

Super Exponential Expansion for Dark Energy Model with Variable Λ in $f(R, T)$ Gravity

Mohammad Moksud Alam^{*}, Mohammad Amjad Hossain, Mohammad Ashraful Islam

Department of Mathematics, University of Chittagong, Chittagong, Bangladesh

Email address:

moksud.math@cu.ac.bd (M. M. Alam), amj62235@cu.ac.bd (M. A. Hossain), ashrafmath1@gmail.com (M. A. Islam)

^{*}Corresponding author

To cite this article:

Mohammad Moksud Alam, Mohammad Amjad Hossain, Mohammad Ashraful Islam. Super Exponential Expansion for Dark Energy Model with Variable Λ in $f(R, T)$ Gravity. *International Journal of Astrophysics and Space Science*. Vol. 5, No. 3, 2017, pp. 41-46.

doi: 10.11648/j.ijass.20170503.11

Received: March 28, 2017; **Accepted:** April 13, 2017; **Published:** June 2, 2017

Abstract: In this paper, we have studied Friedmann-Robertson-Walker (FRW) cosmological model with quadratic equation of state and cosmological constant in the presence of perfect fluid source in $f(R, T)$ gravity. To obtain an exact solution of the field equations of the theory, we have used quadratic equation of state and time dependent deceleration parameter $q(t)$. The physical and geometrical behavior of the model is also discussed.

Keywords: $f(R, T)$ Gravity, Cosmological Model, Quadratic Equation of State, Cosmological Constant, Deceleration Parameter

1. Introduction

The simplest and the most elegant assumption of the universe being homogeneous and isotropic is supported by the observational evidences in the large scale structure (LSS) [1] and cosmic microwave background radiation (CMBR) [2]. The idea, the present universe is expanding with acceleration, put forward by the recent cosmological observations from Supernova [3, 4]. The imposition to the contribution of an exotic component responsible for cosmic acceleration is known as dark energy, which has negative pressure and contributes almost three quarters of the total cosmic density. The most obvious candidate of dark energy is the so-called cosmological constant (Λ) that plays a crucial role in generating an accelerating scale factor. Though, several approaches have made to study the fundamental nature of the dark energy, such as quintessence, phantom, k-essence, tachyon, chaplygin gas and many more [5, 6].

Our primary interest is to search alternative, modified gravity consistent with observational data when general relativity fails to make a natural interpretation of the dark energy epoch of the universe. We resort to Einstein-Hilbert action to obtain modified theories of Einstein such as $f(R)$

gravity [7, 8] and $f(T)$ gravity [9] where, R and T are defined as the curvature scalar and the trace of energy momentum tensor respectively. A newly devised theory known as $f(R, T)$ gravity [10] has been studied to reconstruct some cosmological models. The gravitational Lagrangian is constituted by an arbitrary function of curvature scalar (R) and the trace (T) of the energy momentum tensor. They have investigated FRW model to yield the field equations in this theory of $f(R, T)$ gravity. Very recently, Sahoo et al. [11] studied LRS Bianchi Type-I cosmological model in $f(R, T)$ theory of gravity with $\Lambda(T)$ while cosmological models with linearly varying deceleration parameter in $f(R, T)$ theory has investigated by Ramesh et al. [12]. Akarsu and dereli [13] have first introduced a new law for deceleration parameter varying linearly with time which is congruent with Berman's law where it is constant.

Ananda and Bruni [14] have discussed the general relativistic dynamics of Robertson-Walker models with a non-linear quadratic equation of state and analyzed that the behavior of the anisotropy at the singularity found in the brane scenario can be recreated in the general relativistic context by considering an equation of state in the form

$$P = P_0 + \alpha\rho + \beta\rho^2 \quad (1)$$

Where, P_0 , α and β are parameters. This equation is nothing but the first term of Taylor expansion of any equation of state of the form $p = p(\rho)$ about $\rho = 0$. They have also analyzed the effects of quadratic equation of state in anisotropic homogeneous and inhomogeneous cosmological models in general relativity to isotropize the universe at early times when the initial singularity is approached. Recently, Reddy et al. [15] have thoroughly discussed the Bianchi-I cosmological model with quadratic equation of state in the context of general theory of relativity.

Inspired by the above investigations and discussions, we have studied FRW space-time cosmological model in $f(R, T)$ gravity with quadratic equation state and cosmological constant. We have investigated here one particular class of $f(R, T)$ gravity [10] and solved the surviving field equations satisfying the linearly varying deceleration parameter (q) proposed by Akarsu and Dereli [13]. We have organized this work as follows: In section 2, we reviewed modified $f(R, T)$ theory of gravity in brief. Sect. 3 presents Einstein field equations in terms of Hubble parameter following $f(R, T)$ gravity using $f(R, T) = R + 2f(T)$. An explicit solution of the equations is presented in Sect. 4. We summarize this paper with a brief discussion on the basis of graphical presentation of the results in Sect. 5. The conclusions of the models are given in Sect. 6.

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} + (g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)f_R(R, T) = 8\pi T_{\mu\nu} - f_T(R, T)T_{\mu\nu} - f_T(R, T)\theta_{\mu\nu} \quad (5)$$

where,

$$\theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{lk}} \quad (6)$$

$$\text{Here, } f_T(R, T) = \frac{\partial f(R, T)}{\partial T},$$

$$f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$$

$\square = \nabla^\mu\nabla_\mu$ is the De Alembert's operator, and $T_{\mu\nu}$ is the standard matter energy momentum tensor derived from the Lagrangian L_m . By contracting (5), we obtain the relation between R and T as

$$f_R(R, T)R + 3\square f_R(R, T) - 2f(R, T) = 8\pi T - f_T(R, T)T - f_T(R, T)\theta \quad (7)$$

where, $\theta = g^{\mu\nu}\theta_{\mu\nu}$. From (5) and (7), the gravitational field equations can be written as

$$f_R(R, T)(R_{\mu\nu} - \frac{1}{3}Rg_{\mu\nu}) + \frac{1}{6}f(R, T)g_{\mu\nu} = (8\pi - f_T(R, T))(T_{\mu\nu} - \frac{1}{3}Tg_{\mu\nu}) - f_T(R, T)(\theta_{\mu\nu} - \frac{1}{3}\theta g_{\mu\nu}) + \nabla_\mu\nabla_\nu f_T(R, T) \quad (8)$$

The problem of perfect fluids described by an energy density ρ , pressure p and four velocities u^μ is more complicated, since there is no unique definition of the matter

2. A Brief Review of $f(R, T)$ Gravity

Action for the modified gravity [10] is considered as

$$S = \int \left(\frac{f(R, T)}{16\pi G} + L_m \right) \sqrt{-g} d^4x \quad (2)$$

where $f(R, T)$ is the arbitrary function of Ricci scalar (R) and trace (T) of the stress energy tensor of the matter $T_{\mu\nu}$. The matter Lagrangian density is defined by L_m . The action for the different theories depend on the choice of $f(R, T)$. It is to be noted that when $f(R, T) = f(R)$ then (2) represents the action for $f(R)$ gravity. The stress energy tensor of matter is defined as

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\partial(\sqrt{-g}L_m)}{\partial g^{\mu\nu}} \quad (3)$$

and its trace by $T = g^{\mu\nu}T_{\mu\nu}$ respectively. With the assumption that L_m of matter depends only on $g_{\mu\nu}$ and not on its derivatives, we obtain

$$T_{\mu\nu} = g_{\mu\nu}L_m - 2\frac{\partial L_m}{\partial g^{\mu\nu}} \quad (4)$$

Let us take variation of action (2) with respect to the metric tensor components $g_{\mu\nu}$, so that we obtain the field equations of $f(R, T)$ gravity as

$$f_R(R, T)R_{\mu\nu} - \frac{1}{2}f(R, T)g_{\mu\nu} + (g_{\mu\nu}\square - \nabla_\mu\nabla_\nu)f_R(R, T) = 8\pi T_{\mu\nu} - f_T(R, T)T_{\mu\nu} - f_T(R, T)\theta_{\mu\nu} \quad (5)$$

$$\theta_{\mu\nu} = -2T_{\mu\nu} + g_{\mu\nu}L_m - 2g^{lk}\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{lk}} \quad (6)$$

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The problem of perfect fluids described by an energy density ρ , pressure p and four velocities u^μ is more complicated, since there is no unique definition of the matter

Lagrangian. However, in the present study we assume that the stress-energy tensor of the matter is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu} \quad (9)$$

and the matter Lagrangian can be taken as $L_m = -p$ and we have,

$$u^\mu \nabla_\nu u_\mu = 0, \quad u^\mu u_\mu = 1$$

Now, with the use of (6), we have

$$\theta_{\mu\nu} = -2T_{\mu\nu} - pg_{\mu\nu} \quad (10)$$

It is to note that $f(R, T)$ depends on the physical nature of the matter field through tensor $\theta_{\mu\nu}$. Thus, each choice of $f(R, T)$ leads us to different cosmological models. Harko et al. [10] presented three classes of $f(R, T)$ as follows:

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R)f_3(T) \end{cases} \quad (11)$$

In this present work, we have discussed the class of $f(R, T) = R + 2f(T)$

For this choice and with the help of (9) and (10), (5) reduces to

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = (8\pi + 2f'(T))T_{\mu\nu} + (2pf'(T) + f(T))g_{\mu\nu} \quad (12)$$

which is the gravitational field equation in $f(R, T)$ modified gravity for the class $f(R, T) = R + 2f(T)$.

The gravitational field equation (12), in $f(R, T)$ theory, in the presence of cosmological constant (Λ), is given as

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = (8\pi + 2f'(T))T_{\mu\nu} + (2pf'(T) + f(T) + \Lambda)g_{\mu\nu} \quad (13)$$

where prime stands for differentiation with respect to the argument.

3. Model and Field Equations for $f(R, T) = R + 2f(T)$ Where $f(T) = \lambda T$

The metric of FRW space-time is given by

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right] \quad (14)$$

which is spatially homogeneous and isotropic in standard spherical co-ordinate (t, r, θ, ϕ) , where $a(t)$ is the scale factor and k , the curvature parameter, is used to describe the geometry of the universe with open, flat and closed corresponding to $k = -1, 0, 1$ in some respects.

Perfect fluid energy- momentum tensor can be taken as

$$T^\mu_\nu = (-\rho, p, p, p) \quad (15)$$

where ρ , the energy density and p , the pressure of the fluid are both functions of cosmic time t .

For the choice $f(T) = \lambda T$, the field equations (13) making a connection with (14) and (15), are given by

$$3 \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) = -(8\pi + 3\lambda)\rho + 5\lambda p + \Lambda \quad (16)$$

$$\frac{\ddot{a}}{a^2} + 2\frac{\dot{a}}{a} + \frac{k}{a^2} = (8\pi + 7\lambda)p - \lambda\rho + \Lambda \quad (17)$$

For a flat model, (16) and (17) reduce to

$$3H^2 = -(8\pi + 3\lambda)\rho + 5\lambda p + \Lambda \quad (18)$$

$$2\dot{H} + 3H^2 = (8\pi + 7\lambda)p - \lambda\rho + \Lambda \quad (19)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and an overhead dot

(.) hereafter, denote ordinary differentiation with respect to cosmic time "t" only.

4. Solution of Field Equations

Now, we have a system of two equations (18) and (19) with four unknown functions, namely, $a(t)$, $\rho(t)$, $p(t)$ and $\Lambda(t)$. To obtain an exact solution of the field equations, we consider two physically plausible conditions as:

i. The quadratic equation of state given by

$$p = \alpha\rho^2 - \rho \quad (20)$$

where $\alpha \neq 0$ is a constant quantity to preserve the quadratic nature of the equation.

ii. Akarsu and Dereli[13] and Akarsu et al.[16] proposed the linearly varying deceleration parameter as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -lt + m - 1 \quad (21)$$

where, $l \geq 0$, $m \geq 0$ are constants and law of Berman[17] can be obtained for $l = 0$. Various cosmological models have been discussed executing this law with constant deceleration parameter. As we learn that the universe may exhibit Super exponential expansion for $q < -1$, Exponential expansion for $-1 \leq q < 0$, Expansion with constant rate for $q = 0$ and Decelerated expansion for $q > 0$. We can solve (21) for scale factor and three different forms of solutions are obtained as

$$a = a_1 e^{c_1 t} \text{ for } l = 0 \text{ and } m = 0 \quad (22)$$

$$a = a_2 (mt + c_2)^{\frac{1}{m}} \text{ for } l = 0, m > 0 \quad (23)$$

$$a = a_3 \exp \left[\frac{2}{\sqrt{m^2 - 2c_3 l}} \arctan h \left(\frac{lt - m}{\sqrt{m^2 - 2c_3 l}} \right) \right] \text{ for } l > 0, m \geq 0 \quad (24)$$

where, $a_1, a_2, a_3, c_1, c_2, c_3$ are constants of integration. We will focus on the third one which could be very nice. Particularly, we investigate the solution for $l > 0$ and $m > 0$ because of compatibility with the observed universe. We choose $c_3 = 0$ and the initial time of the universe $t = 0$. Now taking $a_3 = 1$ in (24), the FRW metric can be written in the form

$$H = \frac{\dot{a}}{a} = -\frac{2}{t(lt - 2m)} \quad (26)$$

Now, substituting for the scale factor $a(t)$ from (25) and making use of (20) in (18) and (19), we obtain the energy density

$$ds^2 = -dt^2 + e^{\frac{4}{m} \arctan h \left(\frac{lt - m}{m} \right)} \left[dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (2)$$

5) and pressure of the universe

From (25), we find the Hubble parameter as

$$p = \frac{2}{\sqrt{\alpha(4\pi + \lambda)} t^2 (lt - 2m)^2} \times \left[2\sqrt{\alpha}(lt - m) - \sqrt{4\pi + \lambda} t (lt - 2m) \sqrt{lt - m} \right] \quad (28)$$

Using (27) and (28) in (18), we obtain

$$\Lambda = \frac{12}{t^2 (lt - 2m)^2} + \frac{2(8\pi + 3\lambda)}{\sqrt{\alpha(4\pi + \lambda)} t (lt - 2m)} - \frac{10\lambda}{\sqrt{\alpha(4\pi + \lambda)} t^2 (lt - 2m)^2} \times \left[2\sqrt{\alpha}(lt - m) - \sqrt{4\pi + \lambda} t (lt - 2m) \sqrt{lt - m} \right] \quad (29)$$

The universe has a finite life time according to the assumption of linearly varying deceleration parameter as in equation (21) with $l > 0$ and $m > 0$. It has a big bang singularity at $t = 0$ with large energy density. The scale factor and the energy density diverge very rapidly with the evolution of cosmic time approaching to a Big Rip singularity [18] at a finite time $t = \frac{2m}{l}$. To obtain a proper energy density, the model must satisfy $lt > m$ for which $q < -1$ (Super exponential expansion).

Ricci scalar is found as

$$R = -\frac{24(lt - m + 2)}{t^2 (lt - 2m)^2} \quad (30)$$

Trace of the energy- momentum tensor is given by

$$T = -\frac{2}{\sqrt{\alpha(4\pi + \lambda)} t (lt - 2m)} + \frac{6}{\sqrt{\alpha(4\pi + \lambda)} t^2 (lt - 2m)^2} \times \left[2\sqrt{\alpha}(lt - m) - \sqrt{4\pi + \lambda} t (lt - 2m) \sqrt{lt - m} \right] \quad (31)$$

The function $f(R, T) = R + 2f(T)$ for this model is given by

$$f(R, T) = -\frac{24(lt - m + 2)}{t^2 (lt - 2m)^2} - \frac{4\lambda}{\sqrt{\alpha(4\pi + \lambda)} t (lt - 2m)} + \frac{12\lambda}{\sqrt{\alpha(4\pi + \lambda)} t^2 (lt - 2m)^2} \times \left[2\sqrt{\alpha}(lt - m) - \sqrt{4\pi + \lambda} t (lt - 2m) \sqrt{lt - m} \right] \quad (32)$$

5. Discussions

From the Fig-1, we observe that energy density ρ is decreasing function of time t and ρ approaches towards zero with the evolution of time.

From Fig-2, it can be clearly seen that pressure p is negative and approaches towards zero with the evolution of

time.

According to the Fig-3, the cosmological constant Λ is decreasing function of time and it approaches to a small positive value. This sort of behavior of the cosmological constant is congruent with the present accelerated behavior of the universe.

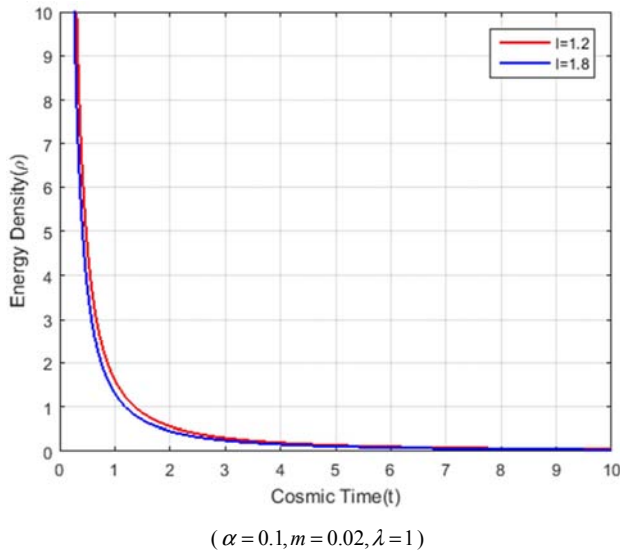


Figure 1. Energy Density vs Cosmic Time.

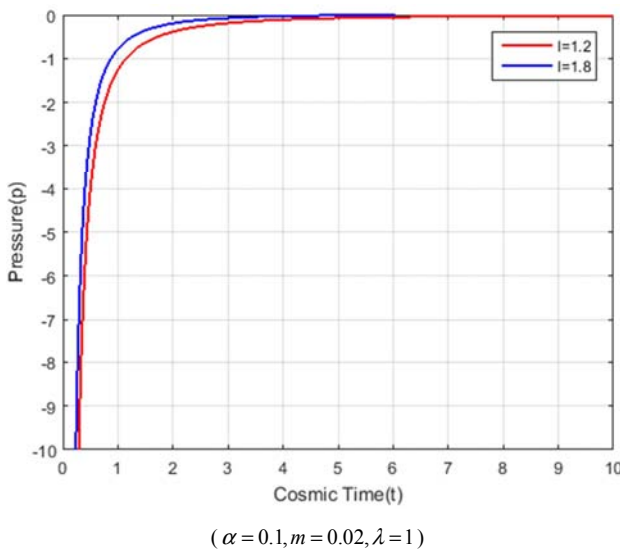


Figure 2. Pressure vs Cosmic Time.

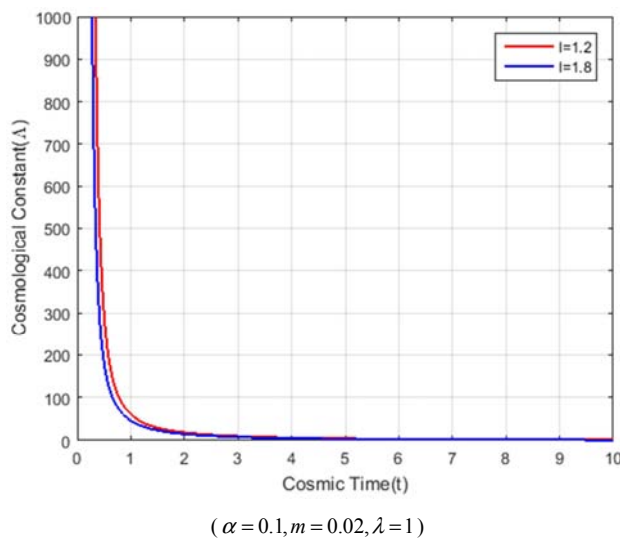


Figure 3. Cosmological Constant vs Cosmic Time.

6. Conclusions

In this work, we have investigated a flat FRW space-time in $f(R, T)$ gravity with cosmological constant in the presence of perfect fluid, formulated by Harko et al. [10]. The field equations of the theory have been solved by using quadratic equation of state for a perfect fluid and time dependent deceleration parameter proposed by Akarsu and Dereli [13]. The physical behavior of energy density, pressure and time dependent cosmological constant of the universe is discussed. It is observed that all the physical quantities diverge at $t = 0$ and vanishes as $t \rightarrow \infty$. The model will also help to explain super exponential expansion of the universe in the presence of cosmological constant (Λ). Here, it should be mentioned that super exponential expansion is rapid rate of expansion if $q < -1$ under linearly varying deceleration parameter.

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