
Statistical Characteristics of New Type Internal Wave in the Ionospheric F Region

Jandieri George^{1,*}, Rahul Kaushik², Tugushi Nika³

¹Physics Dept., Georgian Technical University, GTU, Tbilisi, Georgia

²ECE Dept., Jaypee Institute of Information Technology, JIIT, Noida, India

³Energetic Dept., Georgian Technical University, GTU, Tbilisi, Georgia

Email address:

georgejandieri7@gmail.com (J. George), rahul.kaushik@jiit.ac.in (R. Kaushik), tugushinika96@gmail.com (T. Nika)

*Corresponding author

To cite this article:

Jandieri George, Rahul Kaushik, Tugushi Nika. Statistical Characteristics of New Type Internal Wave in the Ionospheric F Region.

International Journal of Astrophysics and Space Science. Vol. 5, No. 4, 2017, pp. 55-62. doi: 10.11648/j.ijass.20170504.11

Received: June 21, 2017; **Accepted:** July 5, 2017; **Published:** October 31, 2017

Abstract: Second order statistical moments of new internal MHD wave in the ionospheric F region are investigated analytically by geometrical optics approximation. Degree of a curvature of a constant phase surface and the variance of an instant frequency measuring by experiment has been obtained for arbitrary correlation function of electron density fluctuations. Energy exchange between the internal wave and turbulent plasma flow is considered calculating the mean energy flux density in the first and second approximations. Numerical calculations are carrying out for both anisotropic Gaussian correlation function and power law-spectrum characterizing elongated plasma irregularities using experimental data of satellites and incoherent scatter radar observations.

Keywords: Ionospheric Plasma, Internal Wave, Plasma Irregularities, Statistical Moments

1. Introduction

At present time it was established that statistical characteristics of wave depend on as the correlation features of the variable parameters of a chaotically random medium, as well as on the dispersion and the type of waves. The features of transverse electromagnetic waves and low-frequency waves in different turbulent media were investigated quite well [1-3]. However fluctuation features of the longitudinal waves and different MHD waves are not investigated in detail. Propagation of MHD waves in the turbulent plasma flow is of great interest in both cosmic and laboratory plasma.

The Earth magnetic field in the ionosphere generates small and medium-scale MHD waves: Alfvén and magneto-acoustic waves. The last one are fast (propagation velocity more than 1 km/s) and short-period (of the order of 5-20 min) waves generating by elasticity of the geomagnetic lines of force. Alfvén waves are generated due to tension of the geomagnetic lines of force and its velocity depends on the orientation of the wave vector with respect to the geomagnetic field \mathbf{H}_0 . They

can be very slow (10-50 m/s) and long period (1-2 days) when the wave vector \mathbf{k} is almost transversal to \mathbf{H}_0 , and fast, when vectors \mathbf{k} and \mathbf{H}_0 are parallel. Some peculiarities of the second order statistical moments of scattered Alfvén and magneto-acoustic waves in the turbulent plasma flow were considered in [4-6].

We suppose that the external magnetic field \mathbf{H}_0 is directed along the z-axis and turbulent plasma flow propagates with the mean constant velocity \mathbf{V}_0 making angle θ_1 to the vector \mathbf{H}_0 substantially exceeding root-mean-square velocity of turbulent mixing $|\mathbf{v}(\mathbf{r}, t)| \ll \mathbf{V}_0$. Let small amplitude E_0 monochromatic inertial low-frequency MHD wave with the frequency ω_0 and the wave vector \mathbf{k}_0 is generated in the plane $z=0$ making angle θ to the vector \mathbf{H}_0 . Electron density fluctuations surpass freezing-in magnetic field fluctuations and the conditions are fulfilled: $(H_1 / H_0) \ll (|N_1| / N_0) \ll 1$.

In the case of low-pressure plasma neglecting thermal velocities and ions motion under the action of the wave field,

if electron density fluctuations are smooth and plasma nonstationarity is caused by turbulent motion, applying geometrical optics method [7] the following conditions are fulfilled:

$$k_0 l \gg 1, \frac{\omega_0 l}{V_0} \gg 1 \quad (1)$$

where l is characteristic linear scale of electron density irregularities. In this case we will neglect transformation of different waves and consider one of the most interesting case when collision frequency between plasma particles is small with respect to the frequency of an incident wave.

In the absence of temperature stratification, dissipation and compression only internal waves are generated in the ionospheric F-region under the action of geomagnetic field [8, 9]. New Alfvén-type short-wave ($\lambda \leq 10^3$ km) and low-frequency inertial wave generating due to the Ampere electromagnetic force has been discovered in the ionospheric F region [10, 11].

In this paper statistical characteristics of scattered new inertial MHD wave in the ionospheric F-region are considered for the first time. Several specific features are arising at MHD wave propagation in the ionospheric plasma with spatial-temporal chaotically varying parameters, particularly energy exchange between the wave and nonstationary medium.

2. Formulation of the Problem

In the F region where the Hall effect is absent and $H_0 = \text{const}$, the linearized system of Helmholtz-Freedman equations for internal waves has the following form [10, 11]:

$$\begin{aligned} \frac{\partial \text{rot } \mathbf{V}}{\partial t} &= \text{rot} [\mathbf{u} \cdot 2\mathbf{\Omega}_H], \quad \frac{\partial \text{rot } \mathbf{u}}{\partial t} = \text{rot} [\mathbf{V} \cdot 2\mathbf{\Omega}_i], \\ \text{div } \mathbf{V} &= 0, \quad \text{div } \mathbf{u} = 0. \end{aligned} \quad (2)$$

The condition $\text{div } \mathbf{V} = 0$ means that considered below perturbations are transverse waves; \mathbf{V} is perturbation of the neutral particles velocity, \mathbf{u} is vector potential having velocity dimension. Characteristic frequencies of the internal waves caused by geomagnetic field are:

$$\begin{aligned} 2\mathbf{\Omega}_H &= \frac{ck^2}{4\pi eN} \mathbf{H}_0 = 2\mathbf{\Omega}_H \boldsymbol{\tau}, \\ 2\mathbf{\Omega}_i &= \zeta \frac{e}{Mc} \mathbf{H}_0 = 2\mathbf{\Omega}_i \boldsymbol{\tau}, \end{aligned} \quad (3)$$

where: $\boldsymbol{\tau} = \frac{\mathbf{H}_0}{H_0}$ is a unit vector, $\zeta = \frac{N}{N + N_n} \approx \frac{N}{N_n}$ is the iconicity of ionospheric medium varying in 10^{-9} up to 10^{-3} in a height interval 80-600 km, M is the mass of an ion or molecule, N_n is the concentration of neutral particles, e and

N are charge and concentration of electrons. Since the dynamic equations of system (2) are linear and sources and dissipation of energy are not taken into account, the energy of inertial waves cannot change in time and hence the wave frequency ω_0 should be real.

One new branch of the Alfvén-type low-frequency MHD wave has been revealed in the ionospheric F-region [10, 11]:

$$\omega = \pm \omega_a \cos \theta, \quad (4)$$

where: $\omega_a = kV_a$, $V_a = \frac{H_0}{\sqrt{4\pi\rho}} = \sqrt{\zeta} \frac{H_0}{\sqrt{4\pi MN}}$, θ is

angle between vectors \mathbf{k} and \mathbf{H}_0 . Upper sign corresponds to the fast MHD wave, lower sign to the slow MHD wave. The density of neutral particles ρ is important for the frequency ω_a . The phase and group velocities of this wave are:

$$V_{ph} = \frac{\omega}{k} = \frac{\omega_a}{k} \cos \theta, \quad V_{gr} = \frac{\partial \omega}{\partial \mathbf{k}} = \frac{\omega_a}{k} \{ \boldsymbol{\tau} - \mathbf{n} \cos \theta \} \quad (5)$$

The group velocity of this wave (velocity of propagation) is normal to wave vector \mathbf{k} ; its absolute value and projection in direction \mathbf{H}_0 would be written as:

$$|V_{gr}| = \frac{\omega_a}{k} \sin \theta, \quad (\mathbf{V}_{gr} \cdot \boldsymbol{\tau}) = \frac{\omega_a}{k} \sin^2 \theta = |V_{gr}| \sin \theta. \quad (6)$$

From equations (5) and (6) follow, that when the group velocity has a component directed upward, the phase velocity has a component directed downward and vice versa [10, 11]. Short inertial waves in the ionosphere belong to the class of very low-frequency (VLF) electromagnetic waves.

From equation (5) also follows that the information transfer to the Earth by the slow MHD waves by means of geomagnetic lines of force $H_0 = \text{const}$ would not be complete, because a part of the information is lost owing to the second term in (5). From equation (4) follows that numerical values of the frequency of internal waves depend strongly on angle θ . They reach the maximum at $\theta = 0$ and disappear at $\theta = 90^\circ$.

3. Statistical Characteristics of Scattered Internal Wave in the Ionospheric F Region

If the mean velocity \mathbf{V}_0 of turbulent plasma flow makes angle θ_1 to the external magnetic field \mathbf{H}_0 and the wave vector of internal wave is located in the principle yoZ -plane, the dispersion equation (4) can be rewritten as: $\omega - (\mathbf{k} \cdot \mathbf{V}_0) = \pm \omega_a \cos \theta$. Using standard procedure [7] introducing the instant frequency $\omega(\mathbf{r}, t) = \partial \varphi / \partial t$ and wave vector $\mathbf{k}(\mathbf{r}, t) = |k| \mathbf{s} = -\nabla \varphi$ we obtain the eikonal equation for internal wave in the turbulent plasma flow. Phase we submit as: $\varphi(\mathbf{r}, t) = \varphi_0(\mathbf{r}, t) + \varphi_1(\mathbf{r}, t)$, where first is a regular term, the second one is stochastic term caused by electron

density fluctuations, $\varphi_1 \ll \varphi_0$ and is random function of the spatial coordinates and time. Taking into account geometry of the task we obtain stochastic differential equation for the phase fluctuation:

$$\frac{\partial \varphi_1}{\partial t} + A \frac{\partial \varphi_1}{\partial y} + B \frac{\partial \varphi_1}{\partial z} = \mp \frac{k_0 V_{a0} \cos \theta}{2} \tilde{n}_1, \quad (7)$$

where: $A = V_0 \sin \theta_1 \mp \frac{\omega_{a0}}{k_0} \sin \theta \cos \theta$, $\tilde{n}_1 = N_1 / N_0$,

$$B = V_0 \cos \theta_1 \mp \frac{\omega_{a0}}{k_0} \cos^2 \theta \quad , \quad V_{a0} = \sqrt{\zeta} H_0 / \sqrt{4\pi M N_0} .$$

Using the method of characteristics in the region $z > 0$ we obtain:

$$\varphi_1(\mathbf{r}, t) = \mp \frac{\omega_{a0} \cos \theta}{2B} \int_0^{L_z} dz' \tilde{n}_1(x, y', z', t') \quad (8)$$

where: $y' = y - A(z - z') / B$, $t' = t - (z - z') / B$.

If electron density fluctuations are statistically homogeneous and stationary for arbitrary correlation function $W_N(\mathbf{p}, \tau)$ the variance of the phase fluctuation is:

$$\langle \varphi_1^2 \rangle = \left(\frac{\omega_{a0} \cos \theta}{2B} \right)^2 L_z \int_{-\infty}^{\infty} d\rho_z W_N(0, \rho_y, \rho_z, \tau), \quad (9)$$

here: $\rho_y = (A/B) \rho_z$, $\tau = \rho_z / B$; L_z is a distance traversed by wave in the ionospheric plasma along the external magnetic field. Knowledge of this function allows to estimate dumping of the mean field of scattered wave using the well-known formula [12-14]:

$$\langle E \rangle = E_0 \exp\left(-\frac{1}{2} \langle \varphi_1^2 \rangle\right). \quad (10)$$

One of the most important statistical characteristic in nonstationary media is the variance of instant frequency $\langle \omega_1^2 \rangle$ characterizing broadening of the temporal spectrum easily measuring by experiment:

$$\langle \omega_1^2 \rangle = 2\pi \left(\frac{\omega_{a0} \cos \theta}{2B} \right)^2 L_z B \int_{-\infty}^{\infty} d\mathbf{k} \int_{-\infty}^{\infty} d\omega \omega^2 \cdot W_N(\mathbf{k}, Ak_y + Bk_z). \quad (11)$$

The degree of curvature of a constant phase surface in the turbulent plasma is characterized by fluctuations of a unite vector of the wave normal \mathbf{s} :

$$\langle s_{1\perp}^2 \rangle = \frac{1}{k_0^2} \left\langle \left(\frac{\partial \varphi_1}{\partial x} \right)^2 \right\rangle = \left(\frac{\omega_{a0} \cos \theta}{2B} \right)^2 L_z$$

$$\int_{-\infty}^{\infty} d\rho_z \left(\frac{\partial^2}{\partial \rho_x^2} + \frac{\partial^2}{\partial \rho_y^2} \right) W_N(0, \rho_y, \rho_z, \tau). \quad (12)$$

In the geometrical optics approximation neglecting dissipation processes, transport equation for the wave amplitude E or the logarithmic relative amplitude $\chi = \ln(E/E_0)$ can be obtained from the differential equation [7]:

$$\frac{\partial}{\partial t} (\eta E^2) + \text{div} (V_{gr} \eta E^2) = -\frac{\partial \varepsilon}{\partial t} E^2, \quad (13)$$

where: $\varepsilon = \frac{c^2 (\omega - \mathbf{k} \mathbf{V}_0)^2}{V_a^2 \omega^2}$ is the dielectric permittivity,

$\eta = \frac{1}{\omega} \frac{\partial}{\partial \omega} (\omega^2 \varepsilon)$ is the coefficient connecting energy density

and E^2 , $\Phi = \mathbf{V}_{gr} \eta E^2$ is the energy flux density.

Substituting in equation (13) the expressions:

$$\frac{\partial}{\partial t} (\eta E^2) = 2 \frac{c^2}{V_{a0}^2} E_0^2 \eta_0 \left(2 \frac{\partial \chi_1}{\partial t} - \frac{\partial \tilde{n}_1}{\partial t} \right),$$

$$\frac{\partial \varepsilon}{\partial t} E^2 = 2 \frac{c^2}{V_{a0}^2} E_0^2 \varepsilon_0 \frac{\partial \tilde{n}_1}{\partial t} \quad (14)$$

we obtain stochastic differential equation of the logarithmic relative amplitude χ_1 the solution of which is:

$$\chi_1(\mathbf{r}, t) = \frac{1}{V_{20z}} \int_0^{L_z} dz' \left(D_1 \frac{\partial \tilde{n}_1}{\partial t'} + D_2 \frac{\partial \tilde{n}_1}{\partial y'} \right), \quad (15)$$

where: $V_{20z} = V_0 \cos \theta_1 \pm V_{a0} \sin^2 \theta$, $D_1 = \frac{1}{2} \left(1 + \frac{\Delta_0}{4} \right)$,

$\Delta_0 = 1 - g_0 \cos(\theta - \theta_1)$, $g_0 = k_0 V_0 / \omega_0$, ω_0 is determined from the dispersion equation of the internal wave,

$D_2 = \frac{1}{2} \left(V_0 \sin \theta_1 \mp \frac{3}{2} V_{a0} \sin \theta \cos \theta \right)$. Characteristics are:

$y' = y - \frac{A}{V_{20z}} (z - z')$, $t' = t - \frac{1}{V_{20z}} (z - z')$. When

fluctuations of the field are small and the forward scattering approximation is valid, the usually observed scintillation index S_4 is proportional to $S_4^2 \approx 4 \langle \chi_1^2 \rangle$ [15].

The next important problem of wave propagation in nonstationary medium is the energy exchange between the inertial wave and turbulent ionospheric plasma. The mean energy flux density (EFDE) of internal wave along the z direction is the sum of two terms in zero and second approximations:

$$\langle \Phi_z \rangle = \Phi_{z0} + \langle \Phi_z \rangle_2 = 2 \frac{c^2}{V_{a0}^2} E_0^2 \Delta_0 V_{20z} \left[1 - \frac{\Delta_0 L_z}{V_{20z}^2} \right]$$

$$\int_{-\infty}^{\infty} d\rho_z \left(D_1 \frac{\partial^2}{\partial \tau^2} + D_2 \frac{\partial^2}{\partial \rho_y \partial \tau} \right) W_N(0, \rho_y, \rho_z, \tau) \quad (16)$$

Growth of the energy flux along the z -axis means the energy transfer from the medium to the wave and energy decrease vice versa – from wave to the medium. These formulae are valid for near $(L/k_0 l^2) \ll 1$ and far $(L/k_0 l^2) \gg 1$ zones from a plasma slab boundary.

4. Numerical Calculations

Experimental observations using orbital satellite beacons and the EISCAT incoherent scatter radar show [15] that under quite geomagnetic conditions for a cross-field irregularity scale $l_{\perp} \approx 1$ km a plasma drift velocity was basic-

ally westwards and $V_0 = 0.3$ km/s. The field-aligned scale of irregularities was of the order of $l_{\parallel} \approx 30$ km with the plasma drift velocity (1-2) km/s. Since the inertial waves propagate in the F-region of the ionosphere, at $\theta = 0$ and $\lambda = 10^2$ km the frequencies of this wave in the altitudes $h = 150 - 300$ km are in the interval $1.5 \cdot 10^{-2} - 1.5 \cdot 10^{-1}$ [10]. Other experimental data of the slow MHD waves measured by ground-based radar systems are presented in [16]. It was shown that slow MHD waves perturbations can be propagate over distances ~ 1000 km at $\omega \leq 10^{-2}$ sec⁻¹ (period $T \geq 10$ minute), while on global distances at $\omega \leq 10^{-3}$ sec⁻¹ (period $T \geq 100$ minute). Phase velocity of these waves are in the interval $6 \div 10$ km/sec and less.

The irregularity model is described by 3D correlation function of electron density fluctuations. Numerical calculations are carried out for both Gaussian and power-law spectra for elongated ionospheric irregularities. The spectral density function which best describes the irregularities in a randomly inhomogeneous magnetized plasma depends on the particular case. Most widely used the Gaussian spectral function in the Gaussian, which has certain mathematical advantages. However, power-law spectral densities have been receiving more attention as being physically more realistic. Knowledge of the power spectrum of ionospheric refractive index fluctuations can lead to an understanding of the physical processes that characterize the region of the ionosphere under study.

Observations by ‘‘Sura’’ heating facility experiment show [17] that artificial ionospheric irregularities are stretched along the geomagnetic field. It was established that the transversal scale l_{\perp} of elongated irregularities varies in the range of 100 – 500 m; the magnitudes of the drift velocity were within the limits 65-270 m/s (the typical velocities of ionospheric motions $V_0 \approx 60$ -100 m/s). Velocity 100 m/s caused by the steady drifting with the horizontal wind of scattering irregularities embedded in the ionosphere will be used in numerical calculations.

Data obtained from spaced receiver measurements made at Kingston, Jamaica (during the periods August 1967– January

1969 and June 1970–September 1970) show that the irregularities responsible for fluctuations of MHD waves parameters and causing the scintillation of signals from the moving earth satellites (BE–B and BE–C) are between heights of 153 and 617 km closely aligned along the magnetic field lines in the F -region [18]. Orientation of the irregularities in the ionosphere has been measured with respect to the geographic north observing a diffraction pattern of the satellite signals (41 MHz) on the ground. The dip angle of the irregularities with respect to the field lines was within 16° .

In the geometrical optics approximation forward scattering assumption is valid: $\sigma_N^2 k_0 L_z \ll 1 \ll k_0 l$, σ_N^2 is the variance. If the single scattering condition is also fulfilled $\sigma_N^2 k_0^2 l L \ll 1$ a medium is characterized by the Gaussian irregularity spectrum [15].

Numerical calculations are carrying out for the anisotropic Gaussian correlation function of electron density fluctuations [19]:

$$W_N(\mathbf{k}, \omega) = \sigma_N^2 \frac{l_{\perp}^2 l_{\parallel}}{8\pi^{3/2}} \frac{T}{2\sqrt{\pi}} \exp\left(-\frac{k_x^2 l_{\perp}^2}{4} - p_1 \frac{k_y^2 l_{\parallel}^2}{4} - p_2 \frac{k_z^2 l_{\parallel}^2}{4} - p_3 k_y k_z l_{\parallel}^2\right) \exp\left(-\frac{\omega^2 T^2}{4}\right) \quad (17)$$

$$\text{where: } p_3 = \frac{\chi^2 - 1}{2\chi^2} \sin \gamma_0 \cos \gamma_0, \quad p_1 = \frac{1}{\chi^2 \cos^2 \gamma_0 + \sin^2 \gamma_0}$$

$$\left[1 + \frac{(\chi^2 - 1)^2}{\chi^2} \sin^2 \gamma_0 \cos^2 \gamma_0 \right], \quad p_2 = \frac{\sin^2 \gamma_0 + \cos^2 \gamma_0}{\chi^2},$$

$\chi = l_{\parallel} / l_{\perp}$ is the anisotropy factor of field-aligned irregularities defined as a ratio of longitudinal and transversal characteristic liner scales of electron density irregularities, γ_0 is the slop angle of elongated plasma irregularities with respect to the geomagnetic field, T is the characteristic temporal scale of ionospheric plasma irregularities; $1/T$ is the own frequency of irregularities pulsations. The average shape of electron density irregularities has the form of elongate ellipsoid of rotation. The rotation axis is located in the plane of geomagnetic meridian. Anisotropy of the shape of irregularities is connected with the difference of diffusion coefficients in the field align and field perpendicular directions.

Measurements of satellite’s signal parameters passing through ionospheric layer and measurements aboard of satellite show that irregularities in F -region of the ionosphere have power-law spectrum with different spatial scales. The 3D power spectrum of electron density fluctuations adequately explaining observing plasma irregularities having linear scale larger than 800 m has the power-law index $2 < p < 6$ [19]:

$$W_N(\mathbf{k}) = \frac{\sigma_N^2}{2\pi^2} \frac{\Gamma\left(\frac{p}{2}\right)\Gamma\left(\frac{5-p}{2}\right)}{\Gamma\left(\frac{3}{2}\right)} \cdot \sin\left[\frac{(p-3)\pi}{2}\right] \cdot \frac{l_{\perp}^2 l_{\parallel}}{\left[1+l_{\perp}^2(k_{\perp}^2+\chi^2 k_{\parallel}^2)\right]^{p/2}}, \quad (18)$$

where: $\Gamma(x)$ is the gamma function.

Substituting (17) into (9) we obtain:

$$\langle \varphi_1^2 \rangle = \frac{\sqrt{\pi}}{4} \sigma_N^2 \frac{\omega_{a0}^2 L_z T}{\sqrt{q_3} B \chi^2} \cos^2 \theta \left(q_4 - \frac{q_5^2}{q_3}\right)^{-1/2} \quad (19)$$

where: $q_3 = p_1 + (T^2 / l_{\parallel}^2) A^2$, $q_4 = p_2 + (T^2 / l_{\parallel}^2) B^2$, $q_5 = p_3 - (T^2 / 2 l_{\parallel}^2) AB$.

For spectral function (17) variance of an instant frequency (11) and the degree of a curvature of a constant phase surface (12) along the direction of geomagnetic field of lines (z-direction) yield:

$$\frac{\langle \omega_1^2 \rangle_z}{\omega_0^2} = \frac{\sqrt{\pi}}{2} \sigma_N^2 \frac{\omega_{a0}^2 L_z}{\omega_0^2 l_{\parallel}} \cdot \frac{T \cos^2 \theta}{\chi l_{\parallel} B} \left[\frac{A^2}{\sqrt{q_4}} \left(q_3 - 4 \frac{q_5^2}{q_4}\right)^{-3/2} + \frac{B}{\sqrt{q_3}} \left(B + 4 A \frac{q_5}{q_3}\right) \cdot \left(q_4 - 4 \frac{q_5^2}{q_3}\right)^{-3/2} \right], \quad (20)$$

$$\frac{\langle k_{1\perp}^2 \rangle_z}{k_0^2} = \frac{\sqrt{\pi}}{2} \sigma_N^2 \frac{L_z}{k_0 l_{\parallel}^2} \cdot \frac{\omega_{a0}^2 T}{\chi B k_0} \cos^2 \theta \left[\frac{\chi^2}{\sqrt{q_3}} \left(q_4 - 4 \frac{q_5^2}{q_3}\right)^{-1/2} + \frac{1}{\sqrt{q_4}} \left(q_3 - 4 \frac{q_5^2}{q_4}\right)^{-3/2} \right]. \quad (21)$$

Energy exchange between inertial wave propagating in the ionospheric F region and the regular turbulent plasma flow depends on the following parameters: angles θ and θ_1 ; characteristic spatial scale of plasma irregularities l_{\parallel} and the frequency of their temporal pulsation $1/T$; velocities V_0 and V_{a0} ; distance traveling by inertial wave in the turbulent plasma $L_{y,z}/l_{\parallel}$ and $L_{y,z}/(k_0 l_{\parallel}^2) \ll 1$ (this condition should be fulfilled as in the geometrical optics approximation diffraction effects are neglect [12]–[14], however despite of this the obtain results are good approach to the reality (especially phase)); frequencies ω_0 and ω_{a0} ; wavenumber k_0 .

Substituting (17) in equation (16) we obtain:

$$\langle \Phi_z \rangle = 2 \frac{c^2}{V_{a0}^2} E_0^2 \Delta_0 V_{20z} \left\{ 1 - 2\sqrt{\pi} \sigma_N^2 \cdot \frac{\Delta_0 L_z}{V_{20z}^2} \cdot \frac{B T}{l_{\parallel}^2 \chi} \left[\frac{q_0}{\sqrt{q_4}} \left(q_3 - 4 \frac{q_5^2}{q_4}\right)^{-3/2} + \frac{1}{\sqrt{q_3}} \left(-q_1 + 2 \frac{q_2 q_5}{q_3}\right) \cdot \left(q_4 - 4 \frac{q_5^2}{q_3}\right)^{-3/2} \right] \right\}, \quad (22)$$

$$\langle \Phi_y \rangle = 2 \frac{c^2}{V_{a0}^2} E_0^2 \Delta_0 A \left\{ 1 - 2\sqrt{\pi} \sigma_N^2 \cdot \frac{\Delta_0 L_y}{A} \cdot \frac{T}{l_{\parallel}^2 \chi} \left[-\frac{q_6}{\sqrt{q_4}} \left(q_3 - 4 \frac{q_5^2}{q_4}\right)^{-3/2} + \frac{1}{\sqrt{q_3}} \left(q_7 + 2 \frac{q_5 q_8}{q_3}\right) \cdot \left(q_4 - 4 \frac{q_5^2}{q_3}\right)^{-3/2} \right] \right\}, \quad (23)$$

where: $q_0 = A(D_2 - D_1 A)$, $q_1 = D_1 B^2$, $q_6 = D_1 A^2$,

$q_2 = B(D_2 - 2D_1 A)$, $q_7 = B(D_3 - B D_1)$,

$q_8 = A(D_3 - 2D_1 B)$; $D_3 = \frac{1}{2} \left(V_0 \cos \theta_1 \pm \frac{1}{2} V_{a0} \sin^2 \theta\right)$.

If the second term in brace is negative this means that turbulent plasma transfer its energy to the inertial wave increasing its amplitude and intensity propagating along the geomagnetic lines of force; if second term is negative internal wave losses energy transferring it to the plasma flow and its intensity decreases.

Applying the spectral function of the spatial-temporal power spectrum of electron density fluctuation in the “frozen” turbulence $V_N(\mathbf{\kappa}, \Omega) = F_N(\mathbf{\kappa}) \delta(\Omega - \mathbf{\kappa} \mathbf{V}_0)$ [12–14] from equation (9) for the variance of the phase fluctuations in the principle plane we obtain:

$$\langle \varphi_1^2 \rangle = \frac{\pi}{2} \frac{\omega_{a0} k_0 L_z}{B N_0^2} \int_{-\infty}^{\infty} d\kappa_x \int_{-\infty}^{\infty} d\kappa_y F_N(\kappa_x, \kappa_y, \kappa_y \cdot \tan \theta)$$

In this case the mean energy flux density along the z-axis is:

$$\langle \Phi_z \rangle = \pi L_z \frac{c^2}{V_{a0}^2} \cdot \frac{\omega_*^2}{\omega_0^2} \cdot \frac{V_0 (\Lambda_1 - Q_2^2 \Lambda_2 - Q_2 \Lambda_3)}{V_0 \sin \theta_1 + V_{a0} \sin \theta \cos \theta} \int_{-\infty}^{\infty} d\kappa_x \int_{-\infty}^{\infty} d\kappa_z \kappa_z^2 F_N(\kappa_x, -Q_2 \kappa_z, \kappa_z), \quad (24)$$

$$Q_2 = \frac{V_0 \cos \theta_1 - V_{a0} \sin^2 \theta}{V_0 \sin \theta_1 + V_{a0} \sin \theta \cos \theta},$$

where:

$$\Lambda_1 = \cos \theta_1 (3V_{a0} \sin^2 \theta - bV_0 \cos \theta_1), \quad b = 2 - (\omega_* / \omega_0),$$

$$\Lambda_2 = \sin \theta_1 (3V_{a0} \sin \theta \cos \theta + bV_0 \sin \theta_1),$$

$$\Lambda_3 = 3V_{a0} \sin \theta (\sin \theta \sin \theta_1 - \cos \theta \cos \theta_1) - 2bV_0 \sin \theta_1 \cos \theta_1, \quad \omega_* = \omega_0 - (\mathbf{k}_0 \cdot \mathbf{V}_0).$$

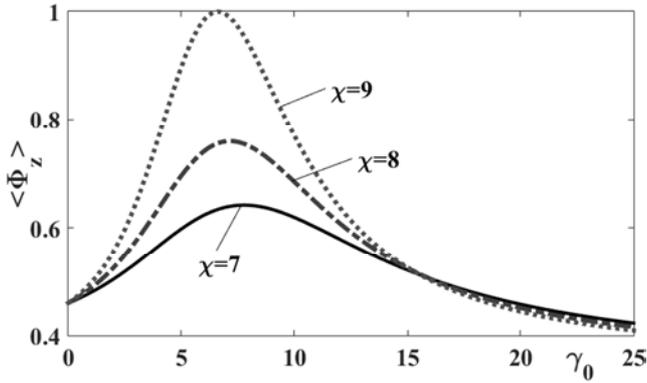


Figure 1. The mean EFDE along z direction versus slope angle γ_0 for different anisotropy factor χ .

Figure 1 depicts normalized mean EFDE of the internal wave along the directions to the external magnetic field versus slope angle of elongated plasma irregularities γ_0 for different anisotropy factor χ . Velocity of the turbulent plasma flow is directed along lines of force of geomagnetic field ($\theta_1 = 0^\circ$) and the angle $\theta = 5^\circ$. Longitudinal characteristic spatial scale of large-scale ionospheric plasma irregularities is $l_{\parallel} = 10$ km. Numerical calculations show that turbulent ionospheric plasma flow transfer energy to the internal wave increasing its intensity in proportion to the parameter χ : traveling distance 10^3 km in the ionospheric F region maximum of the curves is displaced to the direction of an external magnetic field. Particularly, at $T = 10$ sec varying anisotropy factor in the interval $2 \leq \chi \leq 9$ maximum of both functions $\langle \Phi_z \rangle_{\max}$ and $\langle \Phi_y \rangle_{\max}$ (normal to the external magnetic field) shifts from $\gamma_0 = 17^\circ$ to the $\gamma_0 = 8^\circ$. Increasing characteristic temporal scale of electron density irregularities up to $T = 400$ sec maximum of a curve $\langle \Phi_z \rangle_{\max}$ is displaced in the interval $5^\circ \leq \gamma_0 \leq 4^\circ$. Hence, internal wave propagating in the ionospheric F region with large scale electron density irregularities near the turbulent plasma flow pumps energy growing its energy flux; increasing anisotropy factor of elongated plasma irregularities maximum of EFDE is displaced to the direction of lines of force of geomagnetic field; decreasing own frequency ($1/T$) of the turbulent ionospheric plasma irregularities interval of slop angle of elongated plasma irregularities having substantial influence

on the energy flux density substantially narrows.

Figure 2 illustrates normalized mean EFDE as a function of anisotropy factor for different inclination angle of elongated electron density irregularities with respect to the external magnetic field. Numerical analyses show that the mean EFDE of the internal wave is increased inversely proportion to the angle γ_0 due to energy exchange with the plasma flow. MHD wave travelling distance 1000 km near to the lines of force of geomagnetic field pumps energy from the turbulent plasma flow increasing its intensity along and normal directions to the external magnetic field. Varying slop angle in small interval $20^\circ \leq \gamma_0 \leq 23^\circ$ broadening of both functions $\langle \Phi_z \rangle$ and $\langle \Phi_y \rangle$ decreases on 62%. So, increasing the angle of inclination of elongated plasma irregularities even in small interval the width of the EFDE substantially decreases.

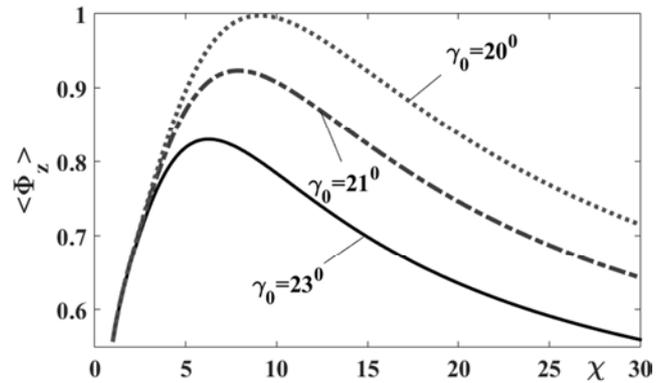


Figure 2. The mean EFDE along z direction versus anisotropy factor χ for different inclination angle γ_0 .

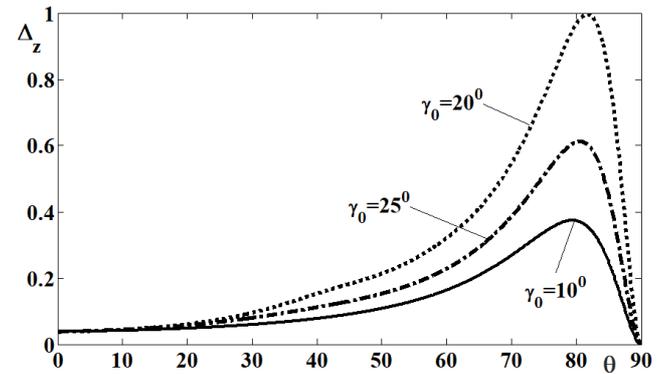


Figure 3. Variance of the instance frequency of scattered inertial wave versus angle of its propagation for different slope angle.

Figure 3 depicts normalized variance of the instance frequency $\Delta_z \equiv \langle \omega_1^2 \rangle_z / \omega_0^2$ determining the broadening of the temporal power spectrum of scattered inertial wave versus angle propagating in the ionospheric F region for different slope angle of elongated plasma irregularities. Analyses show that maximum and shift of maximum of the spectrum depends on both the directions of plasma flow and internal wave propagation with respect to the external magnetic field. Numerical calculations are carried out for fixed parameters $\chi = 2$ and $T = 10$ sec. Increasing angles

γ_0 and θ_1 spectrum narrows and its maximum displaces to the direction of big angle θ . If the angle θ_1 is in the interval $0^\circ \leq \theta_1 \leq 70^\circ$ maximum of the temporal spectrum shifts to the right (to the big angles). Particularly, at $\gamma_0 = 10^\circ$ maximum displaced on 5° degree, for $\gamma_0 = 20^\circ$ - on 8° , while at $\gamma_0 = 30^\circ$ - on 11° ; if γ_0 is in the interval $0^\circ \leq \gamma_0 \leq 30^\circ$ the curve narrows on 71% percent.

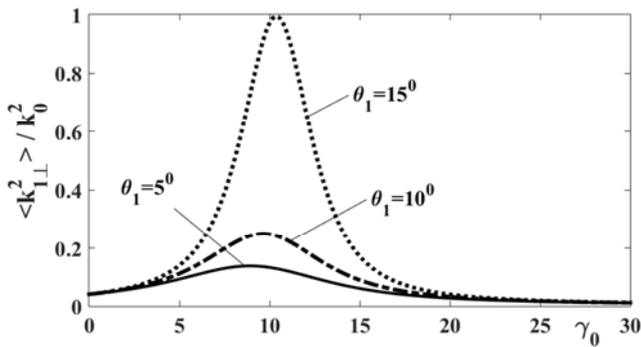


Figure 4. Degree of a curvature of a constant phase surface versus slope angle of elongated plasma irregularities.

Figure 4 illustrates degree of a curvature of a constant surface $\theta = 5^\circ$ in the ionospheric F region. Velocity of a turbulent plasma flow is $V_0 = 150$ m/sec making angle $\theta_1 = 5^\circ, 10^\circ$ and 15° to the external magnetic field; anisotropy factor of elongated plasma irregularities is $\chi = 7$, $T = 10$ sec. Numerical calculations show that internal wave passing distance $L_z = 100$ m in the turbulent plasma flow varying angle from $\theta_1 = 10^\circ$ up to 15° the degree of a curvature increases four times. Maximum of a curve and its curvature degree increases in proportion to the distance travelling by wave in the turbulent plasma.

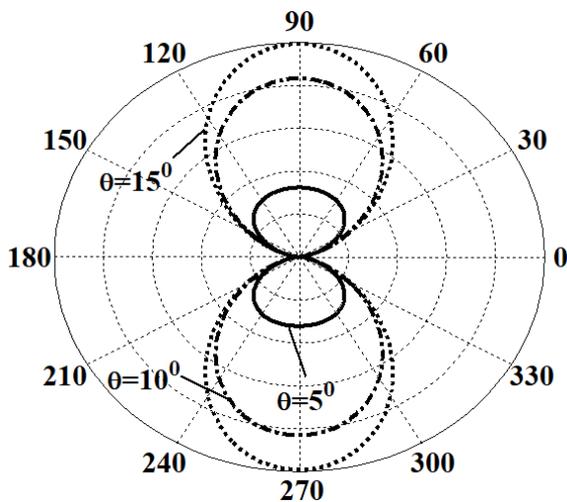


Figure 5. Phase portrait of $\langle \Phi_z \rangle$ in the polar coordinate system for frozen plasma irregularities varying angle θ .

Figure 5 represents phase portrait in the polar coordinate system for the mean EFDE of the internal wave along the z direction in the case of frozen turbulence. Numerical calculations show that in isotropic case ($\chi = 1$) when the directions of plasma flow and external magnetic field coincide ($\theta_1 = 0^\circ$) and the inertial wave propagates at angles $\theta = 5^\circ, 10^\circ, 15^\circ$ with respect to the geomagnetic lines of forces the curves have dipole forms. Analyses show that change of the angle θ_1 has not influence of the phase portrait, while lobes of scattered pattern become bigger in proportion to the angle θ .

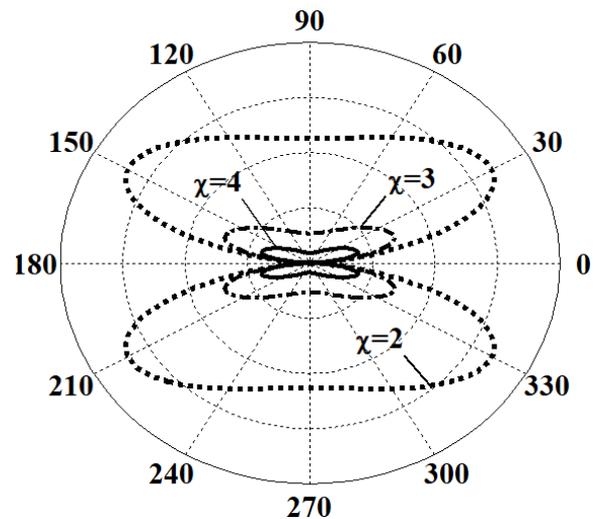


Figure 6. Phase portrait of $\langle \Phi_z \rangle$ in the polar coordinate system for frozen plasma irregularities varying anisotropy factor χ .

Figure 6 depicts phase portrait of the mean EFDE in the frozen magnetized plasma if the inertial wave propagates along the geomagnetic field of lines ($\theta = 0^\circ$) at fixed angle $\theta_1 = 5^\circ$ varying anisotropy factor of elongated plasma irregularities $\chi = 2, 3, 4$. Numerical calculations show that lobes of scattered pattern become flatter increasing angle θ while they become inflated in proportion to the angle θ_1 .

Acknowledgements

The work has been supported by the International Science and Technology Center (ISTC) under Grant # G-2126 and Shota Rustaveli National Science Foundation under Grant # FR/3/9-190/14.

References

- [1] A. I. Akhiezer, I. A. Akhiezer, R. V. Polovin, A. G. Sitenko, and K. N. Stepanov, "Electrodynamics of Plasma," Moscow, Nauka, 1974 (in Russian).
- [2] V. M. Sorokin, and G. V. Fedorovich, "Physics of Slow MHD Waves in the Ionospheric Plasma," Nauka, Moscow, 1982 (in Russian).

- [3] A. F. Aleksandrov, L. S. Bogdankevich, and A. A. Rukhadze, *Electrodynamics of Plasma*, Moscow, Higher Educational Institution, 1988 (in Russian).
- [4] G. V. Jandieri, V. G. Gavrilenko, and A. A. Semerikov, "To the Theory of MHD Waves Propagation in the Turbulent Plasma Flow," *Plasma Physics*, vol. 11, # 10, pp. 1193-1198, 1985 (in Russian).
- [5] G. V. Jandieri, "To the Problem of MHD Wave Propagation in the Ionospheric E-region," *Journal of Applied Science and Research*, vol. 2, # 2, pp. 1-13, 2014.
- [6] G. V. Jandieri, O. Kharshiladze, Zh. M. Diasamidze, M. R. Diasamidze, "Magnetosonic Waves in the Turbulent Ionospheric Plasma," *International Journal of Advancement in Earth and Environmental Science*, vol. 2, # 1, pp. 51-62, 2014.
- [7] Yu. A. Kravtsov, and Yu. I. Orlov, "Geometrical Optics of Inhomogeneous Media," Moscow, Nauka, 1980 (in Russian).
- [8] J. Pedlosky, "Geophysical Fluid Dynamics", (Springer, New York, 1982, Mir, Moscow, 1984).
- [9] L. D. Landau, and I. M. Lifshits, "Fluid Mechanics," (Pergamon Press, Oxford, 1987; Nauka, Moscow, 1988).
- [10] A. G. Khantadze, G. G. Lominadze, and G. V. Jandieri, "Internal Waves in the Ionosphere Related to the Earth's Geomagnetic Field," *Doklady Earth Science*, vol. 420, # 4, pp. 672-675, 2008.
- [11] A. G. Khantadze, and G. V. Jandieri, "General-planetary Character of Three-dimensional Planetary Waves Propagation in Lower and Upper Earth Atmosphere," *J. of Atmospheric and Solar –Terrestrial Physics*, vol. 71, pp. 45-48, 2009.
- [12] A. Ishimaru, "Wave Propagation and Scattering in Random Media, Vol. 2, Multiple Scattering, Turbulence, Rough Surfaces and Remote Sensing," IEEE Press, Piscataway, New Jersey, USA, 1997.
- [13] V. I. Tatarskii, "Wave Propagation in a Turbulent Medium," McGraw-Hill, New York, 1961.
- [14] S. M. Rytov, Yu. A. Kravtsov, and V. I. Tatarskii, "Principles of Statistical Radiophysics. vol. 4. Waves Propagation Through Random Media. Berlin, New York, Springer, 1989.
- [15] A. W. Wernik, and C. H. Liu, "Ionospheric Irregularities Causing Scintillation of GHz Frequency Radio Signals," *J. of Atmospheric and Terrestrial Physics*, vol. 36, pp. 871-879, 1974.
- [16] V. P. Burmaka, L. S. Kostrov L. S., and L. F. Chernogor, "Statistics of Signals of HF Doppler Radar Probing the Bottomside Ionosphere Distributed by Rocket Launches and Solar Terminator," *Radio Physics and Radio Astronomy*, vol. 8, # 2 pp. 143-162, 2003 (in Russian).
- [17] N. V. Bakhmet'eva, V. N. Bubukina, Yu. A. Ignat'ev, G. S. Bochkarev, V. A. Eremenko, V. V. Kol'sov, I. V. Krashennnikov and Yu. N. Cherkashin, "Investigation by Backscatter Radar Irregularities Produced in Ionospheric Plasma Heating Experiments," *Journal of Atmospheric and Terrestrial Physics*, vol. 59, # 18, pp. 2257-2263, 1997.
- [18] A. A., and G. S. Kent, "Determination of the Orientation of Ionospheric Irregularities Causing Scintillation of Signals From Earth Satellites," *Journal of Atmospheric and Terrestrial Physics*, vol. 34, pp. 1411-1414, 1972.
- [19] G. V. Jandieri, A. Ishimaru, V. G. Jandieri, A. G. Khantadze, and Zh. M. "Model Computations of Angular Power Spectra for Anisotropic Absorptive Turbulent Magnetized Plasma," *PIER*, vol. 70, pp. 307-328, 2007.