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Review Article

Relationship Between the Height of a Tank with Pipe Fittings and Efflux Time

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Abstract: The problem of determining the efflux time for tanks of various sizes, shapes, and purposes has received significant attention by engineering literature. Models developed are typically situational, and prove useful in very specific scenarios. Few of these models however, take into consideration tanks with pipe fittings or some kind of tubing aligned. Traditional investigations focus on efflux times of tanks with open orifices, through the usage and manipulation of Torricelli's law. This essay deals with tanks that are connected to pipes, and how the friction between the fluid and pipe walls impedes the usage of Torricelli's law and requires more advanced models. The principal objective of this essay is to develop a model that describes the efflux time of tanks with pipe fittings by investigating its relationship with fluid velocity and flow rate. This includes usage of Bernoulli's Extended equations and undertaking the assumptions of pseudo-steady state flow. The computer software MATLAB will be used to develop the efflux time model and other complex differential equations. The model developed by the paper is then compared to other models in literature, in terms of accuracy of prediction. The most significant conclusion arrived at is that ignorance of minor losses advances the argument over its significance when dealing with the efflux of tanks, and must be considered in any new models in the future.

Keywords: Tanks, Unsteady Flow, Exit Pipe, Continuity

1. Introduction

Fluid mechanics is a branch of physics which deals with fluid behavior and properties, and has the reputation of involving highly complex calculations, yet, the concepts dealt with are very easy to understand, and are for the most part, built into our everyday sense of intuition. The efflux time of a bucket with a protruding tube as this experiment models many real world scenarios, and brings to together many physical and mathematical concepts together.

Over the past 30 years, the problem of determining the efflux time for tanks of various sizes, shapes, and purposes has received significant attention by engineering literature [6]. The models developed are typically situational, and prove useful in very specific scenarios. Few of these models however, take into consideration tanks with pipe fittings or some kind of tubing aligned. Traditional investigations focus on efflux times of tanks with open orifices, through the usage and manipulation of Torricelli's law. This essay, however, deals with tanks that are connected to pipes, and how the friction between the fluid and pipe walls impedes the usage of Torricelli's law and requires more advanced models. Many works have attempted to describe fluid behavior in tanks with pipe fittings, yet make the assumptions that the frictional forces within pipelines are negligible enough to be considered regions of inviscid flow, or that the flow rate of an emptying tank remains relatively constant. Other works do produce relatively accurate models yet make use of overly complicated mathematical or computer software analysis.

This essay seeks to answer the research question, "What is the relationship between the height of a tank with a pipe fitting attached the time needed to drain all the water in the tank?" It attempts to challenge the absonant assumptions that are offered by concurrent literature that arise from usage of Bernoulli's equation.

The principal objective of this essay is to develop a model that describes the efflux time of tanks with pipe fittings by investigating its relationship with fluid velocity and flow rate. This includes usage of Bernoulli's Extended equations and undertaking the assumptions of pseudo-steady state flow. Mathematical manipulations concerning changes in the height of the liquid will be made according to the studies performed by Sianoudis and Drakaki from the University of Athens [11]. The success of the model will be compared to collected experimental data. The reliability of the measurements and assumptions is evaluated against the works and theories presented by concurrent literature.

This essay embarks a mathematical approach combined with scientific theory. The computer software MATLAB will be used to develop the efflux time model and other complex differential equations. The development of the model will be backed by rigorous theory and empirical data, and will also be re-examined when compared to other models. Whilst the essays theoretical scope overarches some typically untouched concepts in high school such as the Reynolds Number and Friction factor, it does limit itself by not considering the effects of minor losses and vortices in pipe flow, as well as the unsettling of water.

This research question proves worthy of investigation due to the practical applications that emerge from the experiment setup. Drainage of oil cisterns, water towers, storage tanks, as well as IV drips can all be analyzed through this experiment as they all feature some sort of gravity fed pipe system. More importantly, the results can be manipulated for more complicated scenarios, particularly in civil engineering, involving pumps and pressure gauges, or even the transport of viscous liquids such as oil or petrol. The conclusions arrived at will also clarify general misconceptions on how hydrostatic pressure can influence fluid behavior, somewhat contrary to the intuition of free fall accrued in classical mechanics.

2. Experiment and Data

2.1. Experimental Setup

The setup is designed to study how long it takes for the water to completely drain (discussed in section 6). Figure 1 represents the apparatus and setup of the experiment.

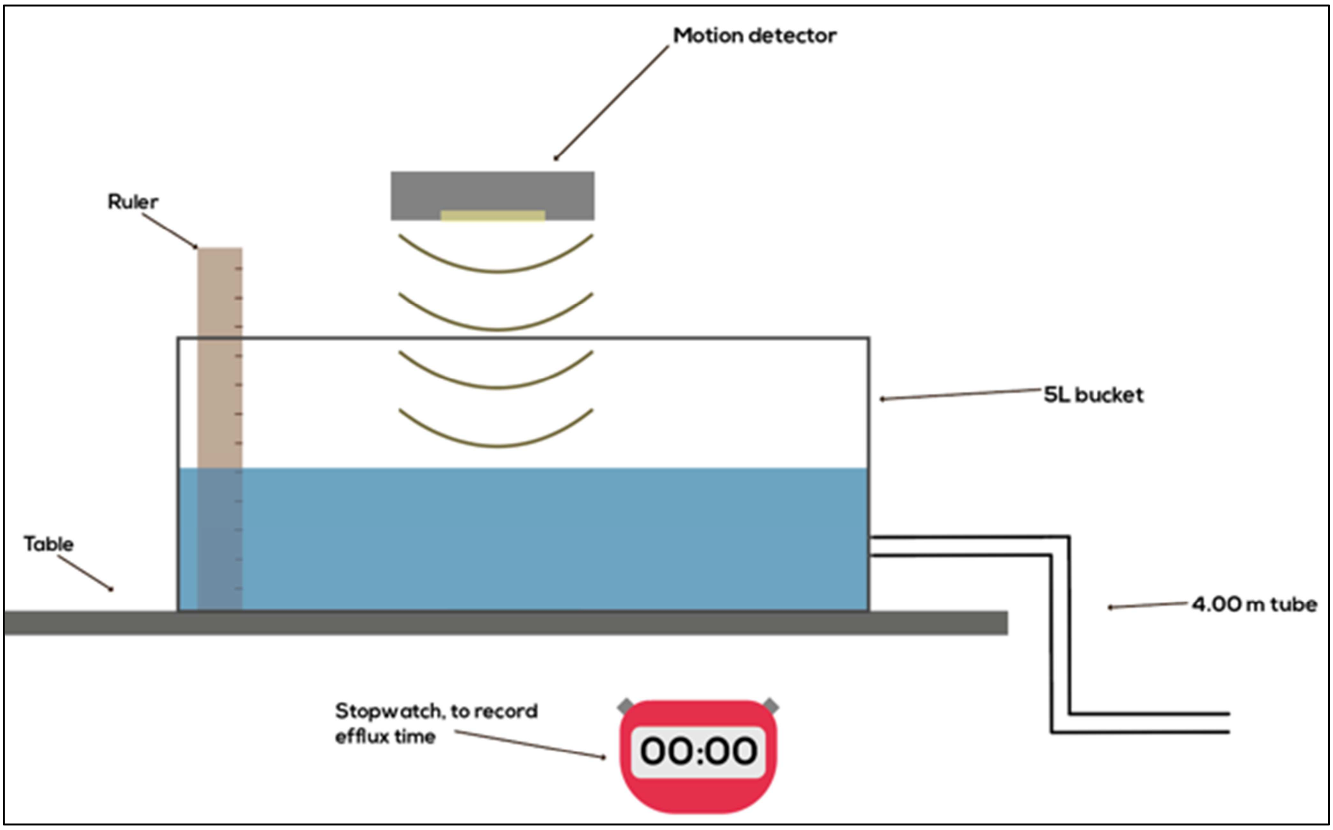


Figure 1. *Experimental Setup.*

A plastic tube with a length of 4 meters and a diameter of 4mm is attached to the bottom of the bucket using silicon glue, ensuring no leakages amidst draining. The bucket has a circumference of 0.69 m, a diameter of 0.21 m, and a cross sectional area of 0.38m^2. A ruler is taped to the side of the bucket ensuring that it is calibrated with the center of the tube.

2.2. Background Knowledge

2.2.1. Mass Balance Equation

Shown in Figure 2 is a stream of the fluid entering at point

X through an area A_x at velocity v_x and leaves through at Y through an area A_y with velocity v_y. In a short time, the fluid leaving X will travel a distance $v_x\,\Delta t$, meaning that a mass of $A_x\, v_x\, \rho_y\, \Delta t$ enters the tube in this time. At the same time, a mass $A_y\, v_y\, \rho_y\, \Delta t$ will leave the stream tube through Y. With steady-state flow, no discontinuities can occur in an ideal liquid, and hence the mass of a fluid entering one side of a stream tube must equal to the same mass exiting. This continuity can be expressed as such:

$$Q = A_x v_x \rho_x \text{-}= A_y v_y \rho_y \tag{1}$$

In the context of the experimental setup, the mass entering the tube at point (1) must equal the mass exiting the tube at point (2).

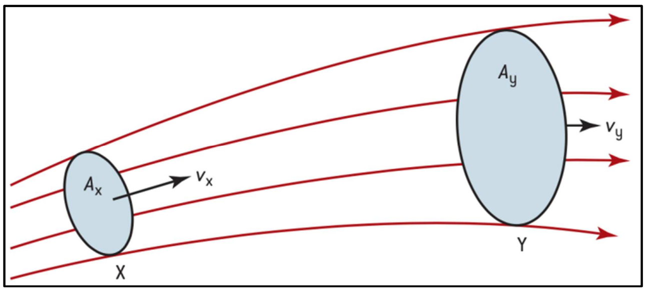


***Figure 2.** Continuity [3].*

2.2.2. Bernoulli's Extended Equation

The Bernoulli equation approximates the relationship between pressure, velocity, and elevation, and is valid in regions of steady, incompressible flow [2, 10]

$$P_1 + \frac{1}{2}\,\rho_1 V_1^2 + \rho_1 g\, h_1 = P_2 + \frac{1}{2}\,\rho_2 V_2^2 + \rho_2 g\, h_2$$

Bernoulli’s equation, in the real world, is traditionally geared towards calculating pressure drops between points in a streamline: examples include venturi meters, pitot tubes… all of which deal with measuring one of the three kinds of pressures a fluid can retain: dyonic, kinetic and hydrostatic pressure [9].

Yet, its applications remain limited due to various factors [2]. Bernoulli's equation limits itself to steady flow, and cannot be applied in scenarios where flow rate is changing. It applies only when viscous and frictional effects are negligible, and assumes there are no frictional energy losses between the fluid and its surroundings [2]. This assumption is invalid when it comes to pipe flow, as the total head possessed by the fluid cannot be fully transferred without losses from one point to another, and therefore Bernoulli's equation can't be used for this experimental setup.

Instead, the Extended Bernoulli Equation is used to analyze the piping system presented by the setup. This revamp accounts for the effects of pressure drops on incompressible fluid-flow. It also accounts for changes in elevation, cross-sections, changes in fluid velocity, sudden contractions or expansions. More importantly, it also accounts for friction loss (H_{friction}) through pipe and fittings such as valves and flow meters as well as any shaft work (W_{device}) done on the liquid [8].

$$\frac{P_1}{\rho_1} + gZ_1 + \frac{1}{2}\alpha V_1^2 + W_{\text{device}} = \frac{P_2}{\rho_2} + \rho_2 g Z_2 + \frac{1}{2}\alpha\, V_2^2 + H_{\text{friction}} \tag{2}$$

Where:

P represents the pressure

Z is the elevation relative to the reference plane

V is the average fluid velocity [4].

Equation (2) arises from the conservation of mechanical-energy. It explains how the sum of a fluid’s pressure (P/ρ), gravitational potential (gZ_1), and kinetic energies $(\frac{1}{2}\alpha V_1^2)$ is constant. The terms W_{device} introduces the work done by any form of machine or pump into a system, and H_{friction} accounts for the thermal energy lost in the system, the energy that was not converted into any of the 3 former energies of water.

The terms α represents what’s known as the kinetic energy correction factor. the kinetic energy of a fluid, we use the mean velocity of a fluid, denoted by $v = Q/A$. However, this neglects the fact that streams that do not touch pipe walls move faster that streams that do, and thus introduces error into our calculation of kinetic energy [7]. To correct this, it’s universally accepted to add α as a coefficient: α is equal to 1 for turbulent flow, and 2 for laminar flow [7]. The two points analyzed will be (1) the surface of the liquid in the bucket and (2) the exit of the tubing with our reference plane being point 2.

2.3. Experiment Procedure and Collected Data

One kind of experimental data will be collected in this experiment: the efflux time, or time elapsed for the water to drain. The height of the water level needs to be collected to extrapolate data after intermittent bursts of water occur near the end of the drainage (Refer to Section 8 for more detailed explanation).

The desired height of the water level is chosen to be 3cm, which will remain constant for all experimental trials. The actual height of the bucket is what's varied over a set of increments, and 6 experimental trials will be performed for each increment. A ruler attached to one of the sides of the bucket is used to measure the desired water level height. The ruler needs to be calibrated with centerline of the orifice [2]. The bucket is allowed to drain and the time will be recorded just before intermittent bursts of water develop.

3. Results and Analysis

3.1. Data Collection

Below in Table 1 is a sample of the recorded data.

Table 1. *Data collected through experiment.*

Height of bucket (±0.0005m)	Time (±0.1s)						
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6	Trial 7
0.425	298.9	283.8	284.4	282.3	282.2	272.2	282.7
1.220	121.1	123.4	11 (4)	122.5	121.6	119.2	122.9
1.067	127.4	126.3	129.6	128.6	128.9	128.6	12 (4)
1.935	71.1	75.3	71.9	68.9	69.9	69.5	71.2

This data is then processed to find average efflux times for each height below in Table 2. Only samples are shown.

Table 2. *Processed data table.*

Height of bucket (±0.0005m)	Average Time (s)	Random error for time (s)
0.425	28 (1)	13.4
1.220	120.7	3.1
1.067	127.6	1.7

3.2. Notation

This essay employs numerous symbols to perform calculations, proofs, and corrections. These are noted down in Figure 3.

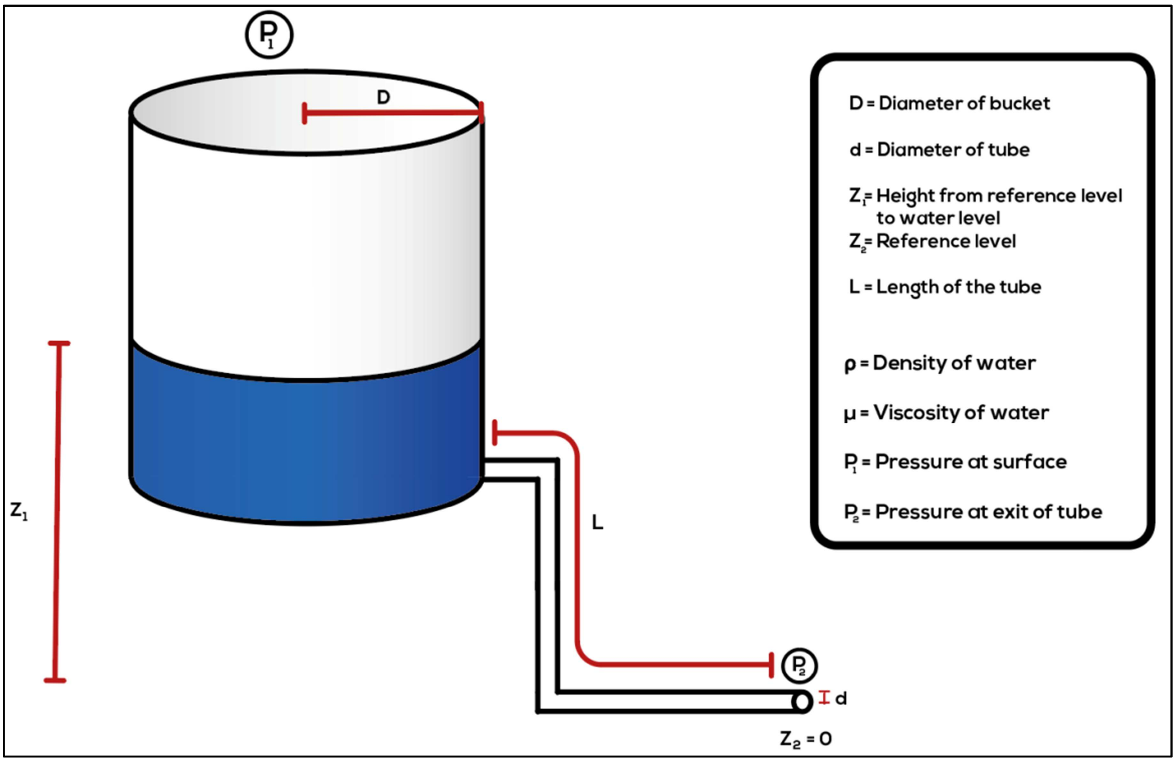


Figure 3. *Symbols and notation.*

3.3. Derivation of Efflux Time

For reference, this theoretical derivation is adapted from [4].

To derive the efflux time of the bucket, the Extended Bernoulli Equation is applied between the surface of the fluid and the end of the tube to find the average discharge velocity at the exit of the tube. This can be converted into a flow rate using the continuity equation which can be used to estimate the efflux time.

Water is incompressible and at both the surface of the liquid and the exit of the tube, the liquid is exposed to atmospheric pressure, and no pumps or machines are installed to perform any work. Thus, Equation (3) is simplified to [1]:

$$\rho g Z_1 + \frac{1}{2}\rho\alpha V_1^2 = \rho g Z_2 + \frac{1}{2}\rho\alpha\, V_2^2 + \rho H_{friction} \quad (3)$$

From section 4, the variable Z_2 is equal to 0. In addition, we make the assumption that the velocity of the water exiting the tube, V_2, is marginally greater than the velocity of the water at the surface of the tube, V_1, and therefore assume $V_1 = 0$.

$$\rho g Z_1 = \frac{1}{2}\rho\alpha\, V_2^2 + \rho H_{friction}$$

Substituting the respective energy correction factors and major loss equation,

$$\rho g Z_1 = \frac{1}{2}\rho(2)V_2^2 + \rho\left(4\frac{fL}{DV_2}\cdot\frac{V_2^2}{2}\right)$$

As well as the Darcy Friction factor and Reynold's number

$$\rho g Z_1 = \frac{1}{2}\rho(2)V_2^2 + \rho\left(4\frac{\left(\frac{64}{Re}\right)L}{DV_2}\cdot\frac{V_2^2}{2}\right)$$

$$\rho g Z_1 = \frac{1}{2}\rho(2)V_2^2 + \rho\left(4\frac{\left(\frac{64}{\left(\frac{\rho V_2 D}{\mu}\right)}\right)L}{DV_2}\cdot\frac{V_2^2}{2}\right)$$

$$\rho g Z_1 = \rho V_2^2 + \left(\frac{32\mu L}{\rho D^2}\right)V_2$$

This equation is rearranged to form a quadratic in terms of the water's average velocity

$$0 = V_2^2 + \left(\frac{32\mu L}{\rho D^2}\right)V_2 - gZ_1$$

Solving for V_2 using the quadratic formula, recognizing that this quantity must be positive, to get

$$V_2 = \frac{\left(-\frac{32\mu L}{\rho D^2}\right) + \sqrt{\left(\frac{32\mu L}{\rho D^2}\right)^2 + 4(gZ_1)}}{2} \tag{4}$$

Equation (4) highlights how the discharge velocity of the water should vary with changes in height, Z_1. In the works of Cengel and Cimabala, as well as Keffer et al, the initial velocity of the tank when the water level is the highest is assumed to be the velocity throughout the drainage. The problems with this assumption are that the hydrostatic pressure of the liquid decreases as the water continues to drain, which would result in the mass leaving the tube constantly being reduced [4]. Whilst Equation 1 highlights how V_2 changes with Z_1, it is unclear how Z_1 varies with time. For this reason, in the equation of continuity between points 1 & 2, the downward velocity of the water is rewritten as a differential of Z_1.

$$A_1\frac{dZ_1(t)}{dt}\rho_1 = A_2 v_2 \rho_2$$

We then substitute equation (4) for V_2 to get the following mass balance equation:

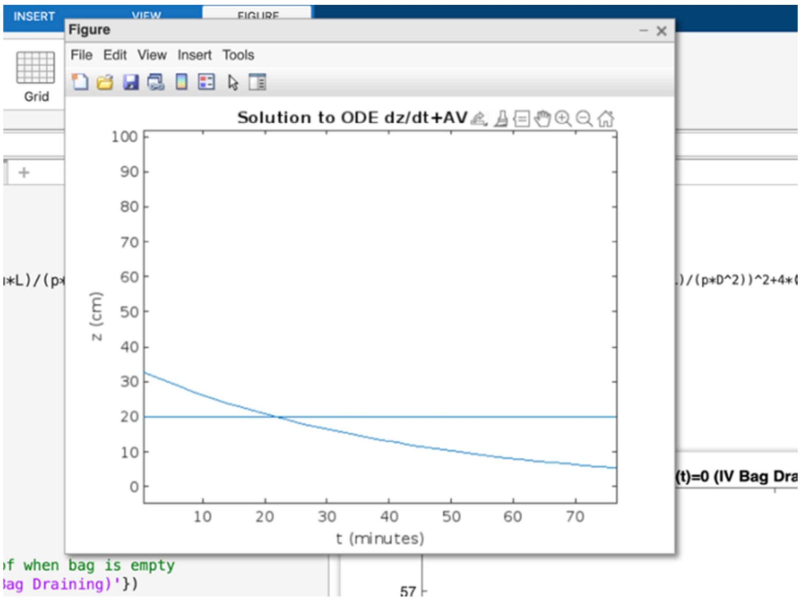
$$A_1\frac{dZ_1(t)}{dt}\rho_1 -= A_2\left(\frac{\left(-\frac{32\mu L}{\rho D^2}\right) + \sqrt{\left(\frac{32\mu L}{\rho D^2}\right)^2 + 4(gZ_1)}}{2}\right)\rho_2 \tag{5}$$

3.4. Development of Model

Solving for $Z_1(t)$ in Equation (5) is not feasible by differential techniques. Thus, the programming software MATLAB was utilized to determine the "theoretical" efflux times of each increment of height, and will be plotted on an excel graph with the experimental data.

```
clc
clf
p=1; %density (g/cm^3)

D=0.4; %diameter (cm)

L=394; %length of tubing (cm)

u=0.01; %viscosity (g/(cm*sec))

g=980; %gravity (cm/sec^2)

P=0; %pressure difference (g/(cm*sec^2))

Sbag= 390;%surface area of bucket (cm^2)

A=((pi*D^2)/4)/Sbag; %constants

dzdt=@(t,z) (-A*(((- 32*u*L)/(p*D^2)+(((32*u*L)/(p*D^2))^2+4*(g*z+p^-1*

%60=conversion factor sec-->min tstart=0;
j = 30;

tend=100;
tstart=00;
zstart=j+3;


[t,z]=ode45(dzdt,[tstart tend],zstart);

plot(t,z) %plot of z vs. t
xlabel('t (minutes)'),ylabel('z (cm)')
refline(0,20) %added to give approximation of when bag is empty
title({'Solution to ODE dz/dt+AVz(t)=0 (IV Bag Draining)'})
```

a



b

Figure 4. *Figure 4a and 4b display a sample of the program used to find the theoretical drainage time.*

Z1 considers the vertical distance from the reference level (the exit of the tube at the bottom) to the surface of the water. The solution to the ordinary differential equation does not factor in at which height is the level of the water equal to 0, in other words, at which point is the level of the water the same as the height of point 1. For this reason, I added a reference line equal to the height of point 1 and determine how long it took for Z_1 to reach Z_2.

The increments of height in experiment vary from 10cm to 200cm. For this reason, 1000 data points were generated from the program, and were graphed on excel to produce a graphical model of the efflux time of the bucket when the it is filled at 3cm height, as shown in Figure 5.

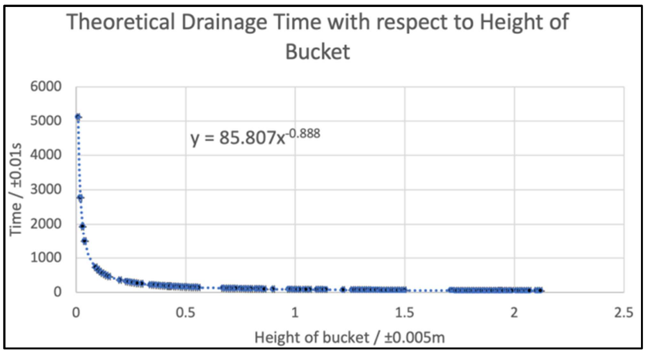


Figure 5. *Theoretical Drainage of Time with Respect to Height of Bucket.*

Excel's computer software represented the function modeling these data points by the power function $y = 85.807x^{-0.888}$. With the sheer number of data points, as well as an R^2 of 1, it is reasonable to accept this value.

The indirect relationship we see in the theoretical model conforms to theory. An increase in the height of the bucket should increase the hydrostatic pressure of the water, as the vertical difference between the water level to the reference level increases, which should hence lead to the water draining faster after a greater pressure drop. The inverse exponential relationship of the two variables matches the conclusions of the works of Storey and Olin as well as Loiacono. The success of this model is determined by how well it models the experimental data, which is done in the next section.

3.5. Data Analysis

The data from Table 1 and the theoretical model in Figure 5 are both plotted on the same axes in Figure 6.

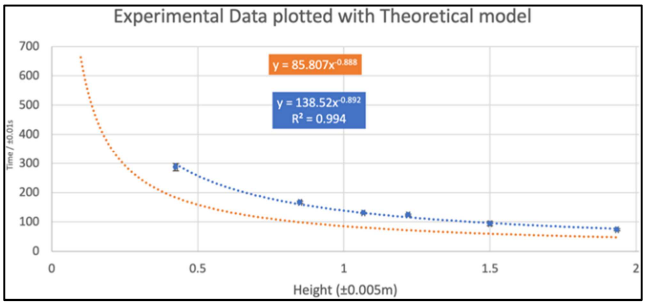


Figure 6. *Experimental Data Plotted with Theoretical Model.*

The error bars for each data point is displayed although their magnitudes are relatively small compared to the scale of the axis to be seen. The general trend in the data displays an inverse exponential proportion, matching the one seen in the theoretical model. However, there is significant disparity between the experiment and the model, particularly near the smaller heights. It's notable that model consistently underestimates the efflux time of the tank.

The discrepancies between the empirical and the theoretical gradients can be attributed to multiple minor reasons. One major source of discrepancy is the parallax error that arose in reading the height of the water level, as shown in Figure 7.

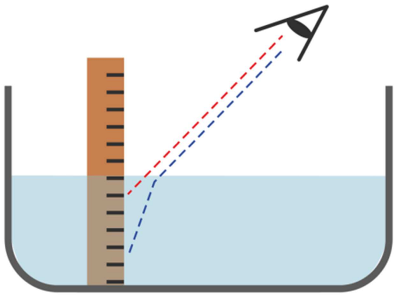


Figure 7. *Student Produced Work.*

When trying to read the water level to begin the timer when it reached the 3cm mark, the refraction caused by the water led to the implementation of significant estimation when the water is actually 3cm high. The impact of this error within increment trials were easy to spot, as incoherencies in the general trend can be attributed to this source of error. Yet when considering data between increments, it is hard to claim whether the timer always began draining at the 3cm mark. This was especially true in the taller increments of height, where I had to stand on the tips of my toes just barely see the ruler. The plugging in of the 3cm water level into the program may have not been an accurate measurement. This problem was also a limitation of the works of Joye et al reporting that specifying which height the water level is can be severely hindered by the parallax error due to the glass and water. This problem apparently mounted to a 15% error or deviation from their theoretical values.

Another source of discrepancy may be linked to the unsettling of the water at the start of the experiment. The extended Bernoulli equation as well as the continuity equation are both applicable in a streamline, where water flows in a constant trajectory. When filling up the bucket, heavy waves and movement amongst the water level began to manifest, which theoretically must've disrupted the streamline water molecules undertook as they were moving in other directions. This problem has only been noted in the works of Larry Glasgow yet still pertains significant implications to the results as it disrupts streamline. Evidently the relevance of the water not being quiescent is why Glasgow prescribes its importance in his paper [5].

However, what is likely to be the significant disparity between the theoretical and the empirical data is the model's unaccountability for the minor losses in the experimental setup. Although it is mentioned that minor losses will not be explored in this essay, they will be evaluated generally as a cause of the discrepancies. According to Cengel and Cimabala, "fluid [s] in a typical piping system pass through various fittings, valves, bends… [for which] interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce" [2]. Quite simply, minor losses refer to the energy lost during the fluid's flow as heat when undergoing twists, turns, contractions or expansions in its trajectory. Contrary to the name, minor losses at times can be greater than the major losses in a pipe system [2]. This is especially the case when a system contains significant twists and turns in a relatively short distance [2]. This is particularly the case in my experimental setup, as shown below in Figure 8.

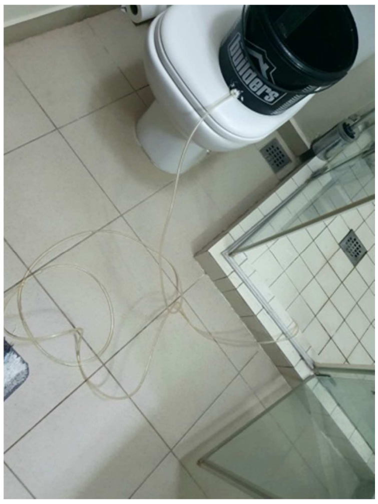


Figure 8. *Picture Taken Personally.*

Due to the length of the piping system, the tubing coiled around itself multiple times, I assume that there is significant loss of mechanical energy, for which it is not accounted for in the model. This limitation is further discussed in the literature review section later.

Admittedly, the model I have developed was successful in following the general trendline of the data displaying the inverse proportion, and its coherence with modern literature is convincing. However, its limitations are linked to either gaps in theory not taken account for or experimental difficulties in carrying out the method. Whilst the precision of the data is not particularly relevant to comparing the models, the minimal random error suggests the method was successful in carrying out precise data. It also suggests my hesitance on the guesswork in determining the water level height remained relatively consistent at least within increments.

4. Literature Review

The model made in Section 8&9 was moderately successful, at least in the extent of modelling the general relationship. In the paper by Joye et al, several models by the works of Loiacono, Keffer et al, Bird et al, and Joye et al themselves are evaluated. The report underscores Loiacono's and Joye et al's models to be the best at fitting experimental data, with a percent error of less than 8%, on average [6]. Hence, these two models will be compared to both the experimental data and my theoretical model' to evaluate their strengths and limitations.

Loiacono's model [12] estimates the efflux time using the following formula:

$$t = \frac{D^2}{d^2}\sqrt{\left(\frac{2}{g}\right)\left(\frac{fl}{d}+1\right)}\left(\sqrt{H_o} - \sqrt{H_f}\right)$$

Where:

t refers to the efflux time

D refers to the diameter of the tank
d refers to the diameter of the tube/pipe
f refers to the pipe friction factor
H_o refers to the initial height of the water level
H_f refers to the final height of the water level

Loiacono's model is derived using the equivalent length method [6]. According to Harvey Wilson, chief engineer at Katmar Software, "[the equivalent length method] is based on the observation that the major losses are also proportional to the velocity" [13]. Loiacano's method ignores the head loss that arises from the twists and turns of a tube, and instead focuses on the head losses due to the length of the tubing, also known as major losses. It's clear Loiacono's model takes into consideration the same kinds of frictional forces as mine, just using a different engineering coefficient. It's worthy to note that Loiacono's method applies strictly to water, as it does not involve any viscosity or density term to account for viscous friction, contrary to my model.

Whilst the model proposed by Loiacano's makes use of the major losses, the model proposed by Joye et al instead utilizes what's known as the restrictive coefficient. The restrictive coefficient is a method that instead considers the friction losses that arise from the twists and turns. The formula proposed by Joye et al is:

$$t = \frac{D^2}{d^2}\sqrt{\frac{2(\frac{4fl}{d}+\sum K}{g}}\cdot(\sqrt{H_o+v}-\sqrt{H_f+v})$$

Where:
t refers to the efflux time
D refers to the diameter of the tank
d refers to the diameter of the tube/pipe
l refers to the length of the tube
K refers to the minor losses of the fluid
f refers to the pipe friction factor
H_o refers to the initial height of the water level
H_f refers to the final height of the water level
v refers to the vertical drop of the exit pipe

The K constant in Joye et al's paper can be derived mathematically by analyzing each twist and turn in the tubing system. Despite the method being scientific, it's highly inefficient and time consuming, and Joye et al proposes the use of a slider for the K constant, setting it equal to the value that makes the graph best fit the experimental data being analyzed. The K value was found to be equal to 90 in this experiment using the slider method.

Shown below in Figure 9 are the two models in addition to the one developed by this paper graphed against the experimental data.

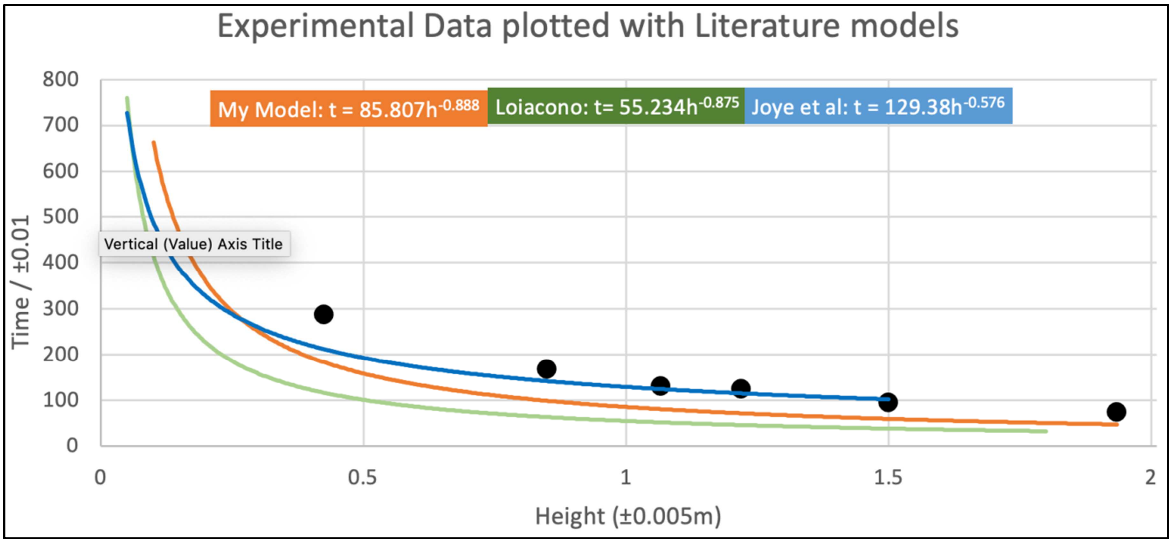


Figure 9. *Experimental Data plotted with Literature Models.*

At first glance, it appears as though the model proposed by Joye et al best fits with the experimental data, followed by mine and then Loiacano's. A more analytical way of measuring the goodness-of-fit of such models is through the correlation coefficient. Shown below are the correlation coefficients of each model with respect to the experimental data:

Table 3. *Correlation coefficients.*

Name of model	Correlation Coefficients
Loiacono	0.73
Joye et al	0.82
Personal	0.71

Admittedly, the model proposed by Joye et al is most accurate, with the highest coefficient, followed by the one proposed by Loiacono and then mine. This finding uncovers an underlying question in Joye et al's paper, of whether minor losses are negligible enough not to be considered in efflux analysis. The correlation coefficients clearly convey that the inclusion of minor losses lead to the most accurate models. Whilst the scope of this essay does not overarch the concepts of minor losses, it's ignorance in the derivation should be noted as a limitation. The equivalent length method performed by Loiacono essentially considers major losses, as does my personal model, which is why the correlation coefficients are very similar.

5. Conclusion

The research question has been successfully answered over the course of this paper, producing the knowledge of the inverse relationship between the height of a bucket and the drainage time. The model developed by this paper has established limits on the concepts it will explore, which inevitably lead to discrepancies with the data and more successful models. The ignorance of minor losses advances the argument over its significance when dealing with the efflux of tanks, and should be considered in any new models in the future. Aside from reducing random error through the improvement of equipment and more trials, future improvements include for any new approaches to deriving the efflux times of tanks would be the consideration of minor losses through the concepts by Joye et al.

The models analyzed in this paper introduce a range of possible real-world applications. The efflux time of oil tanks and cisterns for example, is a significantly important measurement to identify in the realm of distribution and logistics of companies. Intravenous drips found in hospitals can also be analyzed through this method, although most IV drips contain sliders to limit the flow rate and accommodate for nurses who don't understand underlying fluid mechanic principles. Most importantly, the drainage of pipe fitted tanks is most applicable to water towers. The question of how high a water tower should be installed can be successfully answered using the models in this paper, and provides great use in the realm of civil and chemical engineering.

Nevertheless, the approaches proposed my personal model and the literature review provide greater insight into the under-researched properties of unsteady flow.

References

[1] "Bernoulli's Equation | Engineering Library." Engineeringlibrary.org, 2019, engineeringlibrary.org/reference/bernoullis-equation-fluid-flow-doe-handbook. Accessed 18 Apr. 2022.

[2] ÇengelYunus A, and John M Cimbala. Fluid Mechanics: Fundamentals and Applications. Singapore, Mcgraw-Hill Education, 2020, pp. 203, 209.

[3] David, Homer. "Continuity Equation," IB Physics Course Companion, 2014. Accessed 24 Apr. 2022.

[4] Fournier, Ronald L. Basic Transport Phenomena in Biomedical Engineering. Boca Raton, Crc Press, Taylor & Francis Group, 2018.

[5] Glasgow, Larry. EFFLUX TIME for a CYLINDRICAL TANK.

[6] Joye, Donald D., and Branden C. Barrett. "The Tank Drainage Problem Revisited: Do These Equations Actually Work?" The Canadian Journal of Chemical Engineering, vol. 81, no. 5, 19 May 2008, pp. 1052–1057, 10.1002/cjce.5450810516. Accessed 19 Mar. 2022.

[7] "Kinetic Energy Correction Factor: Meaning, Formula, for Laminar Flow [with Pdf] - Mech Content." Mech Content, 27 Nov. 2021, mechcontent.com/kinetic-energy-correction-factor/. Accessed 25 Apr. 2022.

[8] Nguyen, T. K. Transport I (CHE 311 Course Notes), Chemical and Materials Engineering. Vol. 1, Cal Poly Pomona, 2004, pp. 140, 141.

[9] "Pressure." Gsu.edu, 2022, hyperphysics.phy-astr.gsu.edu/hbase/pber.html. Accessed 9 Mar. 2022.

[10] Serway, Raymond A, et al. Holt McDougal Physics. Orlando, Fla., Holt Mcdougal, 2012.

[11] Sianoudis, I A, and E Drakaki. "An Approach to Poiseuille's Law in an Undergraduate Laboratory Experiment." European Journal of Physics, vol. 29, no. 3, 14 Mar. 2008, pp. 489–495, 10.1088/0143-0807/29/3/009. Accessed 21 Feb. 2020.

[12] The Tank Drainage Problem Revisited: Do These Equations Actually Work? Loiacono, N. J., "Time to Drain a Tank with Piping", Chemical Engineering 94 (11), 164–166, August (1987).

[13] Wilson, Harvey. "Pressure Drop in Pipe Fittings and Valves | Equivalent Length and Resistance Coefficient." Www.katmarsoftware.com, Oct. 2012, www.katmarsoftware.com/articles/pipe-fitting-pressure-drop.htm#equivlen. Accessed 27 May 2022.