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Probabilistic Power Flow Model to Study Uncertainty in Power System Network Based Upon Point Estimation Method

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Abstract: In this paper, point estimation approach is used to calculate the statistical moment of a random quantity that is a function of *m* input random variables. In this work, loads of the proposed network is considered as a random variable. Two special cases of point estimation approach are considered such as *2m* and *2m+1* concentration schemes. In *2m* concentration scheme, skewness is considered, but in *2m+1* concentration scheme, both skewness and kurtosis are taken into account for probability density function. The proposed model is investigated using P. M. Anderson 9-bus test system. As a result, by changing the value of a random variable that follows a predefined distribution, expected bus voltage magnitude and expected line loading are identified. For the comparison purpose, *2m* and *2m+1* scheme was compared with deterministic load flow analysis.

Keywords: Probabilistic Power Flow, Point Estimation Method, Random Variable, Uncertainty Analysis

1. Introduction

Power flow study in power system is an important tool. It is mostly used to evaluate the voltage magnitude and angle at different buses in power system, active and reactive power flow through the line. For the expansion, operational planning, real-time operation, and control of power system; power flow has become a key. As the power system becoming more and more complex, several problems such as power system planning, operation, and control are a challenge for engineers. In order to tackle said problem, different studies are carried out in [1], [2].

Due to the integration of renewable energy in power system network, it is becoming more sensitive to above problems. The new challenges for the network are voltage Stability and transient stability. Although, different types of changes are noticed in recently published papers [3], [4]. Among of these problems, uncertainty in renewable sources is a major challenge for engineers. These uncertainties are unpredictable in a renewable generation such as the wind and solar energy, daily load variation, inherent randomness of fault, and failure of the power system.

A classical way of power flow analysis is a deterministic power flow (DPF) analysis. DPF analysis has not the capability to handle the future problems. DPF analysis only calculates the values chosen by the analyst [5]. You can say that DPF analysis provides the variable values in term of certain load and generation characteristics. Its accuracy depends on the input data provided by the analyst. The generation pattern, future load characteristic, and faults in power system network become more unpredictable nowadays.

In DPL analysis, it is impractical to access the probabilistic power flow analysis. For each possible computation need to perform the analysis separately that is not only time consuming but also hectic. To handle the said problem, probabilistic power flow (PPF) analysis is taken into consideration. PPF analysis has the capability to handle almost all kind of uncertainties occurs in power system network. A few decay ago, PPL analysis has become a hot research topic as shown [6], [7], [8], [9], [10] recent publication.

PPF analysis of power system can be categories into three approaches such as simulation approach, analytical approach, and approximate approach. In simulation approach, Monte Carlo simulation (MCS) is very famous and had been used in different papers [11], [12], [13]. In this method, a certain probability distribution is assigned to a certain variable. A random number is selected from the probability distribution, and computation is performed. In this approach, to attain the more precise results, need a large number of iteration that is the result of extensive computation time and storage, which are drawbacks of this approach. To overcome this computational time analytical approach was proposed in [14], [15], [16]. In analytical approach linearization of power flow equation is the main drawback, so that it can work with probability density function.

In approximate practices, no need to linearization the power flow equation, it gives the approximate statistical information about the output random variables. By using several techniques, the evaluation point can be decreased. In this way computation time and storage can be considerably reduced. In approximate techniques, Point Estimation Method (PEM) can reduce the considerable computation time as well as a precise result. PEM calculate the statistical moments of a random variable that is a function of input random variable.

Probabilistic PEM is proposed in this paper. Although, a number of research papers have been published at this method e.g. probabilistic optimal power flow in electricity market based on the *2m* scheme is proposed in [17], and a probabilistic load flow based on nonparametric density estimator is proposed in [18] but in the above research papers *2m* and *2m+1* concentration scheme are used separately.

In this paper, probabilistic power flow model is proposed with point estimation method, two special cases *2m* and *2m+1* concentration schemes are used, a total load of the proposed network is considered as a mean value, and an arbitrary value of standard deviation is selected for creating the uncertainty in load. In this work, few statistical moments (i.e. mean, variance, skewness, and kurtosis) are carried out to determine the effect of uncertainty on network parameters. Especially, by varying the value of skewness and kurtosis different results are observed. These results can be used for future planning, corrective action, and operation of electric power distribution system. The rest of the paper is organised as follow; Section 2 provides an introduction of PEM and corresponding schemes. Section 3 provides the information about test system. The simulation results and discussion is presented in Section 4. Finally, Section 5 conclude the paper.

2. Point Estimation Method

The main categorization of probability and statistic is as,

(I) Descriptive statistic

(II) Probability statistic

(III) Inferential statistic

In inferential statistic, it is used to draw a conclusion about a given population information about a representative in a decision making. It is impractical, expensive, and impossible to measure the large population, by using the samples, it can easily estimate, predict, generalise, and make a decision about large population. Two main methods are employed in this case i.e. point estimator and maximum likelihood method. Here, point estimate method has been considered.

The first PEM was proposed in 1975 by Rosenblueth, at that time, it was used for symmetric variables. It was modified in 1981 for asymmetric variables. Since that, a different method was used to improve the origin Rosenbluth method, but the main difference is to change the types of random variables that they used to improve the performance.

In this method, the sample is used to estimate corresponding population parameter desirable characteristic. Further, classification of this method depends on the selection of estimator value; it may be an unbiased estimator, consistence estimator, and maximum likelihood estimator. In this paper unbiased estimator is selected for analysis.

In unbiased estimator, the mean of the sample distribution should be the same as the target parameter. Simply, the sample mean is an unbiased estimator for the population mean. Variance and minimal variance of the sample is also considered as an unbiased estimator of population variance but in this work mean is consider as an unbiased estimator because mean is the best possible estimator for the population mean if information is given in the sample.

The aim of point estimator is to calculate the statistical moment of the random quantity that is a function of one or more input random variables. Let *Z* denote a random quantity that is a function of *X* input random variables. Then, evaluate the random quantity *Z=F(X)*, to estimating the mean value of *Z*.

2.1. Formulation of PEM

Let p_l be an input random variable having a density function (DF) f_{pl}, *F* is function that relate the input and output variable information about the uncertainty problems. This method concentrate the information provided by the first few central moment of input random variable. *K* is the number of points on the input random variable called concentration. By using *k* and *F* information, uncertainties associated with output random variables are calculated. The concentration of input random variable *k* is compose of two parameter (i.e. $p_{l,k}, \omega_{l,k}$). Here, $p_{l,k}$ is the *kth* value of p_l, it is a location parameter and $\omega_{l,k}$ is weighting factor, both contribute in the output random variable results. The number of *k* evaluation is depends upon the selection of concentration scheme. Total number of evaluation of *F* is $k \times m$, here, m is number of input variables. If *2m* scheme is selected then $k\times m$ will be the evaluation points. If *2m+1* scheme is selected then $k\times m+1$ will be the evaluation points. The input vector form input random variables m with mean is,

$$Z(l,k) = F_i(\mu_{p1}, \mu_{p2,\dots\dots\dots,}\mu_{pl,\dots\dots\dots\dots}\mu_{pm}) \quad (1)$$

K_{th} Concentration of input random variable is evaluated by using statistical input data. The location parameter is calculated by using [22], that is:

$$p_{l,k} = \mu_{pl} + \xi_{l,k}\sigma_{pl} \quad (2)$$

Here,

$p_{l,k}$ = location parameter of input random variable

μ_{pl} = mean of input random variable

$\xi_{l,k}$ = standard location of input random variable

σ_{pl} = standard deviation of input random variable

Here, $\xi_{l,k}$ depending upon the type of concentration scheme, in this paper two concentration scheme are considered.

2.2. 2m Concentration Scheme

In this scheme, standard location of input random variable is calculated by using below equation [22]:

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k}\sqrt{m + (\lambda_{l,3}/2)^2}\ k = 1,2 \quad (3)$$

Here, $\xi_{l,k}$ depend upon the number of input random variables, from the Eq. (2), it is clearly shown that as the *m* increase location of p_l move away from the mean μ_{pl} according to $\sqrt{m}$. In Eq. (3) parameter $\lambda_{l,3}$ denoted the skewness of input random variable that is computed as:

$$\lambda_{l,3} = \frac{E[(p_l - \mu_{pl})^3]}{(\sigma_{pl})^3} \quad (4)$$

The weighting of the concentration located at Eq. (1), then used to estimates to take into account the skewness of probability distribution of *Z*. the value of $\omega_{l,k}$ is range from 0 to 1 and sum of all values of $\omega_{l,k}$is unity.

$$\omega_{l,1} = \frac{-1}{m} * \frac{\xi_{l,2}}{(\xi_{l,1}-\xi_{l,2})}, \omega_{l,2} = \frac{1}{m} * \frac{\xi_{l,1}}{(\xi_{l,1}-\xi_{l,2})} \quad (5)$$

The advantage of the *2m* scheme is related to its simplicity, lesser computation burden, and the real value of concentration.

2.3. 2m+1 Concentration Scheme

In this scheme, three points are taken from each input random variable and one location fixed for mean value. Four statistical moments are carried out for PDF of random variables. Standard location of *k* concentration can be found by using [19], [20], [21] below equation.

$$\xi_{l,k} = \frac{\lambda_{l,3}}{2} + (-1)^{3-k}\sqrt{\lambda_{l,4} - \frac{3}{4} * (\lambda_{l,3})^2} \quad (6)$$

K =1, 2 & $\xi_{l,3} = 0$

The weights are calculated as:

$$\omega_{l,k} = \frac{(-1)^{3-k}}{\xi_{l,k}(\xi_{l,1}-\xi_{l,2})} k=1, 2 \ \&$$

$$\xi_{l,3} = \frac{1}{m} - \frac{1}{(\lambda_{l,4}-\lambda_{l,4}{}^2)} \quad (7)$$

From the Eq. (7), $\lambda_{l,4}$ is kurtosis taken into account that's why *2m+1* scheme is more accurate then *2m* scheme but 1 additional evaluation is needed.

In this method, after estimating the sample point, the fitness function is calculated for all estimated points. The uncertainty is transfer from the input random variables to the output random variables by using *F* function, and *Z (l, k)* is the vector of the output random variables associated with the *Kth* concentration of random variable p_l. The total number of deterministic analyses to be run depends on the concentration scheme considered. Finally, the expected results is computed as:

$$E|Z| = \sum_{l=1}^{m} \cdot \sum_{k=1}^{K}(\omega_{l,k}) * Z(l,k) \quad (8)$$

Here, output random variable is ***Z*** and *E*|*Z*| is the expected value of vector ***Z***, input random variable, and the number of points is denoted by *m* and *k* respectively.

2.4. Skewness

It measures the asymmetry of the distribution data of the random variable about its mean. It is a third central moment of distribution, three different value of the coefficient is used for analysis. Its value may be positive, negative, and even undefined. Fisher Pearson gave an excellent concept about skewness. He gave the constant value of each distribution, by using (9). In this work, random variable follows the normal distribution.

$$Ks = \frac{\sum_{i=1}^{n}(X_i - X^-)^3/N}{\sigma^3} \quad (9)$$

Here, *N* is used rather than *N-1*, for calculating the standard deviation.

Ks = coefficient of skewness

For a normal distribution, if *Ks =0*, it is symmetrical distribution, if *Ks<0*, it mean data distribution is negatively skewed, if *Ks> 0*, it means data is positively skewed.

2.5. Kurtosis

It is a fourth central moment of distribution data set. It measures the peak of distribution. Three different value of the coefficient of kurtosis is used for analysis purpose. Pearson explained that value of kurtosis coefficient is equal to *3* in the case of normal distribution, its formula is given below:

$$K_K = \frac{\sum_{i=1}^{n}(X_i - X^-)^4/N}{\sigma^4} \quad (10)$$

Here, K_K is coefficient of kurtosis, when, $K_K = 3$, it is called Mesohurtic, when the value of kurtosis coefficient is more than 3, this is called Leptokurtic. Leptokurtic distribution mean that data is distributed around the mean value. When the value of kurtosis coefficient is less than 3, it is called Platykurtic distribution. Platyhurtic distribution mean that data is distributed far from the mean value of distribution.

2.6. Flow Chart for PEM

In this section, step-by-step implementation of PEM is presented.

Step 1: First of all, set the parameter such as a probabilistic characteristic of an input random variable (i.e. mean, standard deviation, skewness, and kurtosis) and the number of estimation points *k* is determined. In this paper, *2m* and *2m+1* concentration schemes are used. So, 2 points in case of the *2m* scheme and *3* points in case of the *2m+1* scheme are generated. In this paper, a random variable is a total system load that follows a normal distribution and considered one variable (*m=1)*.

Step 2: In this step, standard central moment and standard location $\lambda_{l,k}$ and $\xi_{l,k}$ are calculated respectively corresponding to selected scheme. Here, $\lambda_{l,1}$, $\lambda_{l,2}$, $\lambda_{l,3}$, $\lambda_{l,4}$ are the mean, standard deviation, skewness, and kurtosis respectively.

Step 3: In this step, input vector for power flow is calculated by using the Eq. (1).

Step 4: In this step, the fitness function is evaluated by the DFL analysis. DFL is derived from the input vector. If the algorithm reached maximum input random variable, it will stop. Otherwise, it follows the step 3.

Step 5: In this step, the algorithm checks the number of point *K* if it is reached to a maximum number of point it will stop. Otherwise, it follows the step 3.

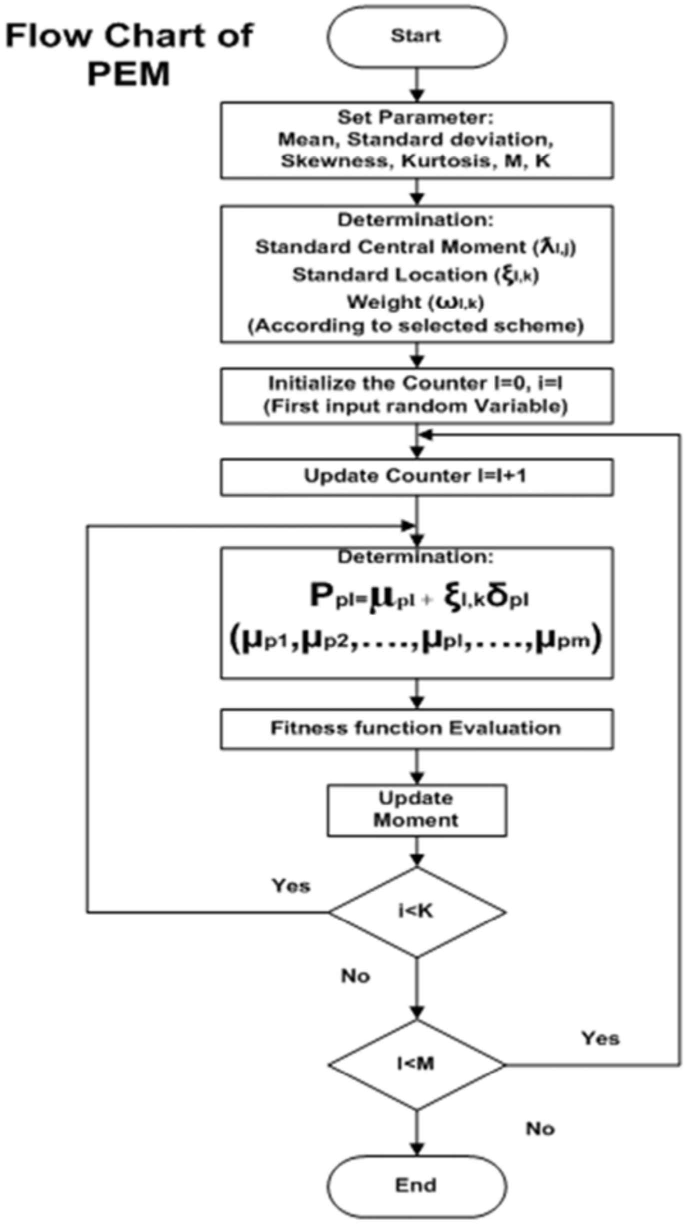

Figure 1. *Flowchart of PEM.*

3. P. M. Anders on 9-Bus Test System

For the results demonstration purpose, P. M. Anderson 9-bus test system is used. All related data to this test system is available in [22], [23]. The total load for base case is 315 MW and 115 Mvar, bus 1 was considered as a reference bus. Some information related to load and line data is given in Tables 1 & 2 respectively.

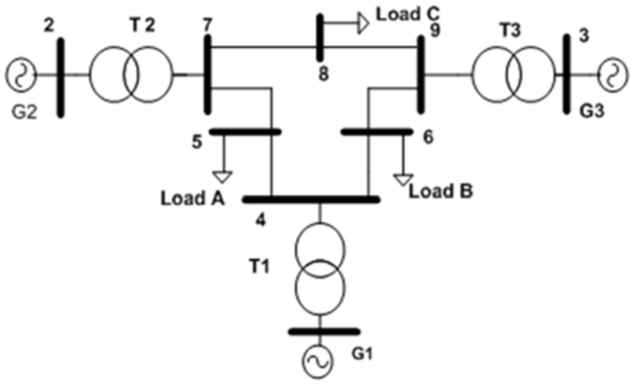

Figure 2. *P. M. Anderson 9-bus test system.*

4. Results and Discussion

In order to verify the accuracy and efficiency of the proposed model, P. M. Anderson 9-bus test system was simulated through DIgSILENT Power Factory software 15.1. In this test system, all PV buses voltage magnitude are set 1.05 (p.u.), and balance Ac load flow analysis was implemented. All results for base case are presented in Tables 2 and 3.

4.1. 2m Concentration Scheme

In 2m point concentration scheme, the standard deviation is considered as an arbitrary parameter. Three different values of coefficient of skewness are set on 0, 0.5, and -0.5, its explanation is presented in Section 2.4. Firstly, standard location, weighting coefficient, Active, and Reactive power will be calculated; these parameter values depend upon the selection of standard deviation and the value of the coefficient of skewness. In this study, when standard deviation was selected 10% and coefficient of skewness 0. The value of standard location and a weighting coefficient for both point was (1 & -1) and (0.5 & 0.5) respectively, then voltage magnitude at each point was calculated (see Fig. 3a). When k_s = 0.5, the value of standard location and a weighting coefficient for both point was (1.368 & -0.868) and (0.326 & 0.515) respectively, then voltage magnitude at each point was calculated (see Fig. 3b). Similarly, when K_s = -0.5, the value of standard location and a weighting coefficient for both point was (0.868 & -1.368) and (0.515 & 0.326) respectively, then voltage magnitude at each point was calculated (see Fig. 3c). From the above analysis, it is clearly shown that as the value of the coefficient of skewness was increased, the magnitude of active and reactive power injection to the buses also increased. This claim can be confirmed by the Table 4.

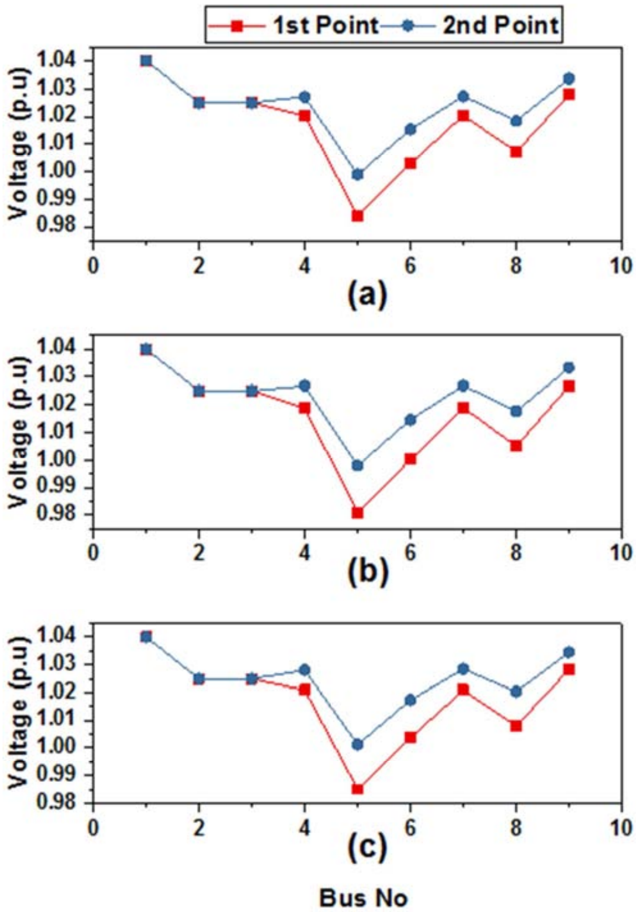

Figure 3. *Voltage magnitude at each bus, when Ks=0, 0.5, & -0.5.*

Table 1. *Base case load data.*

Loads	Load (MVA)
Load A	125+j50
Load B	90+j30
Load C	100+j35

Table 2. *Base Case voltage magnitude.*

Bus Number	Voltage Magnitude (p.u.)
Bus 01	1.040
Bus 02	1.025
Bus 03	1.025
Bus 04	1.024
Bus 05	0.991
Bus 06	1.009
Bus 07	1.024
Bus 08	1.013
Bus 09	1.030

Table 3. *Base Case Line Loading.*

Line Number	Loading (%)
Line 01	16.160
Line 02	20.943
Line 03	19.699
Line 04	09.129
Line 05	14.700
Line 06	10.331

Table 4. *Active and Reactive Power of two points.*

Skewness (K_s)	Points No	Active Power (MW)	Reactive Power (Mvar)
0	1	365.539	133.450
	2	299.077	109.187
0.5	1	377.769	137.915
	2	303.463	110.788
-0.5	1	361.154	131.849
	2	286.847	104.722

4.2. 2m+1 Concentration Scheme

In this scheme, arbitrary parameters such as standard deviation, skewness, and kurtosis (k_k) are set at 10%, 0.5, and 2.5 respectively. The simulation was carried out for three different value of the coefficient of kurtosis such as 2.5, 3, and 3.5 respectively. Its explanation is provided in Section 2.5. Firstly, standard location and weighting coefficient were calculated by using Eq. (3 & 4) respectively. When k_k =2.5, the value of standard location and weighting coefficient related to three point was (1.723, -1.419 & 0) and (0.184, 0.224 & 0.698) calculated respectively, then voltage magnitude at each point was calculated (see Fig. 4a). The voltage magnitude When K_k = *3,* the value of standard location and weighting coefficient related to three points was (1.863, -1.559 & 0) and (0.156, 0.187 & 0.759) calculated respectively, then voltage magnitude at each point was calculated (see Fig. 4b). When K_k = *3.5,* the value of standard location and weighting coefficient related to three points was (2.004, -1.700 & 0) and (0.134, 0.158 & 0.806) calculated respectively, then voltage magnitude at each point was calculated (see Fig. 4c). From the above analysis, it is clearly shown that, as the value of the coefficient of kurtosis was increased, the load injection in buses are increased. This claim can be confirmed by the Table 5.

Table 5. *Active and Reactive Power of three points.*

kurtosis (k_k)	Point No	Active Power (MW)	Reactive Power (Mvar)
	1	389.585	142.229
2.5	2	285.136	104.097
	3	332.308	121.319
	1	394.249	143.932
3	2	280.473	102.394
	3	332.308	121.319
	1	398.911	145.634
3.5	2	275.811	100.692
	3	332.308	121.319

Table 6. *Camparrision of EVM, when Ks= 0 & k_k = 3.*

Bus #	Base Case	2m scheme	2m+1 scheme	SD* 2m & Base	SD* 2m+1 & Base
1	1.04	1.56	1.422	0.26	0.191
2	1.025	1.537	1.401	0.256	0.188
3	1.025	1.537	1.401	0.256	0.188
4	1.024	1.535	1.399	0.255	0.187
5	0.991	1.487	1.354	0.247	0.181
6	1.009	1.513	1.379	0.252	0.184
7	1.024	1.535	1.399	0.255	0.187
8	1.013	1.519	1.384	0.253	0.185
9	1.030	1.546	1.409	0.257	0.189

SD*= standard deviation

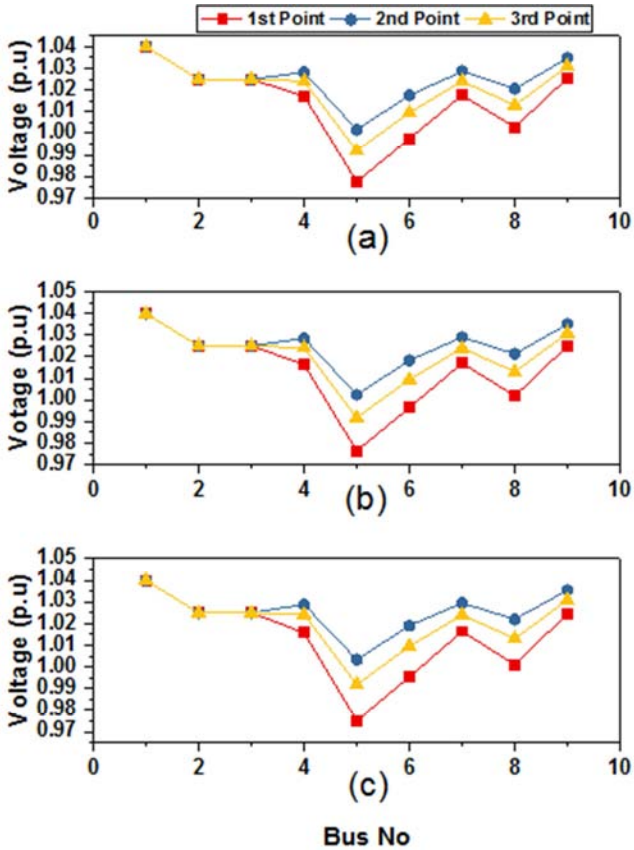

Figure 4. *Voltage magnitude at each bus, when K_k=2.5, 3, & 3.5.*

Finally, the expected voltage magnitude (EVM) at each bus was calculated. The *2m* and *2m+1* scheme was compared with the base case (see Table 6), by looking the value of standard deviation, the results of the *2m* scheme is more deviate than *2m+1* scheme. So the value of EVM computed through the *2m+1* scheme is closer to the base case (see Fig. 5). Similarly, it applies to the line loading case (see Table 7), except those lines that are connected to the PV buses.

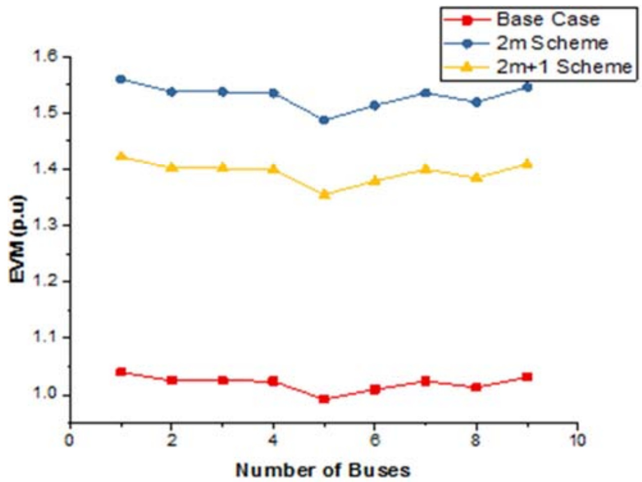

Figure 5. *Comparison of EVM, when Ks= 0 & k_k= 3.*

Table 1. *Comparison of ELL, when Ks= 0 & k_k= 3.*

Line #	Base Case	ELL 2m	ELL 2m+1	SD 2m & Base	SD 2m+1 & Base
1	16.160	16.393	18.064	0.116	0.952
2	20.943	19.711	21.742	0.615	0.399
3	19.699	9.1581	10.099	5.270	4.799
4	9.1294	14.917	16.368	2.894	3.619
5	14.700	10.468	11.544	2.115	1.578
6	10.331	20.979	23.164	5.324	6.416

5. Conclusion

In this paper Probabilistic power flow model used to study uncertainty with power system network. The proposed model based upon the point estimation method, *2m* and *2m+1* concentration scheme were used to evaluate the points of an input random variable. The program was written in DPL code by using the DIgSILENT Power Factory software. This model is used to create the uncertainty in load and compute the effect of this uncertainty in the power system network parameters. Especially, in this paper, expected voltage magnitude and expected line loading are considered for analysis purpose. From the analysis, it is shown that *2m+1* concentration scheme is more reliable than *2m* scheme.

The proposed model can address all possible uncertainties in loads and provide more reliable distribution of output data. Its implementation is investigated by using the P. M. Anderson 9-bus test system, and reliable corresponding results are shown in this paper. These results can be used for future planning, corrective action, and operation of electric power system.

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