Study of variation of temperature of black holes with respect to mass in XRBs

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Abstract: In the present paper, we have derived an expression for the variation of temperature of the black holes with respect to mass using the temperature of black hole at which the black hole radiates to infinity all species of particles with a perfect black body spectrum and surface gravity for Schwarzschild black holes. We have also calculated their values of different test black holes existing only in X-ray binaries.

Keywords: Surface gravity, X-ray binaries, Schwarzschild black holes

1. Introduction

Classically black holes are perfect absorbers, but do not emit anything; their physical temperature is absolute zero (Wald,2001) and they pull all the things towards themselves like materials, messages, information, sound light etc. In 2005, Ram et. al concluded that the black hole is a Bose-Einstein ensemble of quanta of mass equal to twice the Planck mass, confined in a sphere of radius twice the black hole (Ram et al. 2005). Quantum mechanically, however, there is a possibility that one of a particle production pair in a black hole is able to tunnel the gravitational barrier and escapes the black hole's horizon. Thus, a black hole is not really black; it can radiate or evaporate particles (Triyanta et al. 2013). Stephan Hawking provided a theoretical argument for its existence in 1974, and sometimes also after the physicist Jacob Bekenstein who predicted that black holes should have a finite, non-zero temperature and entropy and also stated that it is proportional to the black hole area A (Hawking, 1988). Later on, Bardeen, Carter and Hawking performed calculations using a semi-classical approximation, putting Bekenstein conjecture on a firm basis. They established that the black hole temperature is proportional to its surface gravity (Bardeen et al.1973 & Silva 2006). Shunichiro Kinoshita and Norihiro Tanahashi discussed the Hawking temperature of near-equilibrium black holes using a semi-classical analysis and found that the temperature is determined by the surface gravity of the past horizon (S. Kinoshita & N. Tanahashi, 2012). In 2013, Triyanta & Anike N. Bowaire concluded that in the case of the Schwarzschild black hole, a higher Hawking temperature corresponds to a lower mass of a black hole (Triyanta et al. 2013).

In the present research work, an expression for the variation of temperature of the black holes with respect to mass has been derived using the temperature of black hole at which the black hole radiates to infinity all species of particles with a perfect black body spectrum and surface gravity for Schwarzschild black holes. Thereafter, we have also calculated their values of different test black holes existing only in X-ray binaries.

2. Theoretical Discussion

On the basis of quantum field theory, Stephan Hawking predicted that black holes should radiate energy like a black body with a temperature proportional to the surface gravity(κ) of the black hole given by

$$T = \frac{\kappa}{2\pi} \tag{1}$$

Thus, $\dfrac{\kappa}{2\pi}$ truly is the physical temperature of a black hole, not merely a quantity playing a role mathematically analogous to the temperature in the laws of black hole mechanics (Wald 2001, Mahto et al 2012).

For the Schwarzschild case, the surface gravity (κ) of a black hole is given by (Transchen 2000, Mahto et al. 2011 & 2012)

$$\kappa = 1/4M \tag{2}$$

Putting eqn. (2) in eqn. (1), temperature of a black hole becomes

$$T = 1/8\pi M \tag{3}$$

Using the proper value of four fundamental constants of nature – the speed of light (c), Planck's constant (h), Newton's gravitational constant (G) and Boltzmann constant (k) on right side of the above equation, temperature of a black hole becomes

$$T = \frac{ch^3}{8\pi kGM} \tag{4}$$

$$T = \frac{ch^3}{8\pi kG}\left(M^{-1}\right) \tag{5}$$

The above equation is differentiated with respect to the mass M, we have

$$\frac{dT}{dM} = -\frac{ch^3}{8\pi GkM^2} \tag{6}$$

$$\frac{dT}{dM} = -\frac{ch^3}{8\pi Gk}\left(\frac{1}{M^2}\right) \tag{7}$$

$$\frac{dT}{dM} = -K_D\left(\frac{1}{M^2}\right) \tag{8}$$

$$K_D = \frac{ch^3}{8\pi Gk} \tag{9}$$

$$K_D = 3.767\times10^{-60}\,(S.I.units) \tag{10}$$

$$\frac{dT}{dM} \propto -\frac{1}{M^2} \tag{11}$$

The negative sign in the equations (6) and (7) shows the decrease in mass of the black hole takes place with decrease in the temperature of black hole which occurs due to emission of the Hawking radiations from the black hole. The

equation (11) shows that the variation of the temperature of the black hole is inversely proportional to the square of its mass. K_D is the new constant involving in the equation (8) having value given by equation (10).

When particles escape, the black hole losses small amount of its energy and therefore of its mass according to mass-energy equivalence relation $E=mc^2$. The Hawking radiation process reduces the mass and energy of the black hole and is therefore also known as black holes evaporation. This also leads to the finite life time and lowering the temperature of the black holes.

3. Data in the Support of Black Holes

There are two categories of black holes classified on the basis of their masses clearly very distinct from each other, with very different masses M ~ 5 - 20 $M_\odot$ for stellar – mass black holes in X-ray binaries and M ~ 106 – 109.5 $M_\odot$ for super massive black holes in Active Galactic Nuclei (Narayan, 2005 & Mahto et al. 2012).

4. Data Used in Calculation

G $=6.673\times10^{-11}$m³kg⁻¹s⁻²,   h   = 6.623×10^{-34}Js, k$=1.380\times10^{-23}$J/K , c $= 3\times10^8$m/s, $M_\odot$ = 1.99 x 10^{30} kg.

5. Table

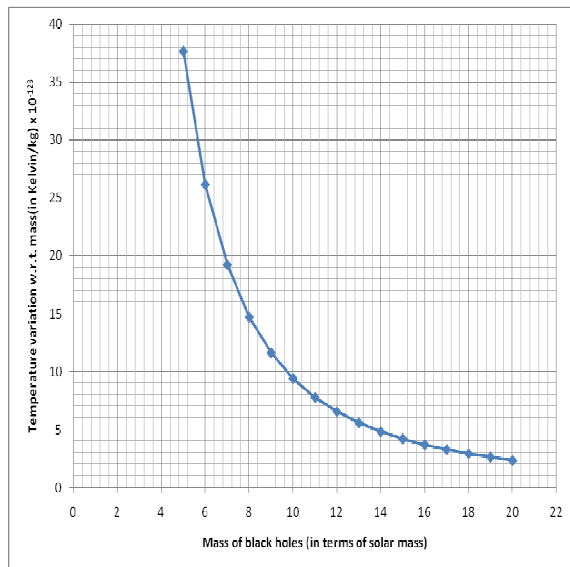| Variation of temp. of black holes w.r.t. mass in XRBs		
S. N.	Mass of black holes (in $M_\square$)	$\frac{dT}{dM} = -\frac{ch^3}{8\pi GkM^2}$ (in Kelvin/kg)
1	5$M_\square$	37.6667×10^{-123}
2	6$M_\square$	26.1622×10^{-123}
3	7$M_\square$	19.2251×10^{-123}
4	8$M_\square$	14.7165×10^{-123}
5	9$M_\square$	11.6255×10^{-123}
6	10$M_\square$	9.4010×10^{-123}
7	11$M_\square$	7.7831×10^{-123}
8	12$M_\square$	6.5405×10^{-123}
9	13$M_\square$	5.5724×10^{-123}
10	14$M_\square$	4.8048×10^{-123}
11	15$M_\square$	4.1859×10^{-123}
12	16$M_\square$	3.6786×10^{-123}
13	17$M_\square$	3.2584×10^{-123}
14	18$M_\square$	2.9064×10^{-123}
15	19$M_\square$	2.6082×10^{-123}
16	20$M_\square$	2.3167×10^{-123}

6. Graph

Fig 1. Graph plotted between mass of different test of black holes and variation of temp. w. r. t. mass in XRBs.

7. Results and Discussion

In the present paper, we have derived an expression for the variation of temperature of the black holes with respect to mass using the temperature of black hole ($T = \kappa / 2\pi$) at which the black hole radiates to infinity all species of particles with a perfect black body spectrum and surface gravity($\kappa = 1/4M$) for Schwarzschild black holes and finally equation (7) is obtained. From the expression (11), it is clear that the variation of the temperature of the black hole is inversely proportional to the square of its mass and the negative sign shows the decrease in the mass of the black hole with decrease in the temperature of different test black holes. The work is further extended to calculate their values of different test black holes existing only in X-ray binaries. To know the nature of temperature variation w.r.t. mass of black holes, the   graph is plotted between the mass of different test black holes and their corresponding temperature variation in XRBs (Figure 1). From the graph plotted in the figure1, it is clear that the rate of change in temperature of black holes with respect to the mass initially falls rapidly with increasing the mass of black holes from $5M_\odot$ to $10M_\odot$ and then gradually falls up to the mass $20M_\odot$ in XRBs. The graph plotted between mass of different test of black holes and variation of temperature with respect to mass in XRBs also shows that the temperature of the black hole is lowering with increasing the mass of different test black holes. This indicates that the heavier black holes have relatively lower temperature than that of the lighter black holes which agrees with the work of Triyanta & A. N. Bowaire, 2013 as discussed in the introduction of the same paper. This gives justification for the formula of the temperature of black hole as proposed Stephan Hawking.

8. Conclusion

In the light of the present work, we can draw the following conclusions:

(1) Heavier the black holes lower is the rate of change in temperature with respect to the mass of black holes.

(2) Lighter black holes radiate energy faster than heavier black holes.

(3) Our results agree with the works of Triyanta & A. N. Bowaire and the Hawking temperature.

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