
Analytical Models for Quark Stars with Van Der Waals Modified Equation of State

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Abstract: Stellar models consisting of spherically symmetric distribution of charged matter locally anisotropic in strong gravitational fields have been widely considered in the frame of general relativity. These investigations require the generation of exact models through the resolution of the Einstein-Maxwell system of equations. The presence of charge produces values for redshifts, luminosity and mass for the stars different in relation to neutral matter. Some applications for dense charged matter we have them in the description of quark stars, spheres with linear or non-linear equation of state, hybrid stars and accreting process in compact objects where the matter acquires large amounts of electric charge. In this paper, we studied the behavior of relativistic compact objects with anisotropic matter distribution considering Van der Waals modified equation of state proposed in 2013 for Malaver and a gravitational potential $Z(x)$ that depends on an adjustable parameter α in order to integrate analytically the field equations. They generalize the ideal gas law based on plausible reasons that real gases do not act ideally. New exact solutions of the Einstein-Maxwell system are generated and the physical variables as the energy density, radial pressure, mass function, anisotropy factor and the metric functions are written in terms of elementary and polynomial functions. We obtained expressions for radial pressure, density and mass of the stellar object physically acceptable with two different values of the adjustable parameter. The proposed models satisfy all physical features of a realistic star.

Keywords: Relativistic Compact Objects, Gravitational Potential, Einstein-Maxwell System, Radial Pressure, Anisotropy Factor, Matter Distribution, General Relativity, Einstein Field Equations

1. Introduction

The study of the relation between ultracompact objects and the gravitational collapse is one of the most fundamental and important factors in astrophysics and has attracted much researchers and scientists due to formulation of the general theory of relativity. On the other hand, one of the most important problems in general theory of relativity is to obtain exact solutions for Einstein field equations [1, 2]. These solutions include many applications in astrophysics, cosmology, string theory and so on [2]. Various types of mathematical formulations permit us solving Einstein's field equations in order to explain behaviour of objects in strong gravitational

fields such as neutron stars, quasars and white dwarfs [3-5].

From the development of Einstein's theory of general relativity, the description of compact objects has been a central issue in relativistic astrophysics in the last few decades [2, 6]. Recent experimental observations in binary pulsars [6] suggest that it could be quark stars. The existence of quark stars in hydrostatic equilibrium was first suggested by Itoh [7] in a seminal treatment. The study of strange stars consisting of quark matter has stimulated much interest so that it could be represented as most energetically favorable state of baryon matter [8].

In the construction of the first theoretical models of relativistic stars, some works are important such as Schwarzschild [9], Tolman [10], Oppenheimer and Volkoff

[11]. Schwarzschild [9] found exact solutions to the Einstein's Field Equations and Tolman [10] proposed a method in order to obtain explicit solutions of static spheres of fluid in terms of known analytical functions. Oppenheimer and Volkoff [11] have deployed Tolman's solutions in order to investigate about gravitational balance of neutron stars. It is noticed that Chandrasekhar's contributions [12] in modelling for production of white dwarfs under relativistic effects and research of Baade and Zwicky [13] establish the concept of neutron stars as relativistic star of very dense matter.

Also, it is noticed that the frame of the general relativity is very important which includes the presence of anisotropy in the pressure [14-26] for description of the behavior of relativistic gravitating matter and is defined as $\Delta = p_t - p_r$ where p_r is the radial pressure and p_t is the tangential pressure. Bowers and Liang [27] extensively discuss the effect of anisotropy in general relativity. The existence of anisotropy within a star can be explained by the presence of a solid core, phase transitions, a type III super fluid, a pion condensation [28] or another physical phenomenon by the presence of an electrical field [29]. Many researchers have used a great variety of mathematical techniques to try in order to obtain solutions of the Einstein-Maxwell field equations since it has been demonstrated by Komathiraj and Maharaj [30], Thirukkanesh and Maharaj [31], Maharaj et al. [32], Thirukkanesh and Ragel [33, 34], Feroze and Siddiqui [35, 36], Sunzu et al. [37], Pant et al. [38] and Malaver [39-42]. These investigations show that the system of Einstein-Maxwell equations plays an important role to describe ultracompact objects.

Thirukkanesh and Maharaj [31], Komathiraj and Maharaj [8], Malaver [43], Bombaci [44], Dey et al [45] and Usov [29] consider linear format of this equation of state for quark stars. Feroze and Siddiqui [35] present a quadratic format equation of state for matter distribution and demonstrate special forms for gravitational potential and electric field intensity. Mafa Takisa and Maharaj [46] found new exact solutions to the Einstein-Maxwell system of equations through a polytropic equation of state. Thirukkanesh and Ragel [6] have studied special models of anisotropic fluids using polytropic equation of state, which have consistency with reported experimental observations. In addition, Malaver [47] obtained new exact solutions to the Einstein-Maxwell system through Van der Waals modified equation of state with polytropic exponent. Mak and Harko [48] presented a relativistic model of strange quark star through the suppositions of spherical symmetry and conformal Killing vector.

The aim of this paper is to generate a new class of anisotropic matter with Van der Waals modified (VDWM) equation of state proposed for Malaver [49] in a static spherically symmetric space-time using a gravitational potential $Z(x)$ which depends on an adjustable parameter α . We have obtained some new classes of static spherically symmetrical models for an uncharged anisotropic matter distribution where the variation of the parameter modifies the radial pressure, energy density and the mass of the compact objects.

This paper has been organized as follows: In Section 2, Einstein's field equations will be presented. In section 3, a particular choice of gravitational potential $Z(x)$, allows us to solve field equations and we have obtained new models for uncharged anisotropic matter. In Section 4, a physical analysis of the new solutions is performed. Finally, in Section 5, we make a conclusion about obtained and discussed results.

2. The Field Equations

Consider a spherically symmetric four-dimensional space-time so that whose line element is given in Schwarzschild coordinates by

$$ds^2 = -e^{2\nu(r)} dt^2 + e^{2\lambda(r)} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

at which $\nu(r)$ and $\lambda(r)$ are considered as two arbitrary functions. Einstein-Maxwell system of field equations in uncharged perfect fluids are formulated as follows:

$$\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\lambda'}{r} e^{-2\lambda} = \rho \quad (2)$$

$$-\frac{1}{r^2}(1 - e^{-2\lambda}) + \frac{2\nu'}{r} e^{-2\lambda} = p_r \quad (3)$$

$$e^{-2\lambda} \left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r} \right) = p_t \quad (4)$$

so that ρ is the energy density, p_r is the radial pressure and p_t is the tangential pressure, Δ is the anisotropy and primes denote differentiations with respect to r . By transformations proposed by Dugapal and Bannerji [50] as $x = cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A^2 y^2(x) = e^{2\nu(r)}$ where A and c are arbitrary constants, then the Einstein-Maxwell system has the equivalent form as follows:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{c} \quad (5)$$

$$4Z \frac{\dot{y}}{y} - \frac{1-Z}{x} = \frac{p_r}{c} \quad (6)$$

$$4xZ \frac{\ddot{y}}{y} + (4Z + 2x\dot{Z}) \frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{c} \quad (7)$$

$$4xZ \frac{\dot{y}}{y} + \dot{Z} \left(1 + 2x \frac{\dot{y}}{y} \right) + \frac{1-Z}{x} = \frac{\Delta}{c} \quad (8)$$

at which dots denote differentiations with respect to x .

According to Durgapal and Bannerji [50], the mass within a radius r in the sphere leads to:

$$M(x) = \frac{1}{4c^{3/2}} \int_0^x \sqrt{x} \rho(x) dx \quad (9)$$

In this paper, we assume the VDWM equation of state

$$\rho = ac \left(3 - \frac{5}{4} ax \right) \tag{11}$$

$$p_r = \alpha \rho^2 + \frac{\gamma \rho}{1 + \beta \rho} \tag{10}$$

proposed by Malaver [49]. In eq. (10) α, β and γ are arbitrary constants and ρ is the energy density.

Substituting (11) into eq.(10), the radial pressure can be written in the form

$$p_r = \frac{1}{2} a^2 c^2 \left(3 - \frac{5}{4} ax \right)^2 + \frac{\gamma ac \left(3 - \frac{5}{4} ax \right)}{1 + \beta ac \left(3 - \frac{5}{4} ax \right)} \tag{12}$$

3. The New Models

In this work, we have chosen the form of the gravitational potential $Z(x)$ as $Z(x) = (1 - \alpha ax)^2$ where a is a real constant and α is an adjustable parameter. This potential is well behaved and regular at the origin in the interior of the sphere. We have considered the particular cases for $\alpha=1/2, 2$. For the case $\alpha=1/2$, using $Z(x)$ in eq.(5), we obtain

Using (11) in (9), the expression of the mass function is

$$M(x) = \frac{a(4 - ax)}{8\sqrt{c}} x^{3/2} \tag{13}$$

With (11) and (12), eq. (6) becomes

$$\frac{\dot{y}}{y} = \frac{5A^2 \beta c}{5a^2 \beta cx - 12a\beta c - 4} + \frac{aB}{ax - 2} - \frac{2Cx + D}{32(a\beta c + 2)(ax - 2)} + \frac{(Cx^2 + Dx + E)a}{32(a\beta c + 2)(ax - 2)^2} \tag{14}$$

where for convenience we have let

$$A^* = \frac{5\gamma}{(a\beta c + 2)^2} \quad B = \frac{(-5a^3 \beta^2 c^3 + 2a^2 \beta^2 c^2 - 20a^2 \beta c^2 + 8a\beta c - 20ac - 40\gamma + 8)}{8(a\beta c + 2)^2} \quad C = -25a^4 \beta c^2 - 50a^3 c, \quad D = 50a^3 \beta c^2 + 100a^2 c$$

and

$$E = 4a^2 \beta c^2 - 16a\beta c + 8ac + 32\gamma - 32$$

Integrating eq. (14), we have

$$y(x) = c_1 \left(5a^2 \beta cx - 12a\beta c - 4 \right)^{A^*} (ax - 2)^B e^{\left[\frac{Cx^2 + Dx + E}{32(a\beta c + 2)(ax - 2)} \right]} \tag{15}$$

c_1 is the constant of integration.

The anisotropy factor is defined as $\Delta = p_t - p_r$ and for $\alpha=1/2$ Δ is given for

$$\Delta = 4cx \left(1 - \frac{1}{2} ax \right)^2 \left[\frac{25A^* a^4 \beta^2 c^2 (A-1)}{(5a^2 \beta cx - 12a\beta c - 4)^2} + \frac{10A^* a^3 \beta c B}{(5a^2 \beta cx - 12a\beta c - 4)(ax - 2)} - \frac{10A^* a^2 \beta c (2Cx + D)}{32(a\beta c + 2)(5a^2 \beta cx - 12a\beta c - 4)(ax - 2)} \right. \\ \left. + \frac{10A^* a^3 \beta c (Cx^2 + Dx + E)}{32(a\beta c + 2)(5a^2 \beta cx - 12a\beta c - 4)(ax - 2)^2} + \frac{B^2 a^2 - Ba^2}{(ax - 2)^2} - \frac{aB(2Cx + D)}{16(a\beta c + 2)(ax - 2)^2} + \frac{a^2 B (Cx^2 + Dx + E)}{(a\beta c + 2)(ax - 2)^3} \right. \\ \left. - \frac{C}{16(a\beta c + 2)(ax - 2)} + \frac{(2Cx + D)a}{16(a\beta c + 2)(ax - 2)^2} - \frac{(Cx^2 + Dx + E)a^2}{16(a\beta c + 2)(ax - 2)^3} \right. \\ \left. + \left[-\frac{2Cx + D}{32(a\beta c + 2)(ax - 2)} + \frac{(Cx^2 + Dx + E)a}{32(a\beta c + 2)(ax - 2)^2} \right]^2 \right] \\ - ac \left(1 - \frac{1}{2} a^2 x \right) \left[1 + 2x \left\{ \frac{5A^* a^2 \beta c}{5a^2 \beta cx - 12a\beta c - 4} + \frac{aB}{ax - 2} - \frac{2Cx + D}{32(a\beta c + 2)(ax - 2)} + \frac{(Cx^2 + Dx + E)a}{32(a\beta c + 2)(ax - 2)^2} \right\} \right] + ac - \frac{1}{4} a^2 cx \tag{16}$$

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as

$$e^{2\lambda} = \frac{1}{\left(1 - \frac{1}{2} ax \right)^2} \tag{17}$$

$$e^{2\nu} = A^2 c_1^2 \left(5a^2 \beta c x - 12a \beta c - 4\right)^{2A} (ax - 2)^{2B} e^{\left[\frac{Cx^2 + Dx + E}{16(a\beta c + 2)(ax - 2)}\right]} \quad (18)$$

With $\alpha=2$, the expression for the energy density is

$$\rho = ac(12 - 20ax) \quad (19)$$

Replacing (19) in (10), we have for the radial pressure

$$p_r = 2c^2(12a - 20a^2x)^2 + \frac{\gamma c(12a - 20a^2x)}{1 + \beta c(12a - 20a^2x)} \quad (20)$$

and the mass function is

$$M(x) = \frac{2a(1-ax)}{\sqrt{c}} x^{3/2} \quad (21)$$

The eq. (6) becomes

$$\frac{\dot{y}}{y} = \frac{20Fa^2\beta c}{20a^2\beta c x - 12a\beta c - 1} + \frac{2Ga}{2ax - 1} - \frac{2Hx + F}{4(2a\beta c + 1)(2ax - 1)} + \frac{(Hx^2 + Ix + J)a}{2(2a\beta c + 1)(2ax - 1)^2} \quad (22)$$

Again for convenience

$$F = \frac{5\gamma}{4(2a\beta c + 1)^2}$$

$$G = \frac{(-160a^3\beta^2c^3 - 4a^2\beta^2c^2 - 160a^2\beta c^2 - 4a\beta c - 40ac - 5\gamma - 1)}{4(2a\beta c + 1)^2}$$

$$H = -800a^4\beta c^2 - 400a^3c, \quad I = 400a^3\beta c^2 + 200a^2c$$

and $J = 8a^2\beta c^2 + 2a\beta c + 4ac + \gamma + 1$

Integrating eq. (22), we have

$$y(x) = c_2 \left(20a^2\beta c x - 12a\beta c - 1\right)^F (2ax - 1)^G e^{\left[\frac{Hx^2 + Ix + J}{4(2a\beta c + 1)(2ax - 1)}\right]} \quad (23)$$

The metric functions $e^{2\lambda}$, $e^{2\nu}$ and the anisotropy factor Δ can be written as

$$e^{2\lambda} = \frac{1}{(1 - 2ax)^2} \quad (24)$$

$$e^{2\nu} = A^2 c_2^2 \left(20a^2\beta c x - 12a\beta c - 1\right)^{2F} (2ax - 1)^{2G} e^{\left[\frac{Hx^2 + Ix + J}{2(2a\beta c + 1)(2ax - 1)}\right]} \quad (25)$$

$$\Delta = 4cx(1 - 2ax)^2 \left[\frac{400F^2 a^4 \beta^2 c^2 (F - 1)}{(20a^2\beta c x - 12a\beta c - 1)^2} + \frac{80Fa^3\beta c BG}{(20a^2\beta c x - 12a\beta c - 1)(2ax - 1)} - \frac{10Fa^2\beta c(2Hx + I)}{(2a\beta c + 1)(20a^2\beta c x - 12a\beta c - 1)(2ax - 1)} \right. \\ \left. + \frac{20Fa^3\beta c(Hx^2 + Ix + J)}{(2a\beta c + 1)(20a^2\beta c x - 12a\beta c - 1)(2ax - 1)^2} + \frac{4(G^2 a^2 - Ga^2)}{(2ax - 1)^2} - \frac{aG(2Hx + I)}{(2a\beta c + 1)(2ax - 1)^2} + \frac{2a^2G(Hx^2 + Ix + J)}{(2a\beta c + 1)(2ax - 1)^3} \right. \\ \left. - \frac{H}{2(2a\beta c + 1)(2ax - 1)} + \frac{(2Hx + I)a}{(2a\beta c + 1)(2ax - 1)^2} - \frac{2(Hx^2 + Ix + J)a^2}{(2a\beta c + 1)(2ax - 1)^3} \right. \\ \left. + \left\{ \frac{2Hx + I}{4(2a\beta c + 1)(2ax - 1)} + \frac{(Hx^2 + Ix + J)a}{2(2a\beta c + 1)(2ax - 1)^2} \right\}^2 \right] \\ - 4ac(1 - 2ax) \left[1 + 2x \left\{ \frac{20Fa^2\beta c}{20a^2\beta c x - 12a\beta c - 1} + \frac{2aG}{2ax - 1} - \frac{2Hx + D}{4(2a\beta c + 1)(2ax - 1)} + \frac{(Hx^2 + Ix + J)a}{2(2a\beta c + 1)(2ax - 1)^2} \right\} \right] + 4ac - 4a^2cx \quad (26)$$

4. Physical Properties of the Models

All physical acceptable solutions must satisfy the following conditions [6,15]:

- (i) Regularity of the gravitational potentials is in the origin.
- (ii) Radial pressure must be finite at the centre and it vanishes at the surface of the sphere.
- (iii) $p_r > 0$ and $\rho > 0$ in the origin.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{27}$$

For the case $\alpha=1/2$, $e^{2\lambda(0)}=1$, $e^{2\nu(0)} = A^2 c_1^2 (-12a\beta c - 4)^{2A} (-2)^{2B} e^{\frac{E}{32(a\beta c + 2)}}$ in the origin $r = 0$ and

$$\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$$

This indicates that the potential gravitational is regular in the origin.

In the centre $r = 0$, $\rho(0) = 3ac$ and

$$p_r(0) = \frac{9}{2} a^2 c^2 + \frac{3ac\gamma}{1+3ac\beta}$$

both are positive if $a > 0$. In the surface of the star $r=R$, we have $p_r(r=R) = 0$ and

$$R = \frac{6}{\sqrt{15ac}}$$

From (13) we have:

$$M(r) = \frac{ac(4 - acr^2)}{8} r^3 \tag{28}$$

and the total mass of the star is

$$0 \leq \frac{3ac + 18a^2c^2\beta + 27a^3c^3\beta^2 + \gamma - (15a^3c^3\beta + \frac{135}{4}a^4c^4\beta^2 + \frac{5}{4}a^2c^2)r^2 + (\frac{225}{16}a^5c^5\beta^2 + \frac{25}{8}a^4c^4\beta)r^4 - \frac{125}{64}a^6c^6\beta^2r^6}{\left(1 + 3\beta ac - \frac{5}{4}\beta a^2c^2r^2\right)^2} \leq 1$$

The metric for this model is

$$ds^2 = -A^2 c_1^2 (5a^2\beta c^2 r^2 - 12a\beta c - 4)^{2A} (acr^2 - 2)^{2B} e^{\frac{[Cc^2r^4 + Dcr^2 + E]}{16(a\beta c + 2)(acr^2 - 2)}} + \frac{dr^2}{\left(1 - \frac{1}{2}acr^2\right)^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \tag{32}$$

With $\alpha=2$, $e^{2\lambda(0)}=1$,

$$e^{2\nu(0)} = A^2 c_2^2 (-12a\beta c - 1)^{2F} (-1)^{2G} e^{\left[\frac{J}{2(2a\beta c + 1)}\right]}$$

(iv) Decreasing of the energy density and the radial pressure with increasing of the radius.

(v) The radial and the tangential pressure are equal to zero at the centre $r=0$.

(vi) In the surface of the sphere, it should be matched with the Schwarzschild exterior solution, for which the metric is given by

$$M(r=R) = \frac{72}{25\sqrt{15ac}} \tag{29}$$

Setting and matching conditions for $r=R$ can be written as

$$\left(1 - \frac{2M}{R}\right) = A^2 y^2 (cr^2) \tag{30}$$

$$\left(1 - \frac{2M}{R}\right)^{-1} = \frac{1}{\left(1 - \frac{1}{2}acR^2\right)^2} \tag{31}$$

With the purpose of satisfying the causality condition, the radial sound speed that is defined as $v_{sr}^2 = \frac{dp_r}{d\rho}$ should be within the limit $0 \leq v_{sr}^2 \leq 1$ in the interior of the star [51]. In this model, we have:

$$v_{sr}^2 = \frac{dp_r}{d\rho} = ac\left(3 - \frac{5}{4}ax\right) + \frac{\gamma}{\left[1 + \beta ac\left(3 - \frac{5}{4}ax\right)\right]^2}$$

and we must impose the condition

again the gravitational potential is regular in $r = 0$.

In addition, we have in the centre $\rho(0) = 12ac$ and

$$p_r(0) = 288a^2c^2 + \frac{12\gamma ac}{1+12\beta ac}$$

In the boundary of the star $r=R$, we have $p(r=R) = 0$

and $R = \frac{3}{\sqrt{15ac}}$.

This is a new value found for the radius of the star. From (21), we obtain

$$M(r) = 2acr^3(1-acr^2) \tag{33}$$

and the total mass of the star is

$$0 \leq \frac{(48ac + 1152a^2c^2\beta + 6912a^3c^3\beta^2 + \gamma - (3840a^3c^3\beta + 34560a^4c^4\beta^2 + 80a^2c^2)r^2 + (57600a^5c^5\beta^2 + 3200a^4c^4\beta)r^4 - 32000a^6c^6\beta^2r^6)}{(1+12\beta ac - 20\beta a^2c^2r^2)^2} \leq 1$$

and the metric for this model is

$$ds^2 = -A^2c_2^2(20a^2\beta c^2r^2 - 12a\beta c - 1)^{2F}(2acr^2 - 1)^{2G} e^{\frac{[Hc^2r^4 + Icr^2 + J]}{2(a\beta c + 1)(acr^2 - 1)}} + \frac{dr^2}{(1-2acr^2)^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{35}$$

In figures 1 and 6, it is observed that the radial pressure is finite and decreasing from the center to the surface of the star in the two studied cases. In figures 2 and 7, the energy density is continuous, also is finite and monotonically decreasing function. In figures 3 and 8, the mass function is strictly increasing, continuous and finite. The variation of the measure of anisotropy in the two cases is shown in figures 4 and 9. In figure 4 for the model with $\alpha=1/2$, the degree of anisotropy reaches a minimum value near at 8 km and then remains finite and continuous throughout the interior of the star. In the figure 9, for the case $\alpha=2$, the measure of anisotropy is increasing and continuous in the stellar interior. In both cases, the anisotropy Δ vanish at the center and this means that the radial and tangential pressures should be equal in $r=0$. The figures 5 and 10 show that the condition $0 \leq v_{sr}^2 \leq 1$ is maintained throughout the interior of the star and satisfy the causality, which is a physical requirement for the construction of a realistic star [51].

The figures 1, 2, 3, 4, 5 represent the graphs of $p_r, \rho, M(x), \Delta$ and v_{sr}^2 , respectively with $\alpha=\gamma=1/2, \beta=1, a=0.024, c=1$ and a stellar radius of $r=10$ km.

$$M(r=R) = \frac{36}{25\sqrt{15ac}} \tag{34}$$

Setting and matching conditions for $r=R$ can be written as

$$\left(1 - \frac{2M}{R}\right) = A^2y^2(cr^2)$$

and

$$\left(1 - \frac{2M}{R}\right)^{-1} = \frac{1}{(1-2acR^2)^2}$$

For this case, the condition $0 \leq v_{sr}^2 \leq 1$, implies that

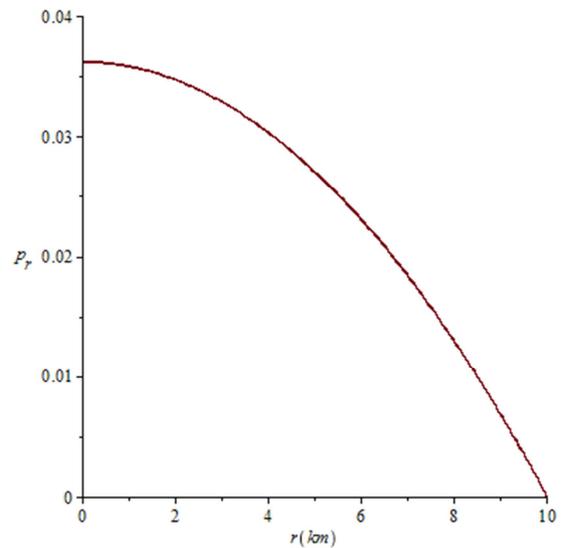


Figure 1. Radial pressure vs radial coordinate for $\alpha=\gamma=1/2, \beta=1$ where $a=0.024$ and $c=1$.

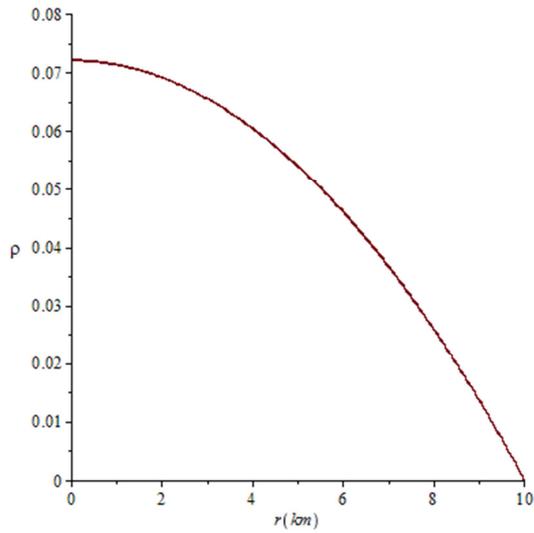


Figure 2. Energy density vs radial coordinate for $\alpha=\gamma=1/2$, $\beta=1$ where $a=0.024$ and $c=1$.

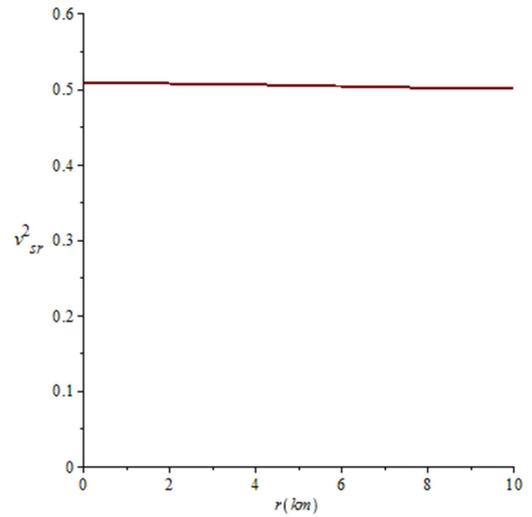


Figure 5. Radial speed sound vs radial coordinate for $\alpha=\gamma=1/2$, $\beta=1$ where $a=0.024$ and $c=1$.

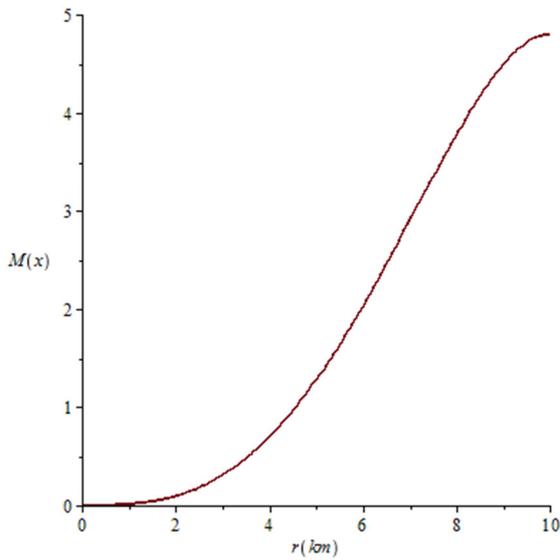


Figure 3. Mass function vs radial coordinate for $\alpha=\gamma=1/2$, $\beta=1$ where $a=0.024$ and $c=1$.

The figures 6, 7, 8, 9, 10 represent the graphs of p_r , ρ , $M(x)$, Δ and v^2_{sr} , respectively with $\alpha=2$, $\gamma=1/4$, $\beta=1$, $a=0.012$ and a stellar radius of $r=7.1$ km.

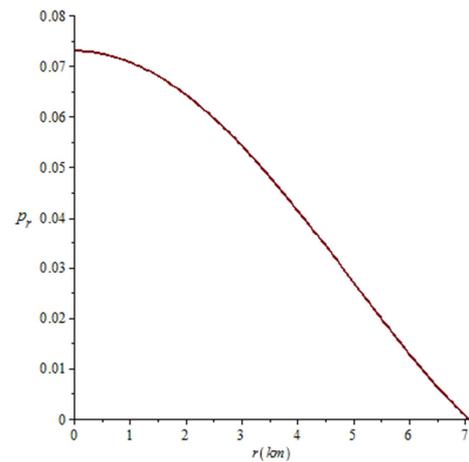


Figure 6. Radial pressure vs radial coordinate for $\alpha=2$, $\beta=1$, $\gamma=1/4$ where $a=0.012$ and $c=1$.

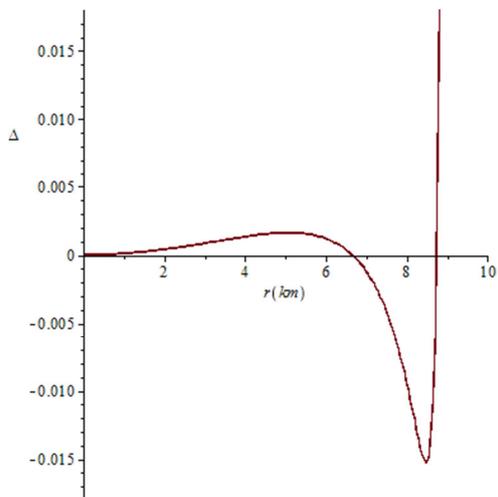


Figure 4. Measure of anisotropy vs radial coordinate for $\alpha=\gamma=1/2$, $\beta=1$ where $a=0.024$ and $c=1$.

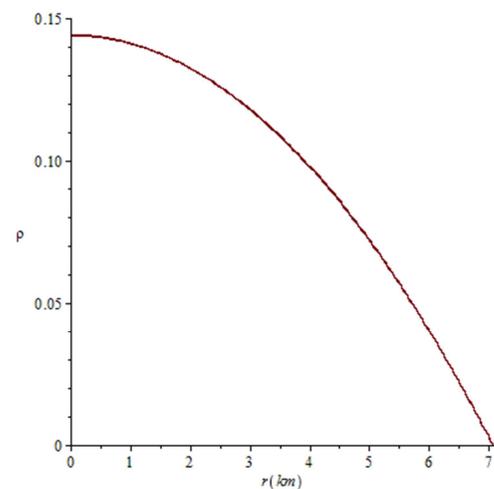


Figure 7. Energy density vs radial coordinate for $\alpha=2$, $\beta=1$, $\gamma=1/4$ where $a=0.012$ and $c=1$.

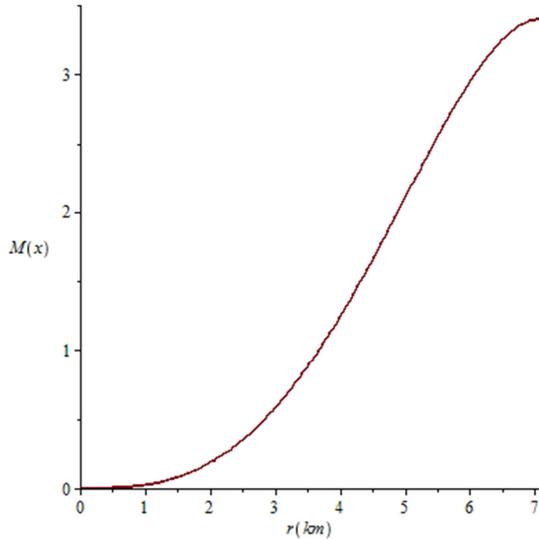


Figure 8. Mass function vs radial coordinate for $\alpha= 2, \beta=1, \gamma=1/4$ where $a=0.012$ and $c=1$.

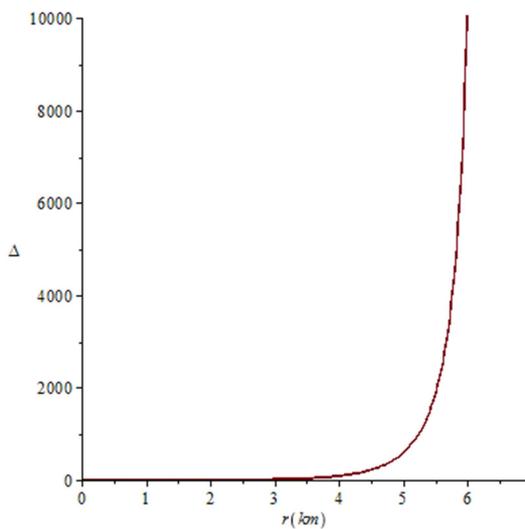


Figure 9. Measure of anisotropy vs radial coordinate for $\alpha= 2, \beta=1, \gamma=1/4$ where $a=0.012$ and $c=1$.

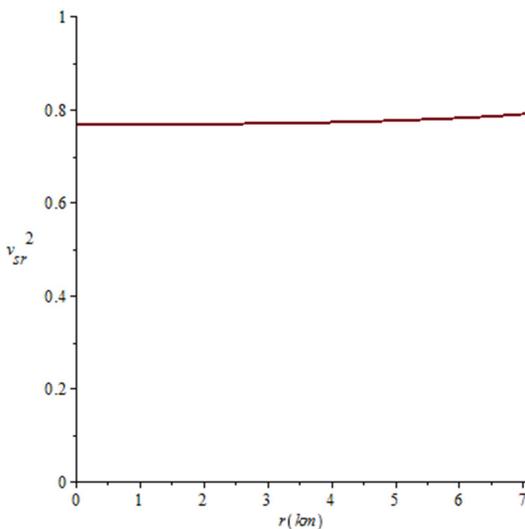


Figure 10. Radial speed sound vs radial coordinate for $\alpha= 2, \beta=1, \gamma=1/4$ where $a=0.012$ and $c=1$.

5. Conclusion

In this paper, we have generated a new class of models with a Van der Waals modified equation of state which could describe the behavior of an anisotropic matter distribution in an static spherically symmetric space-time where the gravitational potential Z depends on an adjustable parameter α . All the obtained models are physically reasonable and satisfy the physical characteristics of a realistic star as are the regularity of the gravitational potentials at the origin, cancellation of anisotropy in $r=0$, radial pressure finite at the centre and decreasing of the energy density and the radial pressure from the centre to the surface of the star. These solutions match with the Schwarzschild exterior metric at the boundary for each value of adjustable parameter.

We can compare the values calculated for energy density, mass and radius with some experimental results. For $\alpha=1/2$, the radius and the total mass of the star is given

$$\text{by } R = \frac{6}{\sqrt{15ac}} \text{ and } M(r=R) = \frac{72}{25\sqrt{15ac}} .$$

A compact object with this mass and radius could have a real existence. Experimental observations suggest a strange star model for 4U 1820-30 which has a radius of 10 km and a central density $\rho_0 = 3.179364432 \times 10^{15} \text{g/cm}^3$ [52]. With $\alpha=2$, we obtained a radius of 7.1 km which could correspond to a model for SAX J 1808.4-3658 with $\rho_0 = 4.808527413 \times 10^{15} \text{g/cm}^3$ [52, 53]. The models presented in this article may be useful in the description of relativistic compact objects, strange quark stars and configurations with anisotropic matter and can be obtained strange stars models of mass and densities comparable to the experimental results.

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