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On a Weibull-Distributed Error Component of a Multiplicative Error Model Under Inverse Square Root Transformation

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Abstract: We first consider the Multiplicative Error Model (MEM) introduced in financial econometrics by Engle (2002) as a general class of time series model for positive-valued random variables, which are decomposed into the product of their conditional mean and a positive-valued error term. Considering the possibility that the error component of a MEM can be a Weibull distribution and the need for data transformation as a popular remedial measure to stabilize the variance of a data set prior to statistical modeling, this paper investigates the impact of the inverse square root transformation (ISRT) on the mean and variance of a Weibull-distributed error component of a MEM. The mean and variance of the Weibull distribution and those of the inverse square root transformed distribution are calculated for σ=6, 7,.., 99, 100 with the corresponding values of n for which the mean of the untransformed distribution is equal to one. The paper concludes that the inverse square root would yield better results when using MEM with a Weibull-distributed error component and where data transformation is deemed necessary to stabilize the variance of the data set.

Keywords: Multiplicative Error Model, Error Component, Weibull Distribution, Inverse Square Root Transformation, Remedial Measure

1. Introduction

The MEM can be classified as an autoregressive conditional duration (ACD) model where the conditional mean of a distribution is assumed to follow a stochastic process. The idea of the MEM is well-known in financial econometrics since its origination from the structure of the Autoregressive Conditional Heteroscedasticity (ARCH) model as proposed by Engle [6] and the stochastic volatility model introduced by Taylor [22], where the conditional variance is dynamically parameterized and multiplicatively interacts with an innovation term.

The MEM is first used within this context by Engle and Russell [5] to model the clustering behavior of waiting times between financial events, for instance market trading and changes in asset prices. Hereafter, let us refer to these waiting times as financial durations. The resulting model is referred to in the literature as the autoregressive conditional duration (ACD) model. According to Engle and Russell [5], the choice of a suitable distribution for the error distribution plays an important role in ACD modeling as many well-known positive support distributions have been used which include exponential and Weibull distributions.

1.1. Rationale

Data transformations such as replacing a variable by its logarithm or by its square root are used to simplify the structure of the data so that they follow a convenient statistical model. Transformations according to Montgomery [9] are used for three purposes:

1) To stabilize response variance.
2) To making the distribution of the response variable closer to a normal distribution, and
3) To improving the fit of the model to the data.

The first and second goals are concerned with simplifying the "error structure" or random component of the data.

The choice of an appropriate transformation depends on the probability distribution of the sample data. More so, the relationship between the mean and the standard deviation can be used in stabilizing the variance.

1.2. Background of the Study

Ohakwe et al [15] studied the effect of square root transformation on the error component of the multiplicative error model whose distribution belongs to the generalized gamma family to determine the effect of the said transformation on the basic assumptions of unit mean and constant variance; and affirmed the unit means assumption is approximately maintained for all the distributions while the result showed variations in the variances of the distributions.

Ohakwe and Ajibade [13] studied the commonly used power transformations on the unit-mean and variance of the error component of a multiplicative error model which has a gamma distribution that requires a variance-stabilization transformation and observed that for the transformations, there was no relative change in the mean between the transformed and the untransformed distributions. While on the variances, it was found that there are relative increases for the inverse, the inverse square and the square root transformations.

Onyemachi [18] investigated the impact of square root transformation on Weibull-distributed error component of a multiplicative error model and confirmed consistence of the unit mean and constant variance assumption in the transformed and untransformed distributions.

This study is focused on investigating the impact of the inverse square root transformation on the unit mean and variance of the error component of a Weibull-distributed random variable under the multiplicative error model with a necessity that requires a variance-stabilization.

1.3. Motivation

Motivated by such strong statistical and empirical relevance, the purpose of this study is to verify the assumed fundamental statistical structure of the error component (unit mean and constant variance) is maintained after the inverse square root transformation and also to investigate what happens to variances of the transformed and untransformed (i.e., σ_1^2 and σ_2^2) in terms of equality and non-equality. To examine this, the Weibull distribution, a non-normal distribution whose distributional characteristics fit $N \sim (1, \sigma^2)$ is chosen considering its flexibility and adaption with asymptotic properties relative to multiplicative error modeling.

1.4. Research Importance

There are various studies on the effects of transformation on the error component of the multiplicative error models whose distributional characteristics is given to follow the unit mean and constant variance criteria. The overall aim of such studies is to obtain the conditions for successful transformations such as [19, 1, 15, 16, 12]. According to Ohakwe et al [15], a successful transformation is achieved when the desirable properties of a data set remain unchanged after transformation.

The choice of a suitable distribution for the error distribution plays an important role in multiplicative error models; thus in the case of MEM, the normality assumption of the error component is out of the question. The study will test the probability distribution of the error term in the MEM.

1.5. Research Problem

In practice many multiplicative time series data do not conform to the basic assumptions of a parametric statistical analysis; they are not normally distributed, their variances are not homogeneous or both. As such, researchers are faced with two choices:(i) Adjusting the data to fit the assumptions through transformation or (ii) Developing new methods of analysis with assumptions which fits the data in its "original" form.

If a satisfactory transformation can be found, it will almost be easier and simpler to use it, rather than developing new methods of analysis Turkey [23]. The study will provide basis to judge the convenience of any data transforming method under which transformation can be made without necessarily altering the desired properties of the original data in the case of non-normally distributed data set. The distribution of failures over the lifetime of the product population is critically important to the MEMs and reliability physicist (small, lightweight and power saving systems compared to conventional systems). Using these concepts, distribution functions can be developed and used for predictive purposes.

1.6. The Standard Weibull Distribution

The Weibull distribution is a continuous probability distribution that can fit an extensive range of distribution shapes. Like the normal distribution, the Weibull distribution describes the probabilities associated with continuous data. However, unlike the normal distribution, it can also model skewed data. In fact, its extreme flexibility allows it to model both left- and right-skewed data. Because of its versatility, analysts use it in a broad range of settings, such as quality control, capability analysis, medical studies, and engineering. It's frequently used in life data, reliability analysis, and warranty analysis to assess time to failure for systems and parts.

There are two forms of the Weibull distribution distinguished by the presence of either two or three parameters. Unlike the normal distribution, the Weibull cumulative density function is expressible in closed form. The probability density function of a Weibull-distributed random variable, X and its k^{th} moment as contained in Walck [24] is given by

$$f(x)=\frac{\eta}{\sigma}\left(\frac{x}{\sigma}\right)^{\eta-1} e^{-\left(\frac{x}{\sigma}\right)\eta},\ x>0, \sigma>0 \text{ and } \eta>0 \quad (1)$$

with

$$E(X^k)=\sigma^k\Gamma\left(1+\frac{k}{\eta}\right) \quad (2)$$

$$E(X)=\sigma\Gamma\left(1+\frac{1}{\eta}\right)=\frac{\sigma}{\eta}\Gamma\left(\frac{1}{\eta}\right) \quad (3)$$

$$E(X^2)=\sigma^2\Gamma\left(1+\frac{2}{\eta}\right)=\frac{2\sigma^2}{\eta}\Gamma\left(\frac{2}{\eta}\right) \quad (4)$$

and

$$Var(X)=\sigma_1^2=\frac{\sigma^2}{\eta}\left[2\Gamma\left(\frac{2}{\eta}\right)-\frac{1}{\eta}\left(\Gamma\left(\frac{1}{\eta}\right)\right)^2\right] \quad (5)$$

The probability density function (*pdf*) of the two parameter Weibull distribution is given by

$$f(x;\eta,\beta)=\frac{dF(x)}{dx}=\frac{\beta}{\eta}\left[\frac{x}{\eta}\right]^{\beta-1}.\exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \quad (6)$$

The two parameter Weibull, has a shape (η) and scale (β) parameters respectively. We shall limit our discussion to the standard two parameter Weibull distribution.

Various applications of the Weibull models for solving a variety of problems from many different disciplines dot the literature as Okorie et al [17] derived the adjusted Fisk Weibull distribution, Meyer [9] presented an insight review with a number of historical facts and many forms of the distribution as used by practitioners, Murthy et al [11] present a monograph containing a wholesome details concerning the Weibull distribution and its extensions, Horst [8] did a valuable handbook and gave a bird's eye view of the Weibull distribution.

2. Materials and Methods

2.1. Theoretical Framework

The proposed model, on which the study devolves, is the multiplicative error model that could be abbreviated as MEM. This model specifies an error that is multiplied by the mean.

Let $\{x_t\}$ denote a discrete time real-valued stochastic process defined on $[0,+\infty)$, $t\in \mathrm{N}$ where N is the set of N natural numbers and let ψ_{t-1} be the information available for forecasting x_t. $\{x_t\}$ according to Brownlees et al [4] is a MEM if it can be expressed as

$$x_t=\mu_t\varepsilon_t \quad (7)$$

where conditionally on ψ_{t-1}, μ_t is a positive quantity that evolves deterministically according to a parameter vector θ.

$$\mu_t=\mu(\theta,\psi_{t-1}) \quad (8)$$

ε_t is an independently, identically distributed ($i.i.d$) innovation series with non negative support density and $E(\varepsilon_t)=1$ with unknown constant variance

$$\varepsilon_t\mid\psi_{t-1}\sim D^+(1,\sigma^2), \quad (9)$$

Irrespective of the specification of the function $\mu(.)$ and of the distribution D^+ (any distribution) equations (7), (8) and (9) according to Engle [7] must evolve

$$E(x_t\mid\psi_{t-1})=\mu_t \quad (10)$$

$$V(x_t\mid\psi_{t-1})=\sigma^2\mu_t^2 \quad (11)$$

The assumption according to Engle and Russell [4] is that the time dependence in the durations can be subsumed in their conditional expectations (10), in such a way that $x_t\mid\psi_t$ is independent and identically distributed. ψ_{t-1} denote the information set available at time $t-1$.

The realization of (9) supports such distributions as Weibull according to Bandi and Russell [2].

The property (10) provides us with a link on (12) which gives

$$\psi_t=\Gamma\left(1+\frac{1}{\alpha}\right)\mu_t \quad (12)$$

where $\Gamma(.)$ is the gamma function. If $\alpha=1$, the Weibull distribution becomes an exponential one. Here, $\mu_t=\psi_t$.

$$x_t=\mu_t\varepsilon_t,\ \varepsilon_t\mid\psi_{t-1}\sim D(1,\sigma_t^2) \quad (13)$$

The range of the disturbance would naturally run from zero to infinity, thereby satisfying (7).

2.2. Ozdemir Transformation

Using the power transformation modified for a class of distributions based on Box and Cox [3], by Ozdemir [20] given as:

$$\mathrm{Y}=\begin{cases}\mathrm{X}^{\mathrm{p}},\ \mathrm{p}\neq 0\\ \log(\mathrm{X}),\ \mathrm{p}=0\end{cases}$$

Suppose $Y_t=X_t^{\mathrm{p}}$, then

$$X_t = Y_t^{\frac{1}{p}} \tag{14}$$

and

$$|J| = \left|\frac{d x_t}{d y_t}\right| = \left|\frac{1}{p} y_t^{\frac{1}{p}-1}\right| = \left|\frac{1}{p}\right| y_t^{\frac{1}{p}-1} \tag{15}$$

where $| J |$ is the absolute value of the Jacobian of the p-th power transformation. The pdf of Y_t, denoted as $f(y_t)$ is then obtained according to Ramachandran and Tsokos [19] as

$$f(y_t) = f\left(x_t = y_t^{\frac{1}{p}}\right)\left|\frac{d x_t}{d y_t}\right| \tag{16}$$

Now, suppose the error component (e_t) of a Multiplicative Error Model (MEM) is assumed to follow a Weibull distribution, then the probability density function (pdf) of e_t denoted as $f(e_t)$ is given as follows:

$$f(e_t) = \left(\frac{\sigma}{n}\right)\left(\frac{e_t}{n}\right)^{\sigma - 1} \exp\left\{-\left(\frac{e_t}{n}\right)^{\sigma}\right\}, e_t > 0 \tag{17}$$

whereσ and n are the shape and scale parameters respectively. The mean ($E(e_t)=\mu_{e_t}$) and Variance ($\mathrm{Var}(e_t)= \sigma^2_{e_t}$) of e_t are given as

$$E(e_t) = \mu_{e_t} = n\,\Gamma\left(1 + \tfrac{1}{\sigma}\right) \tag{18}$$

and

$$\sigma^2_{e_t} = n^2\,\Gamma\left(1+\tfrac{2}{\sigma}\right) - \left[n\,\Gamma\left(1+\tfrac{1}{\sigma}\right)\right]^2 \tag{19}$$

Using (16) and (17), the pdf of the p-th transformed Weibull distribution is obtained denoted by $f(y_t)$

$$f(y_t) = \frac{\sigma}{|p|}\left(\frac{1}{n}\right)^{\sigma} y^{\frac{\sigma}{p} - 1} \exp\left\{-\left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma}\right\}, y_t > 0 \tag{20}$$

If p=1 in (20), we obtain

$$f(y_t) = \left(\frac{\sigma}{n}\right)\left(\frac{y_t}{n}\right)^{\sigma - 1} \exp\left\{-\left(\frac{y_t}{n}\right)^{\sigma}\right\}, y_t > 0 \tag{21}$$

which established that no transformation is required when p=1.

The pdf of the transformed Weibull (Y_t) variable under the inverse square root transformations is thus:

$$f(y_t) = 2\sigma\left(\frac{1}{n}\right)^{\sigma} (y_t)^{-2\sigma - 1} \exp\left\{-\left(\frac{1}{n y^2_t}\right)^{\sigma}\right\}, y_t > 0 \tag{22}$$

2.3. The K-th Uncorrected Moment of Y denoted as [E(Y^k)]

The mean and higher-order raw moments can be used to describe the distribution of any random variable fairly well. Even the celebrated Central Limit Theorem which forms the basis for inferential statistics rely on moments, just to mention a few importance of moments in probability and statistics. Onyemachi [18] shows that $f(y_t)$ is a proper pdf and derived the moments of the transformed Weibull distribution:

The K^{th} uncorrected moment of Y_t denoted as $E(Y_t^k)$ is obtained as follows

$$E(Y_t^k) = \int_0^{\infty} y_t^k f(y_t) d y_t = \frac{\sigma}{|p|}\left(\frac{1}{n}\right)^{\sigma}\int_0^{\infty} y^k y^{\frac{\sigma}{p} - 1} \exp\left\{-\left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma}\right\} d y_t$$

$$= \frac{\sigma}{|p|}\left(\frac{1}{n}\right)^{\sigma}\int_0^{\infty} y^{\frac{\sigma}{p} + k - 1} \exp\left\{-\left(\frac{y^{\frac{1}{p}}}{n}\right)^{\sigma}\right\} d y_t \tag{23}$$

$$E(Y_t^2) = n^{2p}\,\Gamma\left(1+\frac{2p}{\sigma}\right) \tag{24}$$

thus

$$\sigma^2_{y_t} = E(Y_t^2) = \left[E(Y_t)\right]^2 =$$

$$n^{2p}\,\Gamma\left(1+\frac{2p}{\sigma}\right) - \left[n^p\,\Gamma\left(1+\frac{p}{\sigma}\right)\right]^2 \tag{25}$$

The Expressions for the Means and Variances under the inverse square root transformation are given by

$$E(Y_t)=\mu_t= n^{-\frac{1}{2}}\,\Gamma\left(1 - \frac{1}{2\sigma}\right)$$

$$\sigma^2_{y_t} = n^{-1}\,\Gamma\left(1- \frac{1}{\sigma}\right) - \left[n^{-\frac{1}{2}}\,\Gamma\left(1- \frac{1}{2\sigma}\right)\right]^2 \tag{26}$$

3. Relative Change in Means and Variances of the Two Distributions

In this Section, the means and variances of the transformed and the untransformed distributions would be obtained followed by to computations of the relative changes in means and variances between the untransformed and transformed distributions.

The mean $\left(\mu_{\varepsilon_t}\right)$ and variance $\left(\sigma_{\varepsilon_t}{}^2\right)$ of the untransformed Weibull distribution are respectively given (18) and (19).

Considering the unit mean assumption required for modeling, we would calculate the theoretical values of μ_{ε_t}, $\sigma_{\varepsilon_t}{}^2$ μ_{y_t} and $\sigma_{y_t}{}^2$ using values of σ=7, 8,..., 99, 100 and corresponding values of n for which μ_{ε_t} =1.0 Ohakwe and Ajibade [13]. From (18), for μ_{ε_t} =1.0 , without loss of generality, we would adopt $n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$ in order to maintain the values of the shape parameters as positive integers Ohakwe and Ajibade [13]. That is for every value of σ we would use the corresponding values of $n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$ for all the computations involving the untransformed and transformed distributions. Table 1 (see appendix) contains the means of the transformed and the untransformed distributions while their variances are contained in Table 2.

Considering that the interest in this study is to examine the effect of the inverse square root transformations on the Weibull distributed error term as regards the mean and the variance, we will adopt the same measures used in Ohakwe and Ajibade [13] which are the relative change in means and variances between the untransformed and the transformed distributions in measuring the effect of a transformation. The measures are given as follows; for the effect on the mean, the two variables of interest are μ_{ε_t} and μ_{y_t} and the Relative Change in mean (RCIM) is

$$RCIM = \frac{\mu_{y_t} - \mu_{\varepsilon_t}}{\mu_{\varepsilon_t}} = \mu_{y_t} - 1.0 \qquad (21)$$

where RCIM > 0 indicates increase, RCIM=0 indicates no change and RCIM < 0 indicates decrease in mean.

Furthermore, for the effect on the variance, the determinant variables are $\sigma_{\varepsilon_t}{}^2$ and $\sigma_{y_t}{}^2$ and the measure for the Relative Change in Variance (RCIV) between the transformed and the untransformed distributions is

$$\text{RCIV} = \frac{\sigma_{y_t}{}^2 - \sigma_{\varepsilon_t}{}^2}{\sigma_{\varepsilon_t}{}^2} \qquad (22)$$

where RCIV > 0 indicates increase, RCIV=0 indicates no change and RCIV < 0 indicates decrease in variance.

Whereas the theoretical means of the transformed distributions are approximately 1.0 to the nearest whole number for all σ≥ 7 as shown in Table 1, we therefore compute the RCIM and RCIV values for σ≥ 7. Furthermore, the computations of the RCIV are contained in Table 3 (see appendix). Finally, the mean values, minimum and maximum values of the RCIV for the various transformations are given in Table 4.

4. Results and Discussions

The results in Table 1 indicate that the unit-mean assumption is unaffected by the transformations. This result is in agreement with the findings of Ohakwe *et al.*, [15]; and Ohakwe and Ajibade [13].

For the variances given in Table 2, the variances for the inverse square root transformation (VISRT), are lower than that of the untransformed distribution. The RCIV values for ISRT that indicates reduced variances with factors: -0.8867 – (-0.8556), however it is noticeable that the magnitude of decreases are approximately the same. Finally, it is important to mention that stability of the variances for all the transformations is achieved from the point, α≥17, where the variances for all the transformations are all approximately zero.

5. Conclusion

The study discusses inverse square root transformation of the Weibull two-parameter distribution of the multiplicative error model which has importance in presenting a new way of modeling random durations emanating from time varying events. The *pdf* , means and variances of transformed and untransformed distributions were obtained. Results affirmed that decreasing of value of scale parameter is meaningful and effective for inverse and square root transformation Ozdemir [20]. Furthermore, the assumption of unit mean was verified for all transformations, whereas for the variance it was found that there exists relative stability for the inverse square root transformation. The paper finally concludes that the inverse square root would yield better results when using MEM with a Weibull-distributed error component and where data transformation is deemed necessary to stabilize the variance of the data set.

6. Recommendation

The inverse square root transformation is recommended to stabilize variance when dataset requires remedial measures to suit statistical modeling as evidenced in respect of multiplicative error models with class of models considered as stochastic process.

Appendix

Table 1. *Means of the untransformed and Transformed Distributions.*

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	μ_{ε_t}	ISRT
7	1.0690	1	1.0123
8	1.0619	1	1.0094
9	1.0560	1	1.0075
10	1.0511	1	1.0061
11	1.0470	1	1.0050
12	1.0435	1	1.0042
13	1.0405	1	1.0036
14	1.0379	1	1.0031
15	1.0356	1	1.0027
16	1.0335	1	1.0024
17	1.0317	1	1.0021
18	1.0300	1	1.0019
19	1.0286	1	1.0017
20	1.0272	1	1.0015
21	1.0260	1	1.0014
22	1.0249	1	1.0013
23	1.0239	1	1.0012
24	1.0229	1	1.0011

σ	$n=\frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	μ_{ε_t}	ISRT
25	1.0220	1	1.0010
26	1.0212	1	1.0009
27	1.0205	1	1.0008
28	1.0198	1	1.0008
29	1.0191	1	1.0007
30	1.0185	1	1.0007
31	1.0179	1	1.0006
32	1.0174	1	1.0006
33	1.0169	1	1.0006
34	1.0164	1	1.0005
35	1.0137	1	1.0004
36	1.0134	1	1.0003
37	1.0131	1	1.0003
38	1.0128	1	1.0003
39	1.0125	1	1.0003
40	1.0122	1	1.0003
41	1.0120	1	1.0003
42	1.0117	1	1.0003
43	1.0115	1	1.0003
44	1.0113	1	1.0002
45	1.0111	1	1.0002
46	1.0109	1	1.0002
47	1.0107	1	1.0002
48	1.0105	1	1.0002
49	1.0103	1	1.0002
50	1.0101	1	1.0002
51	1.0099	1	1.0002
52	1.0098	1	1.0002
53	1.0096	1	1.0002
54	1.0094	1	1.0002
55	1.0093	1	1.0002
56	1.0091	1	1.0002
57	1.0090	1	1.0002
58	1.0089	1	1.0002
59	1.0087	1	1.0001
60	1.0086	1	1.0001
61	1.0085	1	1.0001
62	1.0083	1	1.0001
63	1.0082	1	1.0001
64	1.0089	1	1.0002
65	1.0087	1	1.0001
66	1.0086	1	1.0001
67	1.0085	1	1.0001
68	1.0083	1	1.0001
69	1.0082	1	1.0001
70	1.0081	1	1.0001
71	1.0080	1	1.0001
72	1.0079	1	1.0001
73	1.0078	1	1.0001
74	1.0077	1	1.0001
75	1.0076	1	1.0001
76	1.0075	1	1.0001
77	1.0074	1	1.0001
78	1.0073	1	1.0001
79	1.0072	1	1.0001
80	1.0071	1	1.0001
81	1.0070	1	1.0001
82	1.0069	1	1.0001
83	1.0069	1	1.0001
84	1.0068	1	1.0001
85	1.0067	1	1.0001
86	1.0066	1	1.0001
87	1.0065	1	1.0001
88	1.0065	1	1.0001
89	1.0064	1	1.0001
90	1.0063	1	1.0001
91	1.0063	1	1.0001
92	1.0062	1	1.0001
93	1.0061	1	1.0001
94	1.0061	1	1.0001
95	1.0060	1	1.0001
96	1.0059	1	1.0001
97	1.0059	1	1.0001
98	1.0058	1	1.0001
99	1.0058	1	1.0001
100	1.0057	1	1.0001

Table 2. *Variance of the untransformed and Transformed Distributions.*

σ	$n=\frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	ISRT
7	1.0690	0.0282	0.0097
8	1.0619	0.0220	0.0072
9	1.0560	0.0177	0.0056
10	1.0511	0.0145	0.0045
11	1.0470	0.0121	0.0037
12	1.0435	0.0102	0.0031
13	1.0405	0.0088	0.0026
14	1.0379	0.0076	0.0022
15	1.0356	0.0067	0.0019
16	1.0335	0.0059	0.0017
17	1.0317	0.0053	0.0015
18	1.0300	0.0047	0.0013
19	1.0286	0.0042	0.0012
20	1.0272	0.0038	0.0011
21	1.0260	0.0035	0.0010
22	1.0249	0.0032	0.0009
23	1.0239	0.0029	0.0008
24	1.0229	0.0027	0.0007
25	1.0220	0.0025	0.0007
26	1.0212	0.0023	0.0006
27	1.0205	0.0021	0.0006
28	1.0198	0.0020	0.0005
29	1.0191	0.0019	0.0005
30	1.0185	0.0017	0.0005
31	1.0179	0.0016	0.0004
32	1.0174	0.0015	0.0004
33	1.0169	0.0014	0.0004
34	1.0164	0.0014	0.0004
35	1.0137	0.0013	0.0003
36	1.0134	0.0012	0.0003
37	1.0131	0.0012	0.0003
38	1.0128	0.0011	0.0003
39	1.0125	0.0010	0.0003
40	1.0122	0.0010	0.0003
41	1.0120	0.0009	0.0002
42	1.0117	0.0009	0.0002
43	1.0115	0.0009	0.0002
44	1.0113	0.0008	0.0002
45	1.0111	0.0008	0.0002
46	1.0109	0.0008	0.0002
47	1.0107	0.0007	0.0002
48	1.0105	0.0007	0.0002
49	1.0103	0.0007	0.0002
50	1.0101	0.0006	0.0002
51	1.0099	0.0006	0.0002
52	1.0098	0.0006	0.0002
53	1.0096	0.0006	0.0001
54	1.0094	0.0005	0.0001
55	1.0093	0.0005	0.0001
56	1.0091	0.0005	0.0001
57	1.0090	0.0005	0.0001
58	1.0089	0.0005	0.0001
59	1.0087	0.0005	0.0001
60	1.0086	0.0004	0.0001
61	1.0085	0.0004	0.0001
62	1.0083	0.0004	0.0001
63	1.0082	0.0004	0.0001
64	1.0089	0.0004	0.0001
65	1.0087	0.0004	0.0001

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	ISRT
66	1.0086	0.0004	0.0001
67	1.0085	0.0004	0.0001
68	1.0083	0.0003	0.0001
69	1.0082	0.0003	0.0001
70	1.0081	0.0003	0.0001
71	1.0080	0.0003	0.0001
72	1.0079	0.0003	0.0001
73	1.0078	0.0003	0.0001
74	1.0077	0.0003	0.0001
75	1.0076	0.0003	0.0001
76	1.0075	0.0003	0.0001
77	1.0074	0.0003	0.0001
78	1.0073	0.0003	0.0001
79	1.0072	0.0003	0.0001
80	1.0071	0.0003	0.0001
81	1.0070	0.0002	0.0001
82	1.0069	0.0002	0.0001
83	1.0069	0.0002	0.0001
84	1.0068	0.0002	0.0001
85	1.0067	0.0002	0.0001
86	1.0066	0.0002	0.0001
87	1.0065	0.0002	0.0001
88	1.0065	0.0002	0.0001
89	1.0064	0.0002	0.0001
90	1.0063	0.0002	0.0001
91	1.0063	0.0002	0.0001
92	1.0062	0.0002	0.0000
93	1.0061	0.0002	0.0000
94	1.0061	0.0002	0.0000
95	1.0060	0.0002	0.0000
96	1.0059	0.0002	0.0000
97	1.0059	0.0002	0.0000
98	1.0058	0.0002	0.0000
99	1.0058	0.0002	0.0000
100	1.0057	0.0002	0.0000

Table 3. *Relative Change in Variance (RCIV) of the untransformed and Transformed Distributions.*

σ	$n = \frac{1}{\Gamma\left(1+\frac{1}{\sigma}\right)}$	$\sigma^2_{\varepsilon_t}$	ISRT
7	1.0690	0.0282	-.6579
8	1.0619	0.0220	-.6713
9	1.0560	0.0177	-.6812
10	1.0511	0.0145	-.6889
11	1.0470	0.0121	-.6951
12	1.0435	0.0102	-.7001
13	1.0405	0.0088	-.7043
14	1.0379	0.0076	-.7078
15	1.0356	0.0067	-.7108
16	1.0335	0.0059	-.7135
17	1.0317	0.0053	-.7157
18	1.0300	0.0047	-.7178
19	1.0286	0.0042	-.7196
20	1.0272	0.0038	-.7212
21	1.0260	0.0035	-.7226
22	1.0249	0.0032	-.7239
23	1.0239	0.0029	-.7251
24	1.0229	0.0027	-.7262
25	1.0220	0.0025	-.7272
26	1.0212	0.0023	-.7281
27	1.0205	0.0021	-.7289
28	1.0198	0.0020	-.7297
29	1.0191	0.0019	-.7304
30	1.0185	0.0017	-.7311
31	1.0179	0.0016	-.7317
32	1.0174	0.0015	-.7323
33	1.0169	0.0014	-.7329
34	1.0164	0.0014	-.7334
35	1.0137	0.0013	-.7339
36	1.0134	0.0012	-.7344
37	1.0131	0.0012	-.7348
38	1.0128	0.0011	-.7352
39	1.0125	0.0010	-.7356
40	1.0122	0.0010	-.7360
41	1.0120	0.0009	-.7363
42	1.0117	0.0009	-.7366
43	1.0115	0.0009	-.7370
44	1.0113	0.0008	-.7373
45	1.0111	0.0008	-.7376
46	1.0109	0.0008	-.7378
47	1.0107	0.0007	-.7381
48	1.0105	0.0007	-.7383
49	1.0103	0.0007	-.7386
50	1.0101	0.0006	-.7388
51	1.0099	0.0006	-.7390
52	1.0098	0.0006	-.7393
53	1.0096	0.0006	-.7395
54	1.0094	0.0005	-.7397
55	1.0093	0.0005	-.7399
56	1.0091	0.0005	-.7400
57	1.0090	0.0005	-.7402
58	1.0089	0.0005	-.7404
59	1.0087	0.0005	-.7406
60	1.0086	0.0004	-.7407
61	1.0085	0.0004	-.7409
62	1.0083	0.0004	-.7410
63	1.0082	0.0004	-.7412
64	1.0089	0.0004	-.7413
65	1.0087	0.0004	-.7414
66	1.0086	0.0004	-.7416
67	1.0085	0.0004	-.7417
68	1.0083	0.0003	-.7418
69	1.0082	0.0003	-.7419
70	1.0081	0.0003	-.7421
71	1.0080	0.0003	-.7422
72	1.0079	0.0003	-.7423
73	1.0078	0.0003	-.7424
74	1.0077	0.0003	-.7425
75	1.0076	0.0003	-.7426
76	1.0075	0.0003	-.7427
77	1.0074	0.0003	-.7428
78	1.0073	0.0003	-.7429
79	1.0072	0.0003	-.7430
80	1.0071	0.0003	-.7431
81	1.0070	0.0002	-.7432
82	1.0069	0.0002	-.7432
83	1.0069	0.0002	-.7433
84	1.0068	0.0002	-.7434
85	1.0067	0.0002	-.7435
86	1.0066	0.0002	-.7436
87	1.0065	0.0002	-.7436
88	1.0065	0.0002	-.7437
89	1.0064	0.0002	-.7438
90	1.0063	0.0002	-.7438
91	1.0063	0.0002	-.7439
92	1.0062	0.0002	-.7440
93	1.0061	0.0002	-.7440
94	1.0061	0.0002	-.7441
95	1.0060	0.0002	-.7442
96	1.0059	0.0002	-.7442
97	1.0059	0.0002	-.7443
98	1.0058	0.0002	-.7444
99	1.0058	0.0002	-.7444
100	1.0057	0.0002	-.7445

Table 4. *Mean values, minimum and maximum of the Relative Change in Variance for the Study Transformations.*

Transformation	Mean Relative Change in Variance	Minimum	Maximum
ISRT	-0.7331	-0.74447	-0.6579

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