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# The Effect of Strong Magnetic Field on Unsteady MHD Nanofluid Flow Through Convergent-Divergent Channel with Heat and Mass Transfer

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**Abstract:** Magnetohydrodynamic Nanofluid flow (Silver-water) through a converging-diverging channel under a strong magnetic field has been investigated. The induction equation is derived from electromagnetism. The non-linear partial differential equations are reduced to first-order non-linear ordinary differential equations using the similarity transformation and dimensionless numbers. The implicit Runge - Kutta fourth-order method via the bvp4c function in MATLAB has been used to generate the graphs of the fluid flow. It was observed that the high value of the Schmidt number leads to an increase in the velocity of the Nanofluid flow. The variation of the Schmidt number leads to a decrease in the temperature profile of the Nanofluid flow in the stretching channel and leads to an increase in the shrinking channel. The higher value of the Schmidt number leads to higher values in the concentration of the Nanofluid flow. Increasing the values of the Schmidt number leads to an augment in the magnetic induction of the Nanofluid flow for the divergent channel and a decrease is observed for a case of the convergent channel. Variation of the nanoparticle volume fraction increases the magnetic induction profiles of the Nanofluid flow for a stretching channel, and a decrease is observed for the case of the shrinking channel. The high value in Reynolds magnetic number leads to a high value in the velocity profile of Nanofluid flow. The change in Reynolds magnetic number leads to a high value in the temperature profiles of the Nanofluid flow for the case of a divergent channel and a decrease is observed for the case of a convergent channel. Varying the Reynolds magnetic number leads to a decrease in the magnetic induction profiles of the Nanofluid, this is due to the effectiveness of the relationship between the fluid flow and the magnetic field. Varying the Reynolds magnetic number leads to an augment in the induction profiles of the Nanofluid. This kind of study has a variety of applications such as geophysics, astrophysics, fire engineering, bio-medical, and blood flow through arteries and capillaries in the human body.

**Keywords:** Induction Equation, Electromagnetic Equations, Unsteadiness, MHD Nanofluid Flow, Divergent-Convergent Channel

## 1. Introduction

Magnetic induction is derived from the electromagnetic equations. These are Maxwell's equations that include:

Faraday's Law says that the induced electromotive force in a conductor proportionally varies to the rate of change of magnetic flux through the conductor. Electromagnetic induction is the process by which a non-uniform magnetism produces an electromotive force. It was observed that the more rapidly the magnetic strength changes, the greater the induced e. m. f., and the magnetic force of the induced e. m. f also depends on the area of the loop of wire (coil) through which the magnetic lines travel.

Ampere's law indicates the source equation of magnetism. This law means that the magnetic lines swirl around electric the current and charge electric flux.

Gauss law states that all magnetic lines must be closed loops.

For high conductivity, Ohm's law determines the finite current density. These equations show the relationship between the fluid laws and magnetism.

Ohm's law is combined with Maxwell's equations to form the magnetic induction.

Lorentz Force is derived from the electromagnetism force. This force opposes the direction of the velocity of the fluid flow and brings the flow retardation for the fluid velocity.

There are numerous studies that have investigated MHD flows and the magnetic field in the connection with MHD Nanofluid flows. These include the following:

Faraday [1] did an experiment with mercury using glass tubes, he placed them between magnet poles and made some observations. He discovered a voltage was generated by the motion of mercury across the magnetism. He observed that the relationship between the current and magnetism retards the velocity of the flow. Ritchie [2] did an investigation on MHD flow and observed that when electricity is applied perpendicular to magnetism and it resulted in the fluid in a direction perpendicular to both fields. Smith [3] did a study on MHD and discovered the means by which he used to measure the speed of a ship. William [4] studied the MHD flow meter and discovered that the induced voltage is proportional to the flow. Young [5] investigated MHD flow and discovered a technology that is used in oceanography.

Alfven [6] investigated MHD flow and found the ways in which the Magnetohydrodynamic waves propagate between fluid and magnetism. Important MHD effects were demonstrated long time ago, most of the practical work in this field has been done 1950s because of the interest in high temperature, gases, nuclear engineering and space technology. Among the applications of MHD theory are meter for liquid-metal heat transfer system employed in nuclear reactors. Soundalgekar [7] studied MHD flow and Laplace Transformed method was used to solve the differential equations governing the flow. It was discovered that varying the values of the magnetic field leads to an augment in the velocity in heated plate, and a decrease was observed in the velocity for the case of cooled plate. Cooling the temperature leads to an augment in the velocity of the fluid. Raptis [8] used Laplace Transform to examine accelerated vertical plate and observed that the effect of magnetic parameter leads to a decrease in the skin friction. The higher values of the magnetic forces lead to lower values of the velocity field.

Helmy [9] used perturbation method to examine MHD free convection. It was observed that velocity profile is affected when varying the magnetic parameter and the temperature distribution is decreased. Thiele [10] did an investigation on MHD flow using viscosity variable and velocity and magnetic profiles were examined. It was observed that the higher values of viscosity lead to lower values of velocity. Ganesh [11] investigated MHD stokes flow of unsteady viscous fluid between two parallel porous plates and it was observed that the magnetic inclination suction/injection rates influence the velocity. Varying the pressure gradient and Hartmann leads to a decrease in the velocity. It was discovered that the velocity is influenced by the magnetic inclination suction/injection rates, pressure gradient, Hartmann. Singh [12] investigated hydromagnetism and used Laplace Transform method. It was found that the velocity and temperature profiles are affected by the variation in the magnetic parameter. Mburu [13] did a study on hydromagnetism flow through parallel plates and it was discovered that variation in the Hartmann leads to a decrease of the velocity distribution, decreasing the angle of inclination leads to a high value of velocity distribution. Varying the angle of inclination leads to a decrease in the velocity of the fluid. It was discovered that higher value of pressure gradient leads to higher value of velocity and an augment in Reynolds number leads to a decrease in velocity and velocity of the fluid is augmented when Reynolds number is decreased.

Manyonge [14] investigated hydromagnetism flow through parallel plates and analytical method was used to solve the differential equations. It was observed that the velocity and temperature are affected by the variation in the magnetic field. Raiham [15] investigated hydromagnetic flow through vertical plate. It was found that varying magnetic field leads to a decrease in the velocity of the fluid. It was discovered that stronger magnetic field, lower the velocity of the fluid. Temperature for cooled plate is augmented and decreased for heated plate when varying the magnetic field. Lakshmi [16] examined hydromagnetic flow involving the heat and mass transfer and Runge-Kutta fourth order method was used to solve the differential equations. It was found that varying Prandtl number leads to a decrease in the velocity profile. The velocity with the boundary layer, thickness of the boundary layer are augmented when varying the radiation parameter and heat source. Bhattachryya [17] made an investigation on hydromagnetic Nanofluid and Brownian motion and thermophoresis were taken into consideration. The differential equations were reduce to ordinary differential equations using similarity transformation and were solved using Fourth order Runge-Kutta method with shooting technique. The magnetic field enhances the temperature and nanoparticle volume fraction and are reduced due to mass transfer. Makinde [18] studied Newtonian fluid and viscous dissipation of Nanofluids and similarity transformation was used to reduce PDEs to ODEs and a Runge Kutta-Fehlberg method with a shooting technique was used to solve the equations numerically. It was found that increasing the value of nanoparticle volume fraction and the Newtonian heating lead to an increase of the transfer rate at the moving plate surface.

Virginia [19] investigated MHD Nanofluid flow. It was observed that variation in the stretching parameter leads to an increase of the velocity of Nanofluid It is observed that the temperature reduces with the increase in the stretching parameter. Varying the stretching parameter leads to an augment of the velocity of Nanofluid which in turn leads to reduction of the thermal boundary layer thickness hence decreased temperature. Heat transfer coefficient increases with the increase in the stretching parameter. An augment in the stretching parameter leads to an augment in the velocity of the Nanofluid. This leads to an augment in the thermal boundary layer, hence increased heat transfer coefficient. Makinde [20] investigated the MHD flow in convergent-divergent channels under the influence of an externally applied homogeneous magnetic field. The results were found using the perturbation technique coupled with a special type of Hermite pade approximation. It was observed that variation in the magnetic Reynolds number leads to a decrease of the fluid velocity around the central region of the channel. The magnetic field strength leads to the flow reversal.

Felicien Habiyaremye et al [21] examined hydromagnetic flow of the nanoparticles via converging-diverging channel. MHD Nanofluid flow through convergent-divergent channel. It was found that varying the nanoparticle volume fraction leads an increase in the velocity of the Nanofluid for both convergent-divergent channel. The higher value of Grashof number leads to higher value in the temperature profile of the Nanofluid for diverging channel and a decrease is observed for converging channel. Akbar [22] did a study on the hydromagnetic flow on Nanofluid motion in an asymmetric channel. Ghosh [23] investigated the influence of the electromagnetic forces on temperature distribution. Beg [24] investigated the influence of magnetic field on the boundary layer flow.

For the best knowledge of the researcher, variable magnetic field on unsteady MHD Nanofluid via convergent-divergent channel is new field to discover. The researcher is interested to discover more in this area of the fluid dynamics. The study is aimed at developing governing equations, determining the influence of the Schmidt, the Reynolds magnetic number on velocity, temperature, concentration and magnetic induction profiles on velocity, temperature and concentration of unsteady MHD Nanofluid flow through convergent-divergent channel.

# 2. MHD Nanofluid Flow Under Variable Magnetic Field

### *2.1. Nomenclature (Abbreviation)*

$\gamma$: Unsteadiness Time Parameter
$R_m$: Reynolds Magnetic Number
$\Psi$: Nanoparticle Volume Fraction
Ag: Silver
$B_0$: Strength of Magnetic Field
$\rho_{nf}$: Density of the Nanofluid
$\rho_f$: Density of the Fluid
$\rho_s$: Density of the solid Nanoparticle
$\mu_{nf}$: Viscosity of Nanofluid
Ha: Hartmann Number
$\vec{J}$: Current Density:
$\mu_f$: Viscosity of the Fluid
$\mu_s$: Viscosity of the solid Nanoparticle
Re: Reynolds Number
$S_c$: Schmidt Number
$\nu_{nf}$: Kinematic Viscosity of Nanofluid
$\sigma$ = Electrical Conductivity
$\omega$ = Non − Dimension Magnetic Induction
e.m.f = Electro Motive Force
MHD: Magneto-hydrodynamics
ODEs: Ordinary Differential Equations
PDEs: Partial Differential Equations
MATLAB: Matrix Laboratory

### *2.2. Mathematical Formulation*

The study focuses on the variable magnetic field on the unsteady MHD Nanofluid via convergent-divergent channel. The Maxwell's equations and Ohm's Law are combined to get the induction equation.

Velocity of the Nanofluid, a magnetic field are given by

$$\vec{q} = \vec{q}(r, \theta, z) \tag{1}$$

$$\vec{b} = \vec{b}\left(b, \frac{B_0}{r}, 0\right) \tag{2}$$

From Figure 1. Relationship between the Nanofluid flow and the magnetic field is observed. The magnetic field is stronger at the walls of stretching/divergent and shrinking/converging and the magnetic field becomes weaker at the mainstream.

Felicien Habiyaremye [21] investigated MHD Nanofluid and have identified the governing equations. The equations governing variable magnetic field on unsteady MHD Nanofluid flow in the cylindrical coordinates are: continuity, momentum, energy and concentration equation in the cylindrical coordinates. In the current study, the induction equation is taken into consideration. The magnetic Induction Equation is derived from the Maxwell's equations which give the relationship among charge density, magnetic field intensity electric field intensity, electric displacement, induction current density vector.

Faraday's Law

$$\frac{\partial \vec{B}}{\partial t} = -\vec{\nabla} \times \vec{E} \tag{3}$$

Ampere's Law

$$\vec{J} = \vec{\nabla} \times \vec{B} \tag{4}$$

Gauss' Law

$$\vec{\nabla} \times \vec{E} = \rho_e \tag{5}$$

Absence of Magnetic Monopoles

$$\vec{\nabla} \times \vec{B} = 0 \tag{6}$$

Ohm's Law

$$\vec{J} = \sigma\left(\vec{E} + \vec{V} \times \vec{B}\right) \tag{7}$$

Ohm's Law and Maxwell's equations are used to derive induction equation. The induction equation is differential equation in space that shows the interaction of fluid velocity and magnetic field of electrically conducting fluid. The induction equation helps to find the influence of magnetic field distribution on the fluid properties. And the induction equation is given by

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma}(\nabla^2 B) + \{\nabla \times (\vec{q} \times \vec{B})\} \tag{8}$$

Using (1) and (2) the induction equation yields

$$\frac{\partial \vec{B}}{\partial t} = \frac{1}{\sigma}\left(\frac{\partial^2 \vec{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vec{B}}{\partial \theta^2}\right)\vec{r} + \frac{1}{\sigma}\left(\frac{\partial^2 \vec{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vec{B}}{\partial \theta^2}\right)\vec{\theta} + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{B_0}{r}q_r - bq_\theta\right)\vec{r} - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{B_0}{r}q_r - bq_\theta\right)\vec{\theta} \tag{9}$$

The magnetic induction equations in r and $\theta$ direction are given by

$$\vec{r}: \frac{1}{\sigma}\left(\frac{\partial^2 \vec{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vec{B}}{\partial \theta^2}\right)\vec{r} + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\frac{B_0}{r}q_r - bq_\theta\right)\vec{r} \tag{10}$$

$$\vec{\theta}: \frac{1}{\sigma}\left(\frac{\partial^2 \vec{B}}{\partial r^2} + \frac{1}{r}\frac{\partial \vec{B}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \vec{B}}{\partial \theta^2}\right)\vec{\theta} - \frac{1}{r}\frac{\partial}{\partial r}\left(\frac{B_0}{r}q_r - bq_\theta\right)\vec{\theta} \tag{11}$$

### *2.3. Thermophysical Characteristics*

Nanofluid flows improve the effectiveness of the temperature differences of the Nanofluid. In the study done by Misra et al, the Thermophysical Properties of the Nanofluid and characteristics of silver-water were discussed [25]. Table 1.

The study considers the two-dimensional unsteady MHD Nanofluid flow and the angle $\theta$ between the walls of the channel and the upper wall is represented by α and the lower wall is represented by – α.

$$\omega = \omega_w \text{ at } \theta = \pm\alpha \tag{12}$$

$$\omega = \omega_\infty \text{ at } \theta = 0 \tag{13}$$

Where $\omega_w$ and $\omega_\infty$ are wall and free stream magnetic induction respectively.

### *2.4. Viscosity of the Nanofluid and Thermal Conductivity*

Xuan et al [26] discussed Nanofluids density as a function of the concentration volume particle, specific heat at constant pressure, and the density of the base fluid were presented.

Brinkman [27] discussed the viscosity correlation equation of suspensions with the moderation of the particle volume fraction. Batchelor [28] showed the influence of Brownian motion of viscosity in a suspension for rigid spherical particles.

$$K_{nf} = \left(\frac{K_s + 2K_f + 2\psi(K_f - K_s)}{K_s + 2K_f + \psi(K_s - K_f)}\right)K_f \tag{14}$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\psi)^{2.5}} \tag{15}$$

$$\rho_{nf} = (1-\psi)\rho_s + \psi\rho_f \tag{16}$$

$$(\rho C_P)_{nf} = (1-\psi)(\rho C_P)_f + \psi(\rho C_P)_s \tag{17}$$

### *2.5. Similarity Transformation of Specific Equations*

#### *2.5.1. Similarity Transformation Relation*

Equations governing the fluid flows are differential equations involving two or more variables. The equations were transformed and reduced to ordinary differential equations in order to solve them numerically. Many researchers have used similarity transformation to simplify continuity, momentum, energy, concentration, and induction equation using dimensionless parameters [29-32].

In the study done by Felicien Habiyaremye et al, the velocity of the Nanofluid in r and $\theta$ directions is given [21].

$$q_r(r,\theta,t) = \frac{Qf(\eta)}{r\delta^{m+1}} \tag{18}$$

$$q_\theta(r,\theta,t) = \frac{Qg(\eta)}{r\delta^{m+1}} \tag{19}$$

While that induction equation as

$$b(r,\theta,t) = \frac{Q\omega(\eta)}{r\delta^{m+1}} \tag{20}$$

Where

$$\eta = \frac{\theta}{\alpha} \tag{21}$$

#### *2.5.2. Transformation of Induction Equation*

To transform the governing equation, relations (10) to (14) are used.

$$(m+1)\frac{r^{m+1}\alpha^2}{2\delta^{m+1}}\frac{R_m}{R_e}\gamma\omega - \alpha^2\omega - \omega'' + \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}\sqrt{\frac{\sigma_{nf}}{\mu_{nf}}}H_a f' - \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}r\alpha\frac{R_m}{R_e}v_0\omega' \tag{22}$$

Reynolds number is defined by $R_e = \frac{rQ}{\nu}$, Hartmann is defined by $(Ha)^2 = \frac{\sigma_{nf}}{\mu_{nf}}r^2(B_0)^2$ unsteadiness time parameter is stated as $\gamma = \frac{\delta^m}{\nu r^{m-1}}\delta'$, $A_1 = \left[\psi\left(\frac{\rho_s}{\rho_f}\right) + (1-\psi)\right] g = v_0$ $R_m = \frac{UL}{\eta}, S_c = \frac{\nu}{D}$

The conditions of the flow are transformed as follows;

$$\eta \to 0: \omega = 0, \omega' = 0 \tag{23}$$

$$\eta \to \infty: \omega = \delta^{m+1}, \omega' = 1 \tag{24}$$

### *2.6. Numerical Method*

In the study done by Felicien Habiyaremye et al, velocity, temperature, and concentration distributions have been given [21].

In the current study, the induction profile is investigated in the connection with the momentum, temperature, and concentration distributions.

$$\omega'' = (m+1)\frac{r^{m+1}\alpha^2}{2\delta^{m+1}}\frac{R_m}{R_e}\gamma\omega - \alpha^2\omega + \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}\sqrt{\frac{\sigma_{nf}}{\mu_{nf}}}H_a f' - \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}r\alpha\frac{R_m}{R_e}v_0\omega' \tag{25}$$

Momentum, temperature, concentration, and induction equation (25) are used to get the numerical simulations, the system (29) is achieved by letting

$$z_8 = \omega, z_9 = \omega' \tag{26}$$

From equation (29), we get

$$z'_8 = z_9 \tag{27}$$

$$z_9 = \omega'' = (m+1)\frac{r^{m+1}\alpha^2}{2\delta^{m+1}}\frac{R_m}{R_e}\gamma z_8 - \alpha^2 z_8 + \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}\sqrt{\frac{\sigma_{nf}}{\mu_{nf}}}H_a z_2 - \frac{(1-\psi)^{2.5}}{A_1}\frac{\mu_f}{\rho_f}r\alpha\frac{R_m}{R_e}v_0 z_9 : M \tag{28}$$

The equations (27) and (28) yield the following systems of equations

$$z' = F(\eta, z) \tag{29}$$

$$z = \begin{bmatrix} z_8 \\ z_9 \end{bmatrix} \; F = \begin{bmatrix} z_9 \\ M \end{bmatrix} \tag{30}$$

# 3. Results and Discussion

***Table 1.*** *Thermophysical Characteristics of Silver and Water.*

| Nanoparticles | $C_p$ | $K$ | $\rho$ | $\sigma$ |
|---|---|---|---|---|
| Water | 997.1 | 4179 | 0.613 | 0.06 |
| Silver | 10500 | 235 | 4291 | $6.3 \times 10^7$ |

***Table 2.*** *Values of Physical Parameters.*

| $Re$ | $R_m$ | $S_c$ | $\gamma$ | $\psi$ |
|---|---|---|---|---|
| 15 | 5 | 0.02 | 0 | 0.2 |
| 20 | 20 | 0.06 | 0.5 | 0.5 |
| 25 | 35 | 0.1 | 1 | 0.7 |

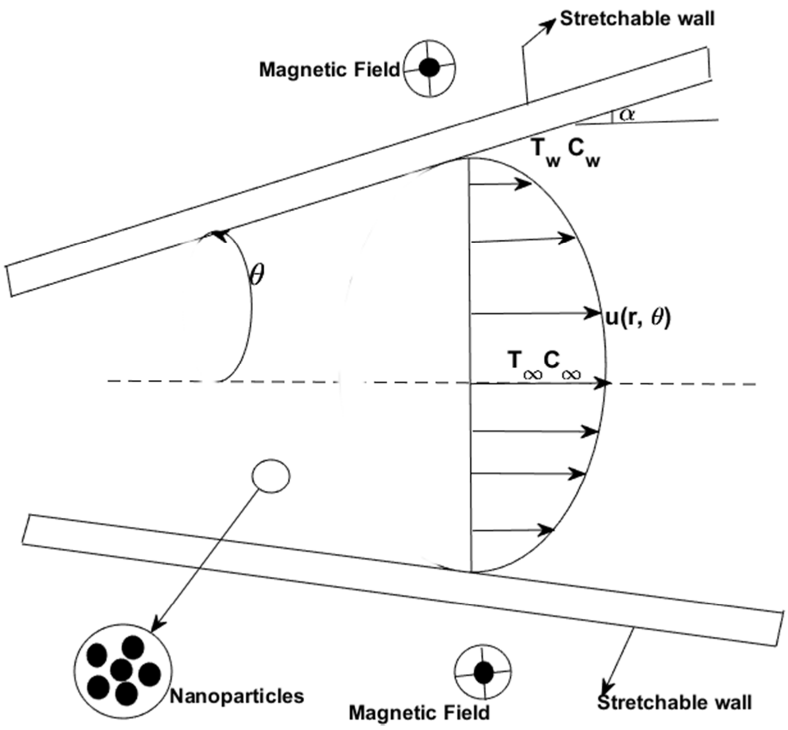


***Figure 1.*** *Flow Diagram. Effect of Strong Magnetic Field on MHD Nanofluid for Unsteady Incompressible Flow via Convergent-Divergent Channel with Heat and Mass Transfer.*

The current study focuses on the effects unsteadiness time parameter, Schmidt number, Reynolds number and nanoparticle volume fraction on the induction distribution and Reynolds Magnetic number and Schmidt number on the velocity, temperature, concentration and induction profiles are examined. The Prandtl number is fixed as 6.2 due the base fluid that is water. Table 2 presents the numerical values of different physical parameter that are utilized to give graphs. Results and discussions on the effect of strong magnetic field on unsteady MHD Nanofluid flow are shown below. The solid lines show the divergent channel and dashed lines show the convergent channel. Many studies have been done on the effect of a strong magnetic Field on unsteady MHD Nanofluid flow through convergent–divergent channel before.

Felicien Habiyaremye et al [21] did a study on the influence of Heat and Mass Transfer on Unsteady MHD Nanofluid flow through the Convergent-Divergent Channel. It was observed that the velocity of the Nanofluid decreases with the increase in the Nanoparticle volume fraction for silver nanoparticles in the case of the divergent channel. For a convergent, the velocity augments with an augment in the Nanoparticle volume fraction. Stretching a divergent channel augments the flow near the walls of the channel. A Shrinking convergent channel decreases the velocity of the fluid near the walls of the channel. It was noted that the temperature augments with an augment in the Grashof number in the divergent channel and temperature decreases with an increase in the Grashof number in the convergent channel. They found that the temperature of Nanofluid increases with an increase in the Eckert number for all the cases. The velocity and temperature increase with an increase in the heat generation parameter for both convergent and divergent channels observed. The concentration of the Nanofluid decreases with an increase in the heat generation parameter was noted for both divergent and convergent.

In the current study, a researcher brings an innovation on the effect of a strong magnetic field on MHD Nanofluid for Unsteady Incompressible Flow through Convergent-Divergent Channel with Heat and Mass Transfer that was not investigated in the previous studies.

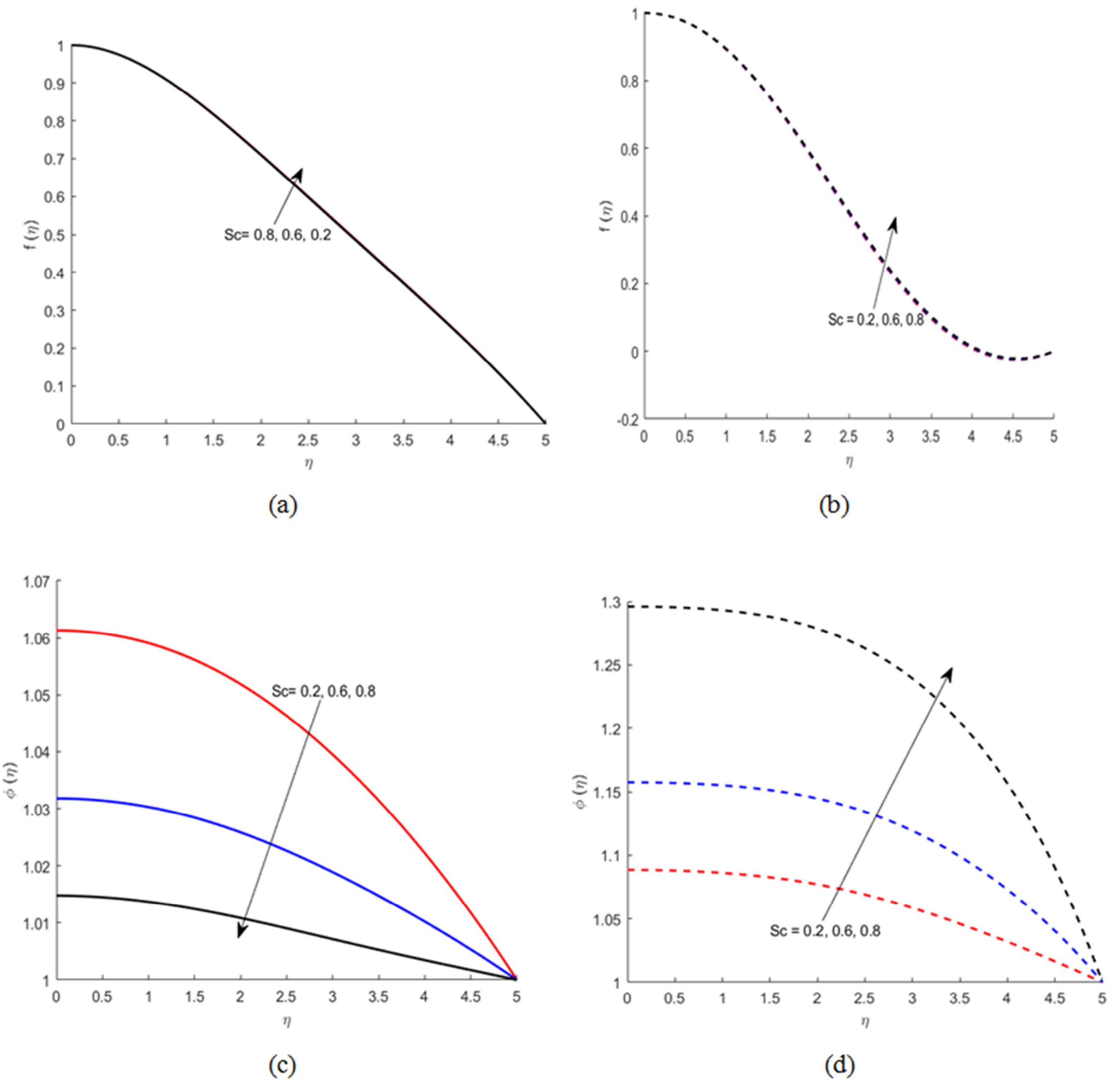


***Figure 2.*** *Effect of the Schmidt number on velocity $(f(\eta))$, temperature $(\phi(\eta))$profiles for a divergent (Left panel) and a convergent (Right panel) channel.*

From Figure 2(a) and 2(b). The velocity increases with an augment in the Schmidt Number. An augment in the Schmidt number leads to a reduction in the viscous forces which in turn gives an increase in the momentum and mass diffusion hence increased velocity. From Figure 2(c). Temperature decreases with an increase in the Schmidt number. an augment in the Schmidt number leads to an augment in the dag force which lead to a decrease in the momentum and mass diffusion hence a decrease in the temperature. From Figure 2(d). The temperature augments with the increase in the Schmidt number. An increase in the Schmidt number leads to a reduction of viscous forces which lead to an increase in the momentum and mass diffusion, thus increasing temperature.

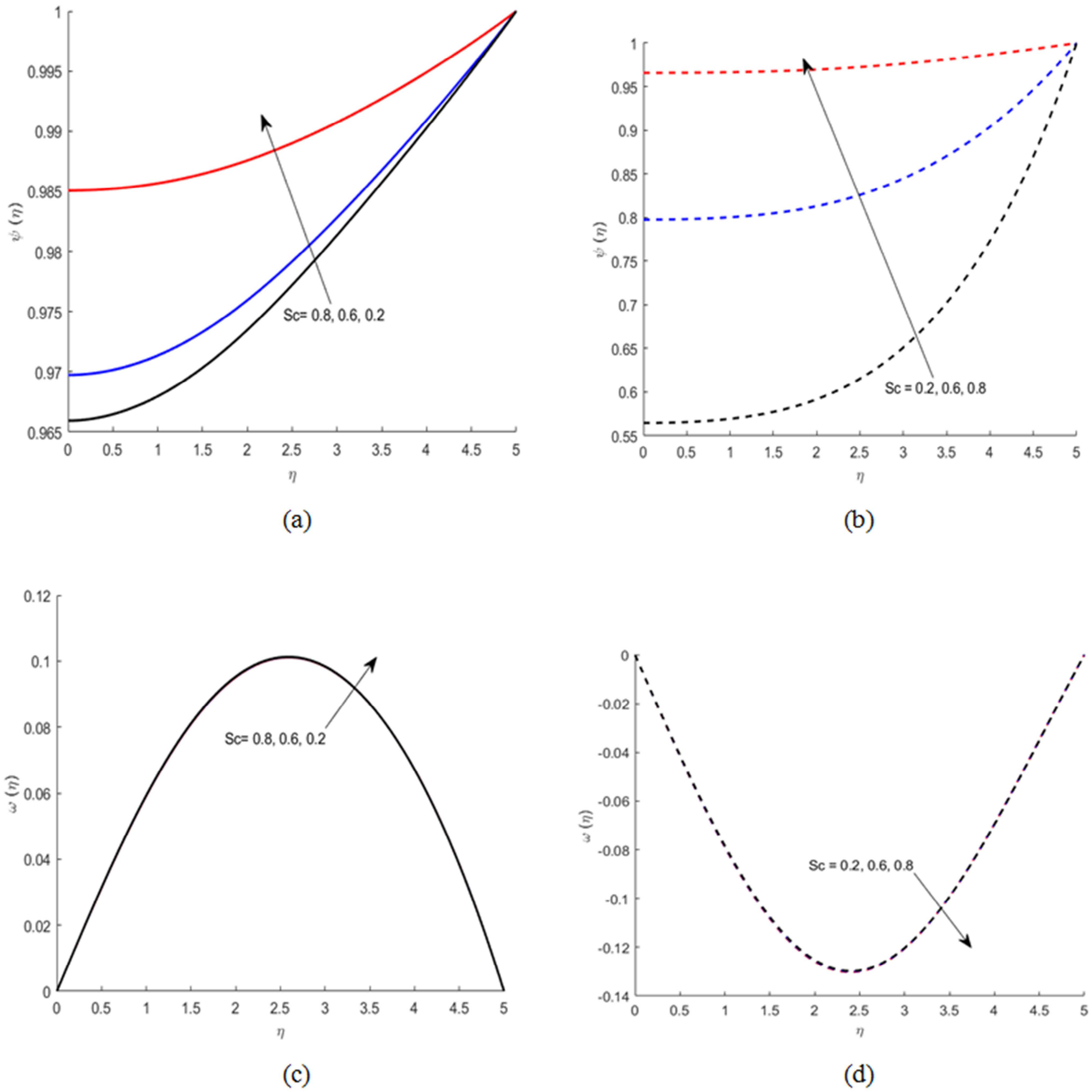


***Figure 3.*** *Effect of Schmidt number on concentration* $(\psi(\eta))$ *and magnetic induction* $(\omega(\eta))$ *profiles for a divergent (Left panel) and a convergent (Right panel) channel.*

From Figure 3(a) and 3(b). The concentration augments with varying the Schmidt Number. The Schmidt number shows the interaction in which there are simultaneous momentum and mass diffusion. An augment in the Schmidt number gives an augment in the viscous forces which in turn leads to increased concentration of the nanofluid. From Figure 3(c). The induction profile increases with the increase in the Schmidt number. An augment in the Schmidt number gives a decrease in the viscous forces which in turn gives an augmented induction. From Figure 3(d). The induction profile reduces with an increase in the Schmidt Number. The Schmidt number shows the interaction in which there are simultaneous momentum and mass diffusion. An augment in the Schmidt number gives an augment in the viscous forces which in turn gives a decrease in the momentum and mass diffusion hence a decreased induction profile.

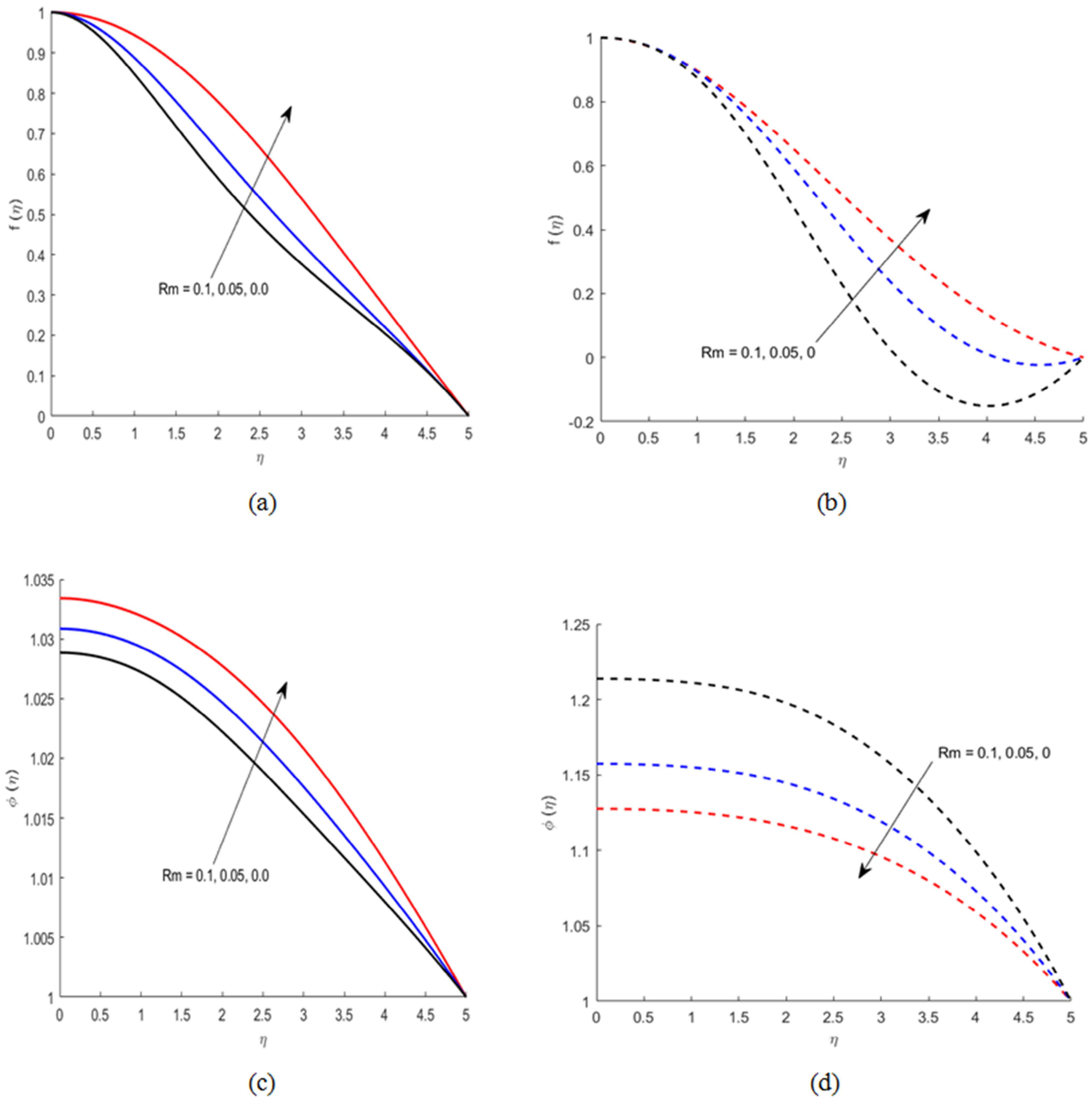


***Figure 4.*** *Effect of the Reynolds magnetic number on velocity $(f(\eta))$, temperature $(\phi(\eta))$ profiles for a divergent (Left panel) and a convergent (Right panel) channel.*

From Figure 4(a) and 4(b). Varying the Reynolds magnetic number augments the velocity profile of the nanofluid. An augment in the Reynolds magnetic number gives a reduction in the viscous forces which in turn gives an increase in the velocity of the nanofluid. From Figure 4(c). Temperature augments with the increase in the Reynolds magnetic number. An increase in the Reynolds magnetic number gives a decrease of viscous forces which give a rise to temperature hence augmenting temperature. From Figure 4(d). The temperature reduces with the augment in the Reynolds magnetic number. an augment in the Reynolds magnetic number gives an increase in the viscous forces which in turn the thickening of the boundary layer takes place hence reducing temperature.

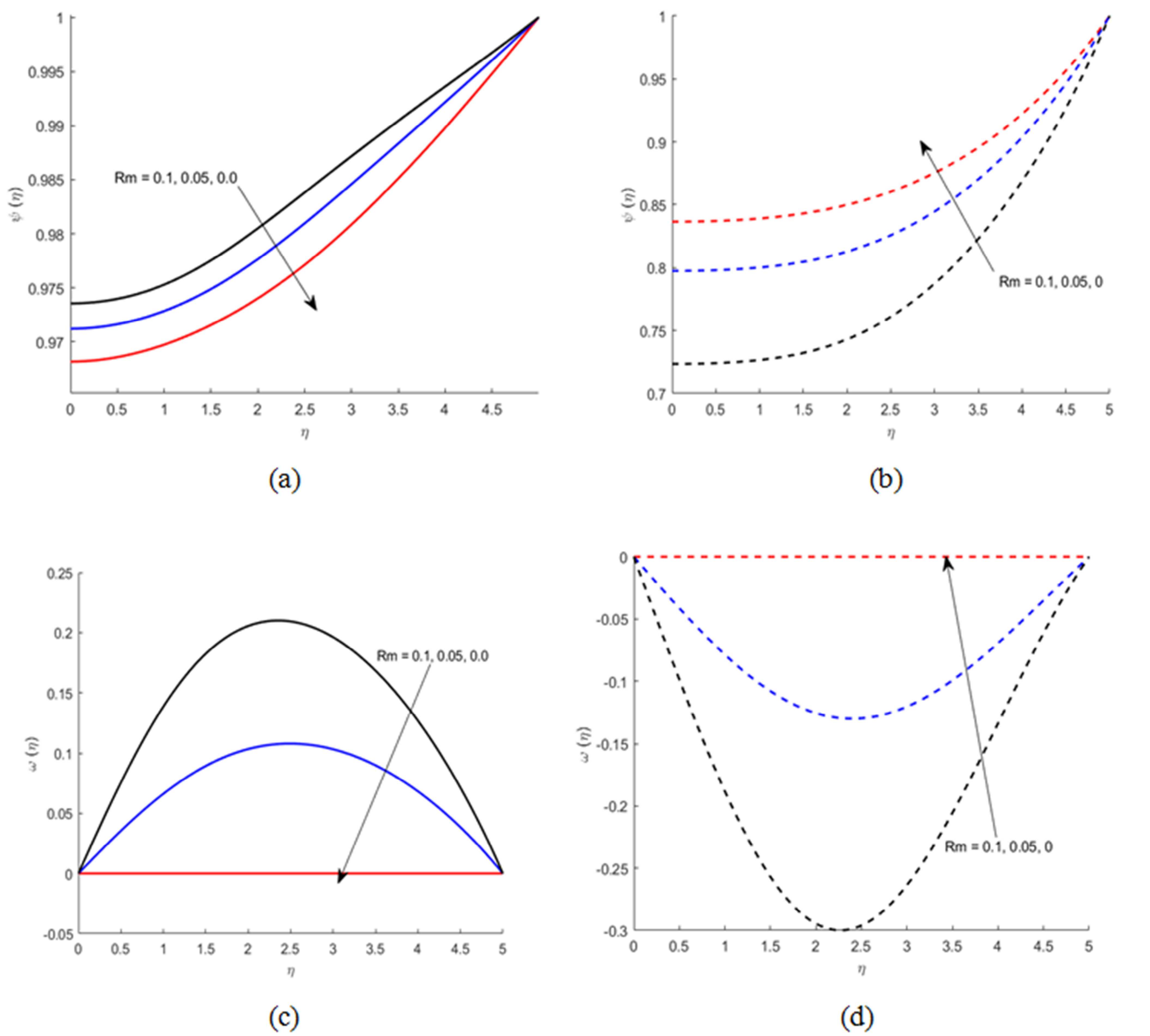


***Figure 5.*** *Effect of Reynolds magnetic number on concentration $(\psi(\eta))$ and magnetic induction $(\omega(\eta))$ profiles for a divergent (Left panel) and a convergent (Right panel) channel.*

From Figure 5(a). Concentration decreases with an augment in the Reynolds magnetic parameter. Increasing the Reynolds magnetic parameter gives an increase in the drag forces hence reduced concentration of Nanofluid. From Figure 5(b). Concentration increases with the increase in the Reynolds magnetic parameter. An increase in the Reynolds magnetic number leads to a reduction of the drag forces in which in turn leads to increased concentration of the Nanofluid. From Figure 5(c). Induction profile reduces with an increase in the Reynolds magnetic parameter. An augment in the Reynolds magnetic number gives a decrease of drag forces hence increased velocity. An increase in the velocity leads to an augment in the interaction of the fluid and magnetic field hence decreased magnetic induction of the Nanofluid. From Figure 5(d). The magnetic induction increases with the increase in the Reynolds magnetic number. An augment in the Reynolds magnetic number gives an augment in the velocity which in turn gives to an increase in the interaction of the fluid flow and magnetic field hence augmented induction profile of the Nanofluid.

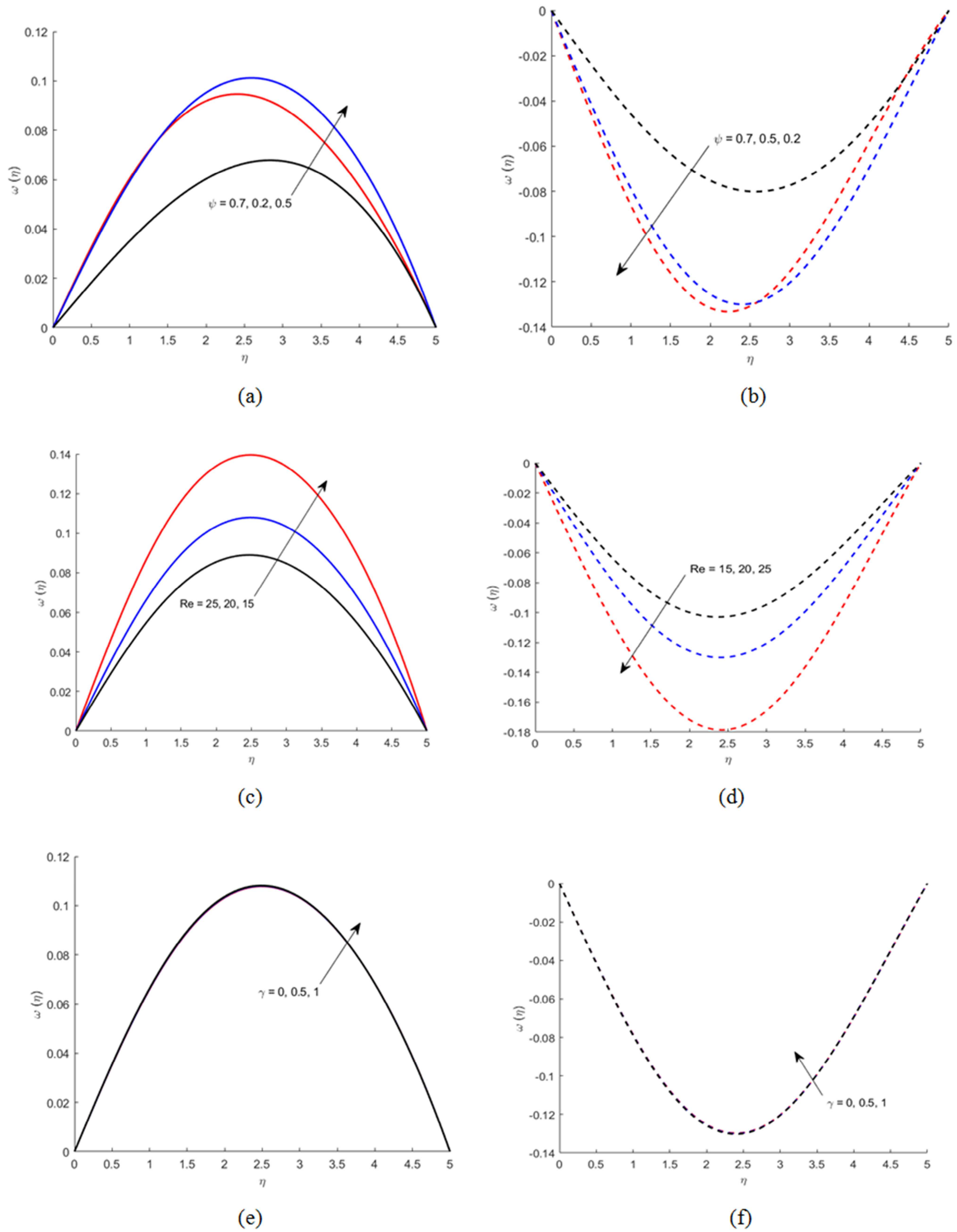


***Figure 6.*** *Effect of Nanoparticle volume fraction, Reynolds Number, and Unsteadiness time parameter on the induction $(\omega(\eta))$ profile for a divergent (Left panel) and a convergent (Right panel) channel.*

From Figure 6(a). The induction of the Nanofluid augments with the increase of the nanoparticle volume fraction. An increase in the nanoparticle volume fraction gives an increase of the velocity and temperature of Nanofluid which in turn gives an augment of the interaction between the Nanofluid flow and magnetic field hence increased induction of the Nanofluid flow. From Figure 6(b). The induction of Nanofluid reduces with the increase in the nanoparticle volume fraction. An augment in the nanoparticle volume fraction gives an increase of the drag forces hence the velocity of Nanofluid flow is reduced which in turn gives a decrease in the interaction between Nanofluid flow and magnetic field. Thus decreased induction of the Nanofluid.

From Figure 6(c). The induction of the Nanofluid increases with the increase in the Reynolds number. An increase in the Reynolds number gives an augment in the velocity distribution. An increase in the velocity enhances the interaction of the flow and magnetic field hence increasing induction of the Nanofluid. From Figure 6(d). The induction of the Nanofluid reduces with an increase in the Reynolds number. An augment in the Reynolds number leads to an increase of drag forces that reduces the interaction of the flow and magnetic field hence decreased induction of the Nanofluid.

From Figure 6(e). The induction of the Nanofluid augments with a variation in the unsteadiness time parameter. An increase in the unsteadiness time parameter gives a widening of the boundary layer which in turn leads to an increase in the velocity. This enhances the interaction between the flow and magnetic field, hence an increased induction of the Nanofluid. From Figure 6(f). The induction of the Nanofluid reduces with an augment in the unsteadiness time parameter. An augment in the unsteadiness time parameter leads to the thickness of the boundary layer which in turn leads to a decrease in the velocity and leads to a decrease in the interaction of the flow and magnetic field hence a decreased induction of the Nanofluid.

# 4. Conclusion

Magnetic Reynolds number on velocity, temperature, concentration, and magnetic induction distributions of unsteady MHD Nanofluid flow.

1) The velocity augments with an increase in the Reynolds magnetic number. An increase in the Reynolds magnetic number gives a reduction in the drag forces which in turn gives an increase in the velocity of the Nanofluid. Temperature increases with the increase in the Reynolds magnetic number. An increase in the Reynolds magnetic number gives a reduction of drag forces which give a rise to temperature hence increased temperature.
2) The temperature decreases with the increase in the Reynolds magnetic number. An augment in the Reynolds magnetic number leads to increase in the viscous forces which in turn the thickening of the boundary layer takes place hence reduced temperature.
3) Concentration decreases with an augment in the Reynolds magnetic number. An augment in the Reynolds magnetic number gives an increase in the drag forces hence decreased concentration of Nanofluid. Concentration increases with the increase in the Reynolds magnetic number. An increase in the Reynolds magnetic number leads to a reduction of the drag forces in which in turn leads to increased concentration of the Nanofluid.
4) The magnetic induction decreases with an increase in the Reynolds magnetic number. An augment in the Reynolds magnetic number leads to a decrease of viscous forces hence increased velocity. An increase in the velocity leads to an augment in the interaction of the fluid and magnetic field hence increased magnetic induction of the Nanofluid. The magnetic induction increases with the increase in the Reynolds magnetic number.
5) An augment in the Reynolds magnetic number leads to an augment in the velocity which in turn leads to an increase in the interaction between the fluid flow and magnetic field hence augmented magnetic induction of the Nanofluid.

# 5. Recommendations

Further research to consider in the future:

1) Influence of variable magnetic field on unsteady MHD Nanofluid flow via convergent-divergent channel considering pressure gradient of the Nanofluid flow.
2) Influence of strong magnetic field on unsteady MHD Nanofluid of compressible flow through the convergent-divergent channel.
3) Influence of chemical reaction and radiation on unsteady MHD Nanofluid flow via convergent-divergent channel under strong magnetic field.

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