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# Blow-up for Semidiscretisations of a Semilinear Schrodinger Equation with Dirichlet Condition

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**Abstract:** Theoretical study of the phenomenon of blow-up solutions for semilinear Schrödinger equations has been the subject of investigations of many authors. It is said that the maximal time interval of existence of the solution blows up in a finite time when this time is finite, and the solution develops a singularity in a finite time. In fact, semilinear Schrhödinger equation models a lot of physical phenomenon such as nonlinear optics, energy transfer in molecular systems, quantum mechanics, seismology, plasma physics. In the past, certain authors have used numerical methods to study the phenomenon of blow-up for semilinear Schrödinger equations. They have considered the same problem and one proves that the energy of the system is conserved, and the method used to show blow-up solutions are based on the energy's method. This paper proposes a method based on a modification of the method of Kaplan using eigenvalues and eigenfunctions to show that the semidiscrete solution blows up in a finite time under some assumptions. The semidiscrete blow-up time is also estimate. Similar results are obtain replacing the reaction term by another form to generalise the result. Finally, this paper propose two schemes for some numerical experiments and a graphics is given to illustrate the analysis.

**Keywords:** Semidiscretization, Blow-up, Schrödinger Equations

## 1. Introduction

This paper concerns the numerical approximation for the following initial-boundary value problem for a semilinear Schrhödinger equation of the form:

$$u_t = iau_{xx} - ib|u|^p, x \in (0,1), t \in (0,T) \tag{1}$$

$$u(0,t) = 0, u(1,t) = 0, t \in (0,T) \tag{2}$$

$$u(x,0) = u_0(x), x \in [0,1], \tag{3}$$

which appears in a lot of models of nonlinear optics, energy transfer in molecular systems, quantum mechanics, seismology, plasma physics, see [4, 21, 28], to cite only a few cases. Here p>1, $a \in IR, a \neq 0, b > 0$. The initial datum $u_0(x)$ is a continuous function in [0, 1]. The conditions $u_0(0) = 0\ and\ u_0(1) = 0$ mean that the temperature is maintained nil on the boundary x=0 and x=1.

Here (0, T) is the maximal time interval of existence of the solution u. The time T may be finite or infinite. When T is infinite, we say that the solution u exists globally. When T is finite, the solution u develops a singularity in a finite time, namely

$$\lim_{t\to T} \|u(x,t)\|_\infty = \infty$$

where $\|u(x,t)\|_\infty = sup_{x\in(0,1)}|u(x,t)|$. In this case, it is say that the solution u blows up in a finite time and the time T is called the blow-up time of the solution u.

The theoretical study of the phenomenon of blow-up and in particular blow-up solutions for semilinear Schrödinger equations has been the subject of investigations of many authors (see [1, 3, 9, 15, 17, 18, 23], and the references cited therein).

This paper is interested by the numerical study of the above problem. Let I be a positive integer and define the grid $x_j = jh, 0 \le j \le I$, where h=1/I. Approximate the solution u of the problem (1)--(3) by the solution

$U_h(t)=\left(U_0(t),U_1(t),\dots,U_I(t)\right)^T$ of the following semidiscrete equations

$$\frac{d}{dt}U_j(t)=ia\delta^2U_j(t)-ib\left|U_j(t)\right|^p, 1\le j\le I-1, t\in(0,T_h) \quad (4)$$

$$U_0(t)=0, U_I(t)=0, t\in(0,T_h) \quad (5)$$

$$U_j(0)=\varphi_j, 0\le j\le I \quad (6)$$

where

$$\delta^2U_j(t)=\frac{U_{j+1}(t)-2U_j(t)+U_{j-1}(t)}{h^2}$$

Here, $(0,T_h)$ is the maximal time interval on which $\|U_h(t)\|_\infty$ is finite, where $\|U_h(t)\|_\infty=\max_{0\le j\le I}\left|U_j(t)\right|$.

When $T_h$ is finite, it is say that the solution $U_h(t)$ of (4)--(6) blows up in a finite time and the time $T_h$ is called the semidiscrete blow-up time of solution $U_h(t)$. It is show that under some assumptions, the solution of the semidiscrete problem defined in (4)--(6) blows up in a finite time and estimate its semidiscrete blow-up time. This paper proposes also some schemes and algorithms for the numerical calculation of the blow-up time. In the past, certain authors have used numerical methods to study the phenomenon of blow-up for semilinear Schrödinger equations but they have considered the problem (1)--(3) in the case where the term $-ib|u(x,t)|^p$ is replaced by $-ib|u(x,t)|^{p-1}u(x,t)$ (see for instance [3, 24]). In this case, one proves that the energy of the system is conserved, and the method used to show blow-up solutions are based on the energy's method. This paper propose a method based on a modification of the method of Kaplan (see [14]) using eigenvalues and eigenfunctions to show that the solution $U_h(t)$ of (4)--(6) blows up in a finite time if $\sum_{j=1}^{I-1}\tan\left(\frac{\pi}{2}h\right)\sin(j\pi h)\,Re(\varphi_j)$ is large enough. The above result is also extend replacing -ib in (4) by b and also by $c-ib$ where b>0 and c>0. One integrates the semidiscrete scheme and obtain some discrete schemes where the convergence and stability have been proved (see for instance [8, 11, 13, 25]). We utilize these schemes to compute the numerical blow-up time by means of appropriate algorithms. In [6, 19, 20], one may find some results about the numerical study of the phenomenon of blow-up and extinction for semilinear parabolic equations.

This paper is written in the following manner. In the next section, the authors give some conditions under which the solution of (4)--(6) blows up in a finite time and estimate its semidiscrete blow-up time. In the last section, we propose some schemes and algorithms to compute the numerical blow-up time. Some numerical values are given.

## 2. Semidiscrete Blow-up Solutions

In this section, under some assumptions, we show that the solution of the semidiscrete problem blows up in a finite time and estimate its semidiscrete blow-up time. One need the following Lemma.

Lemma 2.1. We have $\sum_{j=1}^{I-1}sin(j\pi h)=cotan\,(\frac{\pi h}{2})$.

*Proof.* A routine calculation yields

$$\sum_{j=1}^{I-1}sin(j\pi h)=Im\left(\sum_{j=1}^{I-1}e^{ij\pi h}\right)=Im\left(\sum_{j=1}^{I-1}(e^{i\pi h})^j\right)$$

$$=Im\left(e^{i\pi h}\frac{1-e^{i\pi h(I-1)}}{1-e^{ij\pi}}\right)=Im\left(\frac{e^{i\pi h}-e^{i\pi}}{1-e^{i\pi h}}\right)$$

Because hI=1. Since $e^{i\pi}=-1$, we arrive at

$$\sum_{j=1}^{I-1}sin(j\pi h)=Im\left(\frac{e^{i\pi h}+1}{1-e^{i\pi h}}\right)=Im\left(-\frac{e^{i\frac{\pi h}{2}}+e^{-i\frac{\pi h}{2}}}{e^{i\frac{\pi h}{2}}-e^{-i\frac{\pi h}{2}}}\right)$$

$$=Im\left(icotan\,(\frac{\pi h}{2}\right)=cotan\,(\frac{\pi h}{2})$$

and the proof is complete.

Lemma 2.2 Let $U_h, V_h$ two vectors such that

$$U_0=0, U_I=0, V_0=0, V_I=0$$

Then we have

$$\sum_{j=1}^{I-1}hU_j\delta^2V_j=\sum_{j=1}^{I-1}hV_j\delta^2U_j \quad (7)$$

Proof. A straightforward computation reveals that $\sum_{j=1}^{I-1}hU_j\delta^2V_j=\sum_{j=1}^{I-1}hV_j\delta^2U_j+\frac{V_IU_{I-1}-U_IV_{I-1}+V_0U_1-U_0V_1}{h}$ and the result follows using the assumptions of the lemma.

Now let us state our first result on blow-up.

Theorem 2.1 Assume that $1-\frac{a\lambda_hA^{1-p}}{b(p-1)}>0\ where$

$$\lambda_h=\frac{2-2\cos\pi h}{h^2}$$

and

$$A=\sum_{j=1}^{I-1}tan\left(\frac{\pi h}{2}\right)sin(j\pi h)Re(\varphi_j)$$

Then the solution $U_h$ of (4)—(6) blows up in a finite time $T_h$ which is estimated as follows

$$T_h\le\frac{1}{a\lambda_h}arcos\,\left(1-\frac{a\lambda_hA^{1-p}}{b(p-1)}\right) \quad (8)$$

Proof. Since $(0,T_h)$ is the maximal time interval on which $\|U_h(t)\|_\infty$ is finite, our aim is to show that $T_h$ is finite and obeys the above inequality. Introduce the functions v and w defined as follows

$$v(t)=\sum_{j=1}^{I-1}tan\left(\frac{\pi h}{2}\right)sin(j\pi h)U_j(t)\ and$$

$$w(t)=\sum_{j=1}^{I-1}tan\left(\frac{\pi h}{2}\right)sin(j\pi h)\overline{U}_j(t)$$

Taking the derivative of v in t and using (4), we get

$$v'(t) = ia\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)U_j(t)$$

$$-ib\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p$$

One observes that $\delta^2 sin(j\pi h) = -\lambda_h sin(j\pi h)$. Due to Lemma 2.2, we arrive at

$$v'(t) = -ia\lambda_h v(t) - ib\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p,$$

which implies that

$$\frac{d}{dt}\left(e^{ia\lambda_h t}v(t)\right) = -ibe^{ia\lambda_h t}\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p.$$

We also observe that, taking the derivative of w in t and using (4), we discover that

$$w'(t) = -ia\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right) \sin(j\pi h)\delta^2\overline{U}_j(t)$$

$$-ib\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right) \sin(j\pi h)\left|U_j(t)\right|^p$$

Reasoning as above, we find that

$$\frac{d}{dt}\left(e^{-ia\lambda_h t}w(t)\right) =$$

$$-ibe^{-ia\lambda_h t}\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p.$$

We deduce that

$$Z'(t) = b\sin(a\lambda_h t)\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right) \sin(j\pi h)\left|U_j(t)\right|^p,$$

where Z(t)=$Z(t) = \frac{e^{ia\lambda_h t}w(t)+e^{-ia\lambda_h t}w(t)}{2}$. From Lemma 2.1, we see that $\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)$ equals one. Thus applying Jensen's inequality, we find that $\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\left|U_j(t)\right|^p$ is bounded from below by $\left(\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\left|U_j(t)\right|\right)^p$. Applying the triangle inequality, we discover that $|Z(t)|$ is bounded from above by $\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\left|U_j(t)\right|$. Since $\sin(a\lambda_h t)$ is nonnegative when t is between 0 and $\frac{\pi}{a\lambda_h}$, we deduce that

$$Z'(t) \geq bsin(a\lambda_h t)|Z(t)|^p \ for\ t \in \left(0, \frac{\pi}{a\lambda_h}\right).$$

This inequality implies that the function Z(t) is increasing. Since Z(0) is positive, we find that

$$Z'(t) \geq b\sin(a\lambda_h t)\left(Z(t)\right)^p \ for\ t \in \left(0, \frac{\pi}{a\lambda_h}\right),$$

which implies that

$$\frac{dZ}{Z^p} \geq b\sin(a\lambda_h t)dt \ for\ t \in \left(0, \frac{\pi}{a\lambda_h}\right).$$

Let $T_h^* = \min\left(\frac{\pi}{a\lambda_h}, T_h\right)$. Integrating this inequality over $(0, T_h^*)$, we conclude that

$$\frac{(Z(0))^{1-p}}{p-1} \geq \frac{b}{a\lambda_h}(1 - \cos(a\lambda_h T_h^*)).$$

Therefore, we have

$$\cos(a\lambda_h T_h^*) \geq 1 - \frac{a\lambda_h}{b}\frac{\left(Z(0)\right)^{1-p}}{p-1}.$$

Since the quantity on the right-hand side of the above inequality is positive, we see that the time $T_h^*$ is estimated as follows

$$T_h^* \leq \frac{1}{a\lambda_h} arcos\left(1 - \frac{a\lambda_h}{b}\frac{\left(Z(0)\right)^{1-p}}{p-1}\right).$$

Since $1 - \frac{a\lambda_h}{b}\frac{\left(Z(0)\right)^{1-p}}{p-1}$ is positive, we deduce that $T_h^* \leq \frac{\pi}{2a\lambda_h}$. Consequently $T_h^* = T_h$ is finite. Use the fact that $Z(0) = A$ to complete the rest of the proof.

Now, we consider the following initial-boundary value problem

$$u_t - iau_{xx} = b|u|^p, x \in (0,1), t \in (0,T) \quad (9)$$

$$u(0,t) = 0, u(1,t) = 0, t \in (0,T) \quad (10)$$

$$u(x,0) = u_0(x), x \in [0,1] \quad (11)$$

where $p > 1$, $u_0(0) = 0\ and\ u_0(1) = 0$.

Approximate the solution u of (9)—(11) by the solution $U_h(t) = (U_0(t), \ldots, U_I(t))^T$ of the following semidiscrete equations

$$\frac{d}{dt}U_j(t) = ia\delta^2 U_j(t) + b\left|U_j(t)\right|^p, 1 \leq j \leq I-1, t \in (0, T_h) \quad (12)$$

$$U_0(t) = 0, U_I(t) = 0, t \in (0, T_h) \quad (13)$$

$$U_j(0) = \varphi_j, 0 \leq j \leq I \quad (14)$$

where (0, $T_h$) is the maximal time interval on which $\|U_h(t)\|_\infty$ is finite. Our second result on blow-up is the following.

Theorem 2.2 Assume that $\frac{a\lambda_h A^{1-p}}{b(p-1)} \leq \frac{1}{2}\ where$

$$\lambda_h = \frac{2 - 2\cos\pi h}{h^2}\ and$$

$$A=\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)Re(\varphi_j).$$

Then the solution $U_h$ of (12)—(14) blows up in a finite time $T_h$ which is estimated as follows

$$T_h \le \frac{1}{a\lambda_h} arsin\left(1-\frac{a\lambda_h A^{1-p}}{b(p-1)}\right) \tag{15}$$

Proof. Since $(0,T_h)$ is the maximal time interval on which $\|U_h(t)\|_\infty$ is finite, our aim is to show that $T_h$ is finite and obeys the above inequality. Introduce the functions v and w defined as follows

$$v(t)=\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)U_j(t)\ and$$

$$w(t)=\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\bar{U}_j(t).$$

Taking the derivative of v in t and using (12), we get

$$v'(t)=ia\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)U_j(t)$$

$$+b\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p$$

We observe that $\delta^2 sin(j\pi h)=-\lambda_h sin(j\pi h)$. Due to Lemma 2.2, we arrive at

$$v'(t)=-ia\lambda_h v(t)+b\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p,$$

which implies that

$$\frac{d}{dt}\left(e^{ia\lambda_h t}v(t)\right)=be^{ia\lambda_h t}\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p.$$

We also observe that, taking the derivative of w in t and using (12), we have

$$w'(t)=-ia\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\delta^2\bar{U}_j(t)$$

$$+b\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\left|U_j(t)\right|^p$$

Reasoning as above, we find that

$$\frac{d}{dt}\left(e^{-ia\lambda_h t}w(t)\right)=$$

$$be^{-ia\lambda_h t}\sum_{j=1}^{I-1} tan\left(\frac{\pi h}{2}\right) sin(j\pi h)\left|U_j(t)\right|^p.$$

We deduce that

$$Z'(t)=b\cos(a\lambda_h t)\sum_{j=1}^{I-1} \tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\left|U_j(t)\right|^p,$$

where $Z(t)=\frac{e^{ia\lambda_h t}w(t)+e^{-ia\lambda_h t}w(t)}{2}$. Arguing as in the proof of Theorem 2.1, we deduce that

$$Z'(t)\ge b\cos(a\lambda_h t)\left(Z(t)\right)^p\ for\ t\in\left(0,\frac{\pi}{a\lambda_h}\right),$$

which implies that

$$\frac{dZ}{Z^p}\ge b\cos(a\lambda_h t)dt\ for\ t\in\left(0,\frac{\pi}{a\lambda_h}\right).$$

Let $T_h^*=\min\left(\frac{\pi}{2a\lambda_h},T_h\right)$. Integrating this inequality over $(0,T_h^*)$, we obtain

$$\frac{(Z(0))^{1-p}}{p-1}\ge\frac{b}{a\lambda_h}(\sin a\lambda_h T_h^*),$$

which implies that

$$\sin(a\lambda_h T_h^*)\le\frac{a\lambda_h}{b}\frac{\left(Z(0)\right)^{1-p}}{p-1}.$$

We deduce that

$$T_h^*\le\frac{1}{a\lambda_h} arsin\left(\frac{a\lambda_h}{b}\frac{\left(Z(0)\right)^{1-p}}{p-1}\right).$$

Since $\frac{a\lambda_h}{b}\frac{(Z(0))^{1-p}}{p-1}\le\frac{1}{2}$, we have $T_h^*\le\frac{\pi}{6a\lambda_h}$. This implies that $T_h^*=T_h$ is finite. Therefore $T_h$ is finite and use the fact that Z(0)=A to complete the rest of the proof.

*Remark 2.1* Consider the following initial-boundary value problem

$$u_t-iau_{xx}=(c-ib)|u|^p, x\in(0,1), t\in(0,T) \tag{16}$$

$$u(0,t)=0, u(1,t)=0, t\in(0,T) \tag{17}$$

$$u(x,0)=u_0(x), x\in[0,1]0 \tag{18}$$

where $c>1, b>0\ and$ approximate the solution u of (16)—(18) by the solution $U_h(t)=(U_0(t),\ldots,U_I(t))^T$ of the following semidiscrete equations

$$\frac{dU_j(t)}{dt}=ia\delta^2U_j(t)+(c-ib)\left|U_j(t)\right|^p, 1\le j\le I-1, t\in(0,T_h) \tag{19}$$

$$U_0(t)=0, U_I(t)=0, t\in(0,T_h) \tag{20}$$

$$U_j(0)=\varphi_j, 0\le j\le I \tag{21}$$

Combining the methods developed in the proofs of Theorems 2.1 and 2.2, we easily prove that if $\sum_{j=1}^{I-1}\tan\left(\frac{\pi h}{2}\right)\sin(j\pi h)\,Re(\varphi_j)$ is large enough, the solution $U_j(t)$ of the above semidiscrete problem blows up in a finite time.

## 3. Numerical Results

In this section, one present some numerical approximations of the blow-up time for the solution of the problem (1)-(3). Consider the following explicit and implicit schemes

Scheme I

$$\frac{U_j^{(n+1)} - U_j^{(n)}}{\Delta\, t_n} = ia\frac{\left(U_{j+1}^{(n)} - 2U_j^{(n)} + U_{j-1}^{(n)}\right)}{h^2} - ib\left|U_j^{(n)}\right|^p, 1 \le i \le I-1,$$

$$U_0^{(n)} = 0, U_I^{(n)} = 0,$$

$$U_j^{(0)} = \varphi_i, 0 \le i \le I.$$

Scheme II

$$\frac{U_j^{(n+1)} - U_j^{(n)}}{\Delta\, t_n} = ia\frac{\left(U_{j+1}^{(n+1)} - 2U_j^{(n+1)} + U_{j-1}^{(n+1)}\right)}{h^2}$$

$$-ib\left|U_j^{(n)}\right|^p, 1 \le i \le I-1,$$

$$U_0^{(n+1)} = 0, U_I^{(n+1)} = 0,$$

$$U_j^{(0)} = \varphi_i, 0 \le i \le I,$$

where $n \ge 0, \Delta t_n = min\left\{\frac{h^2}{2|a|}, \tau\left\|U_h^{(n)}\right\|_\infty^{1-p}\right\}$ $with\ \tau = const\ \in (0,1)$. We need the following definition.

*Definition 3.1* One say that the solution $U_h^{(n)} = (U_0^{(n)}, \dots, U_I^{(n)})^T$ of Scheme I or II blows up in a finite time if $\lim_{n\to\infty}\left\|U_h^{(n)}\right\|_\infty = \infty$ and the series $\sum_{n=0}^{\infty}\Delta\, t_n$ converges. The quantity $\sum_{n=0}^{\infty}\Delta\, t_n$ is called the numerical blow-up time of $U_h^{(n)}$.

In the tables 1 and 2 in rows, we present the numerical blow-up times, the numbers of iterations, the CPU times and the orders of the approximations corresponding to meshes of 16, 32, 64, and 128. We take the numerical blow-up time $t_n = \sum_{n=0}^{n-1}\Delta\, t_j$ which is computed at the first time when $\Delta\, t_n = |t_{n+1} - t_n| \le 10^{-16}$. The order (s) of the method is computed from

$$s = \frac{\log((T_{4h} - T_{2h})/(T_{2h} - T_h)}{\log(2)}.$$

For the numerical values, we take p=2, $U_j^{(0)} = 20\sin(\pi\, j\, h)$ a=1, b=1 and $\tau = h^{3/2}$.

***Table 1.*** *Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with Scheme I.*

| I | $t_n$ | N | CPUt | S |
|---|---|---|---|---|
| 16 | 0.078223 | 18837 | - | - |
| 32 | 0,078229 | 72612 | 4 | - |
| 64 | 0,078232 | 279341 | 45 | 1.00 |
| 128 | 0,078233 | 6962549 | 14611 | 1.58 |

In this graphics, one can see that the norm of the solution u of the problem (1)—(3) is increasing and develops a singularity in a finite time. Also, we see that the blow-up rate occurs at the middle of the solution for the mesh i=I/2. This graphics respect $U_0(t) = 0, U_I(t) = 0, t \in (0, T_h)$. But this condition doesn't prevent the blow-up of the solution.

***Table 2.*** *Numerical blow-up times, numbers of iterations, CPU times (seconds) and orders of the approximations obtained with Scheme II.*

| I | $t_n$ | N | CPUt | S |
|---|---|---|---|---|
| 16 | 0.078280 | 14807 | 1 | - |
| 32 | 0,078244 | 56510 | 6 | - |
| 64 | 0,078236 | 214935 | 95 | 2,1 |
| 128 | 0,078234 | 6962549 | 14611 | 2,0 |

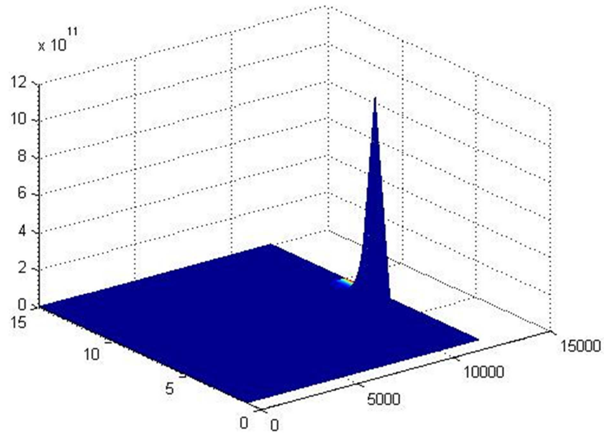


***Figure 1.*** *Evolution of discrete solution* $\left|U_j^{(n)}\right|$ *for a=b=1.*

## 4. Conclusion

Under some assumption, and using a method based on a modification of the method of Kaplan, it is show that the semidiscete solution of the semilinear solution of the problem (1)-(3) blows up in a finite time and the semidiscrete blow-up time is estimate. The result obtains with the problem (1)-(3) is generalize considering a reaction term more complex. At the end, two schemes proposed, permit to illustrate the estimation of the numerical blow-up time which converge to 0,0782 (see Tables 1 and 2). But the convergence of the schemes proposed was not proof and can be the subject of another investigation.

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