/kT}) \\ &= e^{-2hv/kT} \end{aligned} \tag{12}$$

Formula (12) means the probability of vibrators' absorbing at least two photons. Therefore, any vibrator whose energy level is above E_1 belongs to this category. Likewise, we call G_2 the probability of the vibrators' absorbing radiation whose energy state is E_1. Generally, the radiation absorbing probability of the vibrators whose energy level is E_{m-1} is

$$\begin{aligned} G_m &= 1 - D\sum_{i=0}^{i=m-1} e^{-ihv/kT}\Delta\varepsilon = 1 - \frac{\sum_{i=0}^{i=m-1} e^{-ihv/kT}\Delta\varepsilon}{\sum_{i=0}^{i=m-1} e^{-ihv/kT}\Delta\varepsilon} \\ &= 1 - \left(\frac{1-e^{-mhv/kT}}{1-e^{-hv/kT}}\right) \Big/ \left(\frac{1-e^{-(n+1)hv/kT}}{1-e^{-hv/kT}}\right) \\ &= \frac{e^{-mhv/kT} - e^{-(n+1)hv/kT}}{1-e^{-(n+1)hv/kT}} \end{aligned} \tag{13}$$

In the formula, *n, n+1* is the highest energy level of vibrators. Please notice, when $n\rightarrow\infty$, $e^{-(n+1)hv/kt}\rightarrow 0$. Therefore, Formula (13) may be expressed as

$$G_m = e^{-mhv/kT} \tag{14}$$

Formula (14) is true with vibrators of various frequencies. It indicates that the radiation absorbing probability of vibrators is directly proportional to their energy distribution probability. Also because

$$\begin{aligned} &G_0 + G_1 + G_2 + \cdots + G_m + \cdots + G_n + \cdots \\ &= 1 + e^{-hv/kT} + e^{-2hv/kT} + \cdots + e^{-mhv/kT} + \cdots + e^{-nhv/kT} + \cdots \\ &= \sum_{m=0}^{m=\infty} e^{-mhv/kT} \end{aligned} \tag{15}$$

the normalized factor remains to be *D*. After normalization, the radiation absorbing probability of vibrators whose energy level is E_{m-1} is $De^{-mhv/kT}$. However, vibrators with E_{m-1} energy level are those which have absorbed *(m-1)* number of photons. Therefore, the radiation absorbing probability of vibrators with E_{m-1} energy level is exactly the probability of vibrators which have absorbed *m* number of photons. So the average probability of vibrators' absorbing radiation may be written as

$$\overline{G} = \frac{\sum_{m=1}^{m=\infty} me^{-mhv/kT}}{\sum_{m=1}^{m=\infty} e^{-mhv/kT}} = \frac{1}{e^{hv/kT}-1} \tag{16}$$

Formula (16) is Formula (9), referring to the average radiation absorbing probability of vibrators. Likewise, the probability of radiation absorption of vibrators is also related to the density of state of the same frequency in the cavity. Multiply the average absorbing probability with the density of state, we get the number of vibrators' absorption of radiation. We know $\varepsilon = hv$. Multiply it with *hv*, we get the Planck formula of black-body radiation. Now this formula is obtained through the opposite process of absorption. In this way we can explain in theory the law of energy exchange during thermal equilibrium radiation.

4. Fundamental Laws of Thermal Radiation

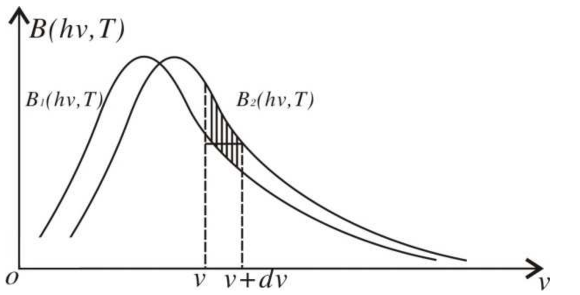
Boltzmann's law of energy distribution may be derived on the basis of laws of vibrators' stimulating and absorbing radiation. We start with the investigation into vibrators' stimulating radiation. *B(ε, T)* still stands for the energy distribution coefficient. The probability of vibrators' stimulating radiation is directly proportional to their state energy levels. In the *v→v+dv* area, the probability relationship of vibrators' stimulating radiation is

$$p_i(v,T)\mathrm{d}v \propto iB(hv,T)\mathrm{d}v \tag{17}$$

Multiply Formula (17) with *C* so as to write it into an equation. Then

$$p_i(v,T) = CiB(hv,t)\mathrm{d}v \tag{18}$$

However, we also know that the *v* frequency vibrators and *hv* energy vibrators correspond with each other. They show the same distribution. When vibrators stimulate radiation, their energy distribution inevitably changes. We can infer that $p_i(v, T)$d*v* is exactly the full differential of energy distribution coefficient *B(ε, T)* at *ÄT→0*.



The shadow part of the figure shows the change of energy *D(B(hv, T))* within *Ät* time.

Fig. 2. *Sketch to Show the Relationship between Energy Distribution and Frequency.*

The relationship between vibrators' energy spectrum and frequency is shown in Fig.2. In the formula $B_1(hv, T)$ is the spectrum line at *t* and $B_2(hv, T)$ is the spectrum line at *t+Ät*. Because *Ät* is sufficiently small, $B_2(hv, T)$ may be deemed as a tiny parallel shift of $B_1(hv, T)$. Radiation reduces the energy of matter, the spectrum lines move to lower frequency. The shadow part of the figure shows the change of energy *D(B(hv, T))* within *Ät* time. Given that energy only exchanges by way of radiation, when *ÄT→0* and further *ÄT→0*, the change is expressed by *B(hv, T)* versus *v* differential. Then

$$\begin{aligned} p_i(v,T)dv &= \lim_{\Delta t\to 0} D(B(hv,T)) = \frac{\partial B(hv,T)}{\partial v}dv + \lim_{\Delta t\to 0}\frac{\partial B(hv,T)}{\partial T}\mathrm{d}T \\ &= \mathrm{D}_r B(hv,T) \end{aligned} \tag{19}$$

Noticing that vibrators' stimulating radiation reduces the energy, hence the following relational expression

$$p_i(v,T) = -\mathrm{D}B_i(hv,T) = CiB_i(hv,T)\mathrm{d}v \tag{20}$$

After re-organization, the formula becomes

$$\frac{DB_i(hv.T)}{B_i(hv,T)} = -Ci\mathrm{d}v \tag{21}$$

The integrals of both sides of Formula (21) result in

$$\mathrm{In}B_i(hv,T) = -Civ \tag{22}$$

So

$$B_i(hv,T) = De^{-civ} \tag{23}$$

D stands for normalized constant. Formula(23) already has the form of Boltzmann energy distribution function. Coefficient *C* awaits further analysis.

The energy distribution function is the result of both vibrators' stimulation and absorption of radiation. Therefore, we still have to analyze the influence of vibrators' radiation absorption on the distribution function. From the level of vibrators' absorbing radiation, similar conclusion comes about. We use $J_{i-1}(v, T)$ to represent the probability of vibrators with E_{i-1} energy level absorbing photons. Now we know that the probability of vibrators (E_{i-1} energy level) absorbing photons is directly proportional to $B_i(e, T)$. We also know that the probability of vibrators (E_{i-1} energy level) absorbing radiation is exactly the probability of absorbing *i* number of photons. So in the *dv* area, the probability relationship of vibrators' absorbing radiation is

$$J_{i-1}(v,T)\mathrm{d}v = CiG_i\mathrm{d}v = CiB_i(hv,T)\mathrm{d}v \tag{24}$$

The absorption of radiation by vibrators may also change the original distribution function, which is equal to looking at the relationship shown in Fig.2 the other way round. The vibrators' absorption of radiation increases energy, so

$$J_{i-1}(v,\mathrm{T}) = \mathrm{D}B(hv,T) = CiB_i(hv,T) \tag{25}$$

Compared with Formula (20), one negative symbol is missing.

The functions of average probabilities of vibrators' stimulating and absorbing radiation are the same. The average probability is the function of frequency. The frequency changes in the *dv* area, which inevitably influences the average probability. In return the change of average probability also influences the vibrators' absorption radiation. Therefore, we have to investigate the change ratio of frequency by average probability. So we do differential on Formula(9).

$$\frac{\mathrm{d}}{\mathrm{d}v}\left(\frac{1}{e^{hv/kT}-1}\right) = -\frac{\frac{h}{kT}e^{hv/kT}}{(e^{hv/kT}-1)^2} \tag{26}$$

The derivative of average absorption ratio is negative, which means the lowering of absorption ratio as frequency rises. This conforms to fact. To find the change ratio of absorption, we approximate Formula (26) in normal temperature. In normal temperature T=300K, after Planck constant and Boltzmann constant are put in

$$\frac{hv}{kT}=\frac{6.626\times10^{-34}v}{1.38\times10^{-23}\times3\times10^{2}}=1.36\times10^{-13}v \qquad (27)$$

In normal temperature, thermal energy is concentrated in the infra-red area in which the characteristic frequencies are higher than 10^{13}, $hv/kT>1$, $e^{hv/kT}>>1$. In high temperature, e.g. T=3000K, the thermal energy moves towards the violet end and the characteristic frequencies are higher than 10^{14}, $e^{hv/kT}>>1$.[8] So Formula (26) may be expressed as

$$\frac{\mathrm{d}}{\mathrm{d}v}(1/e^{hv/kT}-1)=-\frac{h}{kT}\frac{e^{hv/kT}-1}{(e^{hv/kT}-1)^2}=-\frac{h}{kT}\frac{1}{e^{hv/kT}-1} \qquad (28)$$

i.e.

$$\frac{\mathrm{d}(1/e^{hv/kT}-1)}{1/(e^{hv/kT}-1)}=-\frac{h}{kT}\mathrm{d}v \qquad (29)$$

Formula(29) means that the change of frequency by average absorption ratio is directly proportional to *(-h/kT)*. So, when frequency changes in the *dv* area, the probability of vibrators' absorbing radiation is directly proportional to *(-h/kT)*. Here we replace *C* in Formula(25) with *(-h/kT)*.

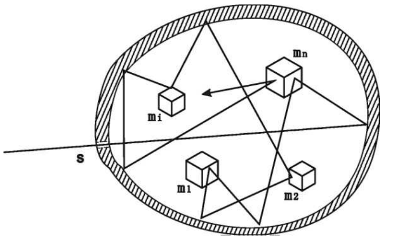
$$\frac{DB(hv,T)}{B(hv,T)}=-\frac{ih}{kT}\mathrm{d}v$$

After integral

$$\mathrm{In}B_i\left(hv,T\right)=-\frac{ihv}{kT},\quad B_i(hv,T)=De^{-ihv/kT} \qquad (30)$$

Formula (30) is Boltzmann distribution function of vibrators' energy levels. But we know that Planck formula is also an experiment law. If we don't regard vibrators' average energy and the average probability as the theoretic result of Boltzmann's energy distribution law, only as an experiment law, we may claim that Boltzmann's energy distribution law is the result of energy exchange law in the radiation field.

5. Analysis of Thermal Equilibrium Radiation Process



The $e_v(v, T)$ in the cavity is determined by the average $r_v(v, T)$ of *n* bodies.

Fig. 3. *Thermal Equilibrium Radiation of Heat-insulated carity.*

We conduct an analysis of thermal radiation process on the basis of the fundamental law of thermal radiation. An isolated material system is bound to develop into a thermal equilibrium system. If the energy change of the system is effected only by way of radiation, it is a system of thermal equilibrium radiation. Again let's take the special case of cavity radiation as an example. Suppose the cavity is heat insulated and there are *n* kinds of bodies including the cavity which are $m_1, m_2, \ldots, m_i, \ldots, m_n$. The temperatures are $T_1, T_2, \ldots, T_i, \ldots, T_n$ and the monochromatic radiation degrees are $r_1(v, T_1), r_2(v, T_2), \ldots, r_i(v, T_i), \ldots, r_n(v, T_n)$. The holes of the cavity are meant to measure the cavity's radiation energy. The heat-insulated cavity is an isolated system, as is shown in Fig.(3).Suppose the body temperatures of the cavity bear the following relationship

$$T_1>T_2>\cdots>T_i>\cdots>T_n \qquad (31)$$

The stimulation and absorption of radiation by a body is related to its structure, especially to its surface conditions (color, cleanness and brightness). Different bodies have different monochromatic radiation degrees $r_i(v, T)$ and different monochromatic irradiation degrees $e_i(v, T)$. But on the other hand the $r_i(v, T)$ of the same body is closely related to its $e_i(v, T)$. C.R. Kirchhoff law tells us

$$\frac{r_1(v,T)}{\alpha_1(v,T)}=\frac{r_2(v,T)}{\alpha_2(v,T)}=\cdots=\frac{r_i(v,T)}{\alpha_i(v,T)}=\frac{r(v,T)}{1}=e(v,T) \quad (32)$$

In the formula $á_i(v, T)$ stands for the absorption coefficient of the body; $r(v, T)$ and $e(v, T)$ respectively represent the monochromatic radiation degrees and the irradiation degrees of the absolute black-body; its absorption coefficient is $á(v, T)=1$. Formula(32) is true with an temperature *T*. It is an expression of the body's features and has nothing to do with radiation itself.[9]

In Formula(32) the monochromatic radiation degrees of different bodies are simplified into the monochromatic radiation degrees of black-bodies. Noticing $r(v,T)=cu(v,T)/4$, [10]then

$$\begin{aligned} r_i(v,T)&=\alpha_i r(v,T)=\frac{c}{4}\alpha_i u(v,T)\\ &=\frac{c}{4}\alpha_i\frac{8\pi v^2}{c^3}\frac{hv}{e^{hv/kT}-1}\\ &=\frac{2\pi\alpha_i v^2}{c^2}\frac{hv}{e^{hv/kT}-1} \qquad (i=1,2,\cdots,n) \end{aligned} \qquad (33)$$

Formula (33) is a general formula of monochromatic radiation degrees. It may be predicted that given long enough time, this isolated system tends to be thermal equilibrated. Suppose its equilibrium temperature is T_a. At this moment the monochromatic radiation degrees of the cavity is

$$r(v,T_a)=\frac{c}{4}u(v,T_a)=\frac{2\pi v^2}{c^2}\frac{hv}{e^{hv/kT_a}-1} \qquad (34)$$

The probability of vibrators' absorbing radiation is directly proportional to that of vibrators' energy distribution. This relationship is true with any heat-exchange process, regardless of the strength of radiation on the surface. The probability of radiation absorption by vibrators of low energy is always higher than those of high energy level. But obviously, how many photons are absorbed by a body is directly proportional to the number of photons transmitted to the surface of the body, i.e. to the monochromatic radiation degrees $e_v(v, T)$. The $e_v(v, T)$ in the cavity is determined by the average $r_v(v, T)$ of n bodies. For this purpose we calculate the average degrees of the bodies in the cavity. Suppose the number of vibrators of body m_i and frequency v is N_i and the absorbing coefficient is $á_i$. The probability of stimulating radiation is of the same effect with $á_iN_v$ black-body vibrators. So the average probability of stimulating radiation by body m_v is

$$F_i(v,T_i) = \alpha_i N_i f_i(v,T_i) = \alpha_i N_i \frac{1}{e^{hv/kT_i}-1} \qquad (i=1,2,\cdots,n) \tag{35}$$

The total probability of stimulating radiation by n bodies is

$$F(v,T_i) = \sum_{i=1}^{i=n} \alpha_i N_i \frac{1}{e^{hv/kT_i}-1} \tag{36}$$

The average probability of stimulating radiation by n bodies is

$$\overline{F}(v,T_i) = \frac{\sum_{i=1}^{i=n} \alpha_i N_i (1/e^{hv/kT_i}-1)}{\sum_{i=1}^{i=n} \alpha_i N_i} \tag{37}$$

For the purpose of brevity, only two body systems are investigated. Because $T_1 > T_2$, $f_1(v, T_1) > f_1(v, T_2)$. Let's agree that $f_2(v,T_2) = f_1(v,T_1) - a$, and a is a real number smaller than $f_1(v, T_1)$. So the average probability of stimulating radiation of the system is

$$\overline{f}(v,T) = \frac{\alpha_1 N_1 f_1(v,T_1) + \alpha_2 N_2 f_2(v,T_2)}{\alpha_1 N_1 + \alpha_2 N_2} \tag{38}$$

Because

$$\begin{aligned}\overline{f}(v,T) &= \frac{\alpha_1 N_1}{\alpha_1 N_1 + \alpha_2 N_2} f_1(v,T_1) + \frac{\alpha_2 N_2}{\alpha_1 N_1 + \alpha_2 N_2} f_2(v,T_2) \\ &= \frac{\alpha_1 N_1}{\alpha_1 N_1 + \alpha_2 N_2} f_1(v,T_1) + \frac{\alpha_2 N_2}{\alpha_1 N_1 + \alpha_2 N_2} \left(f_1(v,T_1) - a\right) \\ &= f_1(v,T_1) - \frac{\alpha_2 N_2 a}{\alpha_1 N_1 + \alpha_2 N_2} > f_2(v,T_2)\end{aligned} \tag{39}$$

At the same time

$$\begin{aligned}\overline{f}(v,T) &= \frac{\alpha_1 N_1}{\alpha_1 N_1 + \alpha_2 N_2} f_1(v,T_1) + \frac{\alpha_2 N_2}{\alpha_1 N_1 + \alpha_2 N_2} f_2(v,T_2) \\ &= \frac{\alpha_1 N_1}{\alpha_1 N_1 + \alpha_2 N_2} \left(f_2(v,T_1) + a\right) + \frac{\alpha_2 N_2}{\alpha_1 N_1 + \alpha_2 N_2} f_2(v,T_2) \\ &= \frac{\alpha_1 N_1 a}{\alpha_1 N_1 + \alpha_2 N_2} + f_2(v,T_2) < f_1(v,T_1)\end{aligned} \tag{40}$$

From Formula (39) and (40) we know

$$f_1(v,T_1) > \overline{f}(v,T) > f_2(v,T_2) \tag{41}$$

Noticing the characteristic that the radiation probability keeps changing with the temperature, we believe there must exist a temperature T_a in $[T_1, T_2]$ degree area. Satisfy

$$\overline{f}(v,T) = f(v,T_a) = \frac{1}{e^{hv/kT_a}-1} \tag{42}$$

So that

$$\left(\frac{\alpha_1 N_1}{e^{hv/kT_1}-1} + \frac{\alpha_2 N_2}{e^{hv/kT_2}-1}\right) = \frac{\alpha_1 N_1 + \alpha_2 N_2}{e^{hv/kT_a}-1} \tag{43}$$

This result may be applied to a system with many bodies. For the system shown in Formula (31), there is

$$\begin{gathered} f_1(v,T_1) > f(v,T_a) > f_n(v,T_n), \\ \sum_{i=1}^{i=n} \alpha_i N_i \left(\frac{1}{e^{hv/kT_i}-1}\right) = \sum_{i=1}^{i=n} \frac{\alpha_i N_i}{e^{hv/kT_a}-1} \end{gathered} \tag{44}$$

Since $f(v, T_a)$ is the average probability of stimulating radiation by bodies in the cavity, the monochromatic radiation degree in the cavity is

$$e(v,T_a) = \frac{c}{4}\frac{8\pi v^2}{c^3}\frac{hv}{e^{hv/kT_a}-1} = \frac{2\pi v^2}{c^8}\frac{hv}{e^{hv/kT_a}-1} \tag{45}$$

Noticing that the radiation degree of any body may be equally expressed by the radiation degree of the black-body and that the radiation absorbing ability of a body is directly proportional to $á_i$, we think it provides a basis for judgment. Compared with Formula(33), at any time for any body whose T_i is bigger than T_a, its radiation degree is bigger than its radiation illuminance; its radiation energy is bigger than its absorbing energy; the body temperature drops. For any body whose T_i is smaller than T_a, its radiation degree is smaller than its radiation illuminance; its absorbing energy is bigger than its radiation energy; the body temperature rises. If we continuously measure the temperatures of m_1 and m_n, the T_1 shows a continuous decline sequence while the T_n shows a rising series. Besides,

$$T_1 > \cdots T_{1s} > T_a > T_{ns} > \cdots > T_n \tag{46}$$

According to the principle of nested intervals in mathematics, the system is bound to become equilibrated at a certain temperature, say T_a, in the temperature area.

At the first glance, the equilibrated temperature is a process quantity which changes with the process of equilibrium. Further analysis proves it is not true. If the number of bodies in the cavity is great enough and the temperature difference is big enough, the temperature relationship shows the following change if the variation dT_a of the equilibrium temperature is bigger than T_a:

Moment t $T_1 > T_2 > \cdots > T_i > T_a > T_{i+1} > \cdots > T_n$,

Moment t+dt $T_1 > T_2 > \cdots > T_a > T_i > T_{i+1} > \cdots > T_n$.

From the above temperature relationship, we see that if T_a is a process quantity, the body m_i may probably release heat a *t* time and absorb heat at *t+dt* time. But the fundamental of thermal radiation tells us that this phenomenon won't take place spontaneously.

This is because, if any body m_i in the radiation field changes from releasing heat to absorbing heat, it inevitably goes past the equilibrium point of heat change $dQ=0$, at which the energy radiated by the body *(á$_i$r(v, T$_i$) ds dt)* equals its absorbed energy *(á$_i$e(v, T$_a$) ds dt)*. When the body shifts from releasing heat to absorbing heat, it indicates that the relationship between radiation degree and radiation illuminance undergoes a reverse. However, the radiation degree is determined by the energy distribution of m_i vibrator and the radiation illuminance depends on the system. The influence of the radiation degree of m_i body is insignificant. Relative to the radiation illuminance in cavity, if the radiation degree of m_i reverses, it must be a spontaneous act, which is obviously impossible.

If body m_i changes from absorbing heat to releasing heat, it likewise goes past the balance point of $dQ=0$. The change shows the change of radiation degree from being smaller to being bigger than the radiation illuminance. But the illuminance depends on the system, so the strength of any body in the cavity in absorbing radiation cannot possibly be greater than the illuminance. Therefore, for any body whose radiation degree is smaller than illuminance, the reverse phenomenon cannot possible take place.

Our analysis shows that, a certain body will always maintain a balance state if it is in a balance state with the radiation field, unless interfered by outside conditions. This characteristic is very much revealing for us to understand the characteristics of thermal equilibrium radiation. As is stated above, if the number *n* of the body samples in the cavity is big enough and the temperature difference dT is sufficiently small, if the balance temperature T_a of the system is located between T_i and T_{i+1}, no variation of absorbing and releasing heat of the bodies will take place in the equilibrium process. As time goes on, the $[T_i, T_{i+1}]$ area narrows down, which indicates that the equilibrium temperature *Ta* of the system is the immobile point of the $[T_i, T_{i+1}]$ area, and also the immobile point of the $[T_1, T_n]$ area. Therefore, we arrive at the following conclusion about the characteristics of thermal equilibrium process: the equilibrium temperature T_a of the thermal equilibrium process is only determined by the initial conditions of the system, is characteristic of an immobile point and has nothing to do with the equilibrium process.

This is true microscopically. On the non-equilibrium point the average energy of vibrators and the energy of the radiation field are in a state of non-balance, so that the energy transfer is maintained. On the equilibrium point the average energy of vibrators and the energy of the radiation field are in a state of balance and any subtle change in the distribution state of vibrators' energy is resisted by the balance state. The shift of a body from the balance state to the state of stimulation bigger than absorption implies that the energy distribution of vibrators hops towards higher energy level. And immediately after, the radiation energy increases, so that it returns to the low level. The shift of a body from the balance state to the state of absorption bigger than stimulation implies that the energy distribution of vibrators hops towards lower energy level. The result is inevitably the increase of absorbed energy, so that it returns to the high level. Here, the fundamental law of thermal radiation maintains the balance of the radiation field and prevents the system state from develop to the opposite direction. Therefore, we claim that the irreversibility of thermal equilibrium radiation is a form of expression of the fundamental law of thermal radiation.

The conditions for thermal equilibrium radiation may be derived from the average probability. The average probabilities of vibrators' stimulating and absorbing radiation can both be expressed by Formula(9). The two equal each other in the state of thermal equilibrium. At this moment the changing possibility of average probabilities tends to be zero, i.e. *Df(v, T)→0*.

$$\begin{aligned}\mathrm{D}f(v,T) &= \mathrm{D}\left(1/(e^{hv/kT}-1)\right)\\ &= \frac{\partial}{\partial v}\left(1/(e^{hv/kT}-1)\right)\mathrm{d}v + \frac{\partial}{\partial T}\left(1/(e^{hv/kT}-1)\right)\mathrm{d}T\\ &= \frac{-h\mathrm{d}v}{kT}\frac{e^{hv/kT}}{\left(e^{hv/kT}-1\right)^2} + \frac{hv\mathrm{d}T}{kT^2}\frac{e^{hv/kT}}{\left(e^{hv/kT}-1\right)^2}\end{aligned} \tag{47}$$

Let Df(v, T)=0, so

$$\frac{h\mathrm{d}v}{kT} = \frac{hv\mathrm{d}T}{kT^2} \tag{48}$$

After re-organization

$$\frac{\mathrm{d}v}{v} = \frac{\mathrm{d}T}{T} \tag{49}$$

Formula (49) shows the basic conditions for thermal equilibrium radiation.

We have conducted an investigation into the thermal equilibrium process of the isolated system in the special form of empty cavity radiation. But the material systems are universally associated and the absolute isolated system is rare. Real and concrete thermal equilibrium processes have their concrete complexity, but this does not hinder our understanding of the natural characteristics of thermal equilibrium process.

6. Conclusion

A complete fundamental law of thermal radiation should contain two levels of energy distribution and exchange. Our conclusion on the fundamental law of thermal equilibrium radiation is: the energy distribution of matter vibrators in the thermal equilibrium radiation field follows Boltzmann's law of energy distribution; the probability of vibrators' stimulating radiation is directly proportional to their state of energy levels and the probability of vibrators' absorbing radiation is directly proportional to the probability of their energy distribution. Boltzmann's law of energy distribution and the law of vibrators' stimulating and absorbing radiation serve as causality condition for each other. The former reveals the law of vibrators' energy distribution and is characteristic of static, while the latter reveals the law of energy exchange between vibrators and the radiation field and is characteristic of dynamic. Both constitute the basic laws of thermal equilibrium radiation. The above conclusion is deduced from thermal equilibrium radiation and is also true with non-equilibrium radiation.

As for the process of thermal equilibrium radiation, our conclusion is that the thermal equilibrium temperature in the isolated system is determined only be the initial conditions of the system and has nothing to do with the equilibrium process. The irreversibility of thermal radiation is the expressing form of the fundamental law of thermal radiation.

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