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Chiral Symmetry Breaking and Quark Mass Generation of Fermions

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Abstract: It has been shown earlier that the measure of entanglement between two nearest neighbor spins ina spin system given by concurrence is related to the Berry phase acquired by the ground state whenit evolves in a closed path. The significant aspect of this quantization procedure is that it has the specific property of coordinate independence and is governed by geometry. It has been pointed out that this formulation is equivalent to the geometric quantization where the Hermitian line bundle takes a significant role. Also it has been shown that this procedure has its relevance in the quantization of a fermionin the framework of Nelson's stochastic quantization procedurewhen a spinning particle is endowed with an internal degree of freedom through a direction vector (vortex line) which is topologically equivalent to a magnetic flux line. In view of this specific feature of the role of magnetic field in all these formulations of quantization procedure it is expected that the peculiar property of entanglement in quantum mechanics has its relevance with the magnetic flux associated with the quantization procedure. In a seminal paper Berry has shown that when a quantum particle moves in a closed path in a parameter space it attains a geometric phase apart from the dynamical phase. It is here argued that as the Berry phase is related to chiral anomalyentanglement leads to topological mass generation through this anomaly. It is pointed out thatwhen a spin 1 state is considered to be an entangled system of two spin 1/2 states, the maximallyentangled state corresponds to the longitudinal component and gives rise to mass leading to gaugesymmetry breaking.

Keywords: Berry Phase, Chiral Anomaly, Berry Phase, Quantization, Topological Mass Generation, Gauge Symmetry, Entanglement

1. Introduction

It is observed in the context of quantum field theory that the Berry phase is related to chiralanomaly [1]. Indeed the origin of anomaly lies in the fact that some symmetries which arehonored classically are violated quantum mechanically and the anomaly is a manifestationofquantum mechanical symmetry breaking. The fact that the Berry phase is related to chiral anomaly arises from the relationship of the space-time integral of the anomaly with thePontryagin index which is given by twice the magnetic monopole charge [2]. It is noted thatthe quantization procedure of a fermion where spinning degrees of freedom are endowed witha direction vector (vortex line) attached to a space-time point effectively depicts a fermionas a scalar particle attached with a magnetic flux line. In this formalism a massive fermionappears as a styrmion [3]. This helps us to consider entanglement of spin systems as to becaused by the deviation of the magnetic flux line associated with one particle under the influence of the magnetic flux line in the configuration of the other particle. This implies thatentanglement appears as a consequence of chiral symmetry breaking which is manifestedthrough chiral anomaly. The measure of entanglement of a bipartite spin system in a mixedstate given by concurrence is related to the Berry phase acquired by a spin state when it evolves in a closed path and the relationship of Berry phase with chiral anomaly associatesentanglement with chiral anomaly.

The chiral anomaly is found to be instrumental in topological mass generation [4-6]. In viewofthis we may consider that entanglement has a specific role in topological mass generation. In this note we shall explore this and shall show that topological mass generation can indeedbe achieved through quantum entanglement. In this section we shall consider therelevance of maximally entangled state in chiral anomaly and topological mass generation.

2. Maximally Entangled State, Chiral Anomaly and Topological Mass Generation

In a recent paper [5] it has been shown that a spin 1 state can be represented as an entangled system of two spin 1/2 states. As we know a gauge boson with spin J=1 has only transverse components $J_z = \pm 1$ which suggests that in the system of two spin 1/2 statesboth spins are in the up or down direction. But for a massive spin 1 boson there is the longitudinal component $(J_z = 0)$ also where in the two spin 1/2 system one is in the up directionand other in the down direction. From our analysis in the previous section we note that forthe transverse components the two spins are in the product state where the entanglemententropy vanishes. The relationship between concurrence and the Berry phase as well as withthe chiral anomaly suggests that in this case there is no chiral anomaly. However for the longitudinal component $J_z = 0$ where two spins are such that their orientations are oppositeto each other we have the maximally entangled state (MES) with concurrence corresponding to the effective monopole charge $\tilde{\mu} = 1$ and the system exhibits a combination of chiralspinors ψ_R and ψ_L represented by up and down spins which generates chiral anomaly [7-10]. Thefact that the very presence of this component is an indicator of mass for the spin 1 bosonimplies that MES and chiral anomaly takes a significant role in mass generation.

In an earlier paper [4] it has been shown that the weak interaction gauge bosons attainmass from the topological current given by:

$$\vec{J}_\mu = \varepsilon^{\mu\nu\lambda a} \vec{a}_\nu \times \vec{F}_{\lambda\sigma} = \varepsilon^{\mu\nu\lambda a} \partial_\nu \vec{f}_{\lambda a} \qquad (1)$$

which is associated with the axial vector current.

The Lagrangian for the interaction of the Dirac field with the $SL(2,C)$ gaugefield (neglecting the mass term) is given by

$$L = -\bar{\psi}\gamma_\mu D_\mu \psi - \frac{1}{4} Tr \varepsilon^{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} \qquad (2)$$

where D_μ is the $SL(2,C)$ gauge covariant derivative given by $D_\mu \equiv \partial_\mu - igA_\mu$ where is some coupling constant. Here D_μ is the gauge covariant derivative defined by $D_\mu \equiv \partial_\mu - igA_\mu$, g being the coupling constant. If we split the Dirac massless spinor in chiral form and identify the internal helicity+1/2 (-1/2) with left (right) chirality corresponding to $\psi_L(\psi_R)$ we can write:

$$\bar{\psi}\gamma_\mu D_\mu \psi = \bar{\psi}\gamma_\mu \partial_\mu \psi = ig\bar{\psi}\gamma_\mu A_\mu^a g^a \psi = \bar{\psi}\gamma_\mu \partial_\mu \psi - \frac{ig}{2}(\bar{\psi}_R \gamma_\mu A_\mu^1 \psi_R - \bar{\psi}_R \gamma_\mu A_\mu^2 \psi_R + \bar{\psi}_L \gamma_\mu A_\mu^2 \psi_L + \bar{\psi}_L \gamma_\mu A_\mu^3 \psi_L) \qquad (3)$$

This gives rise to the following three conservation laws [2]

$$\partial_\mu [\frac{1}{2}(-ig\bar{\psi}_R \gamma_\mu \psi_R) + J_\mu^1] = 0$$

$$\partial_\mu [\frac{1}{2}(-ig\bar{\psi}_L \gamma_\mu \psi_L + ig\bar{\psi}_R \gamma_\mu \psi_R) + J_\mu^2] = 0$$

$$\partial_\mu [\frac{1}{2}(-ig\bar{\psi}_L \gamma_\mu \psi_L) + J_\mu^3] = 0 \qquad (4)$$

where $J_\mu^i (i = 1,2,3)$ are gauge field currents given by eqn.(1).

These three equations represent a consistent set of equations if we choose $J_\mu^1 = -\frac{1}{2} J_\mu^z$, $J_\mu^3 = +\frac{1}{2} J_\mu^z$. This guarantees the vector current conservation. From this we can write

$$\partial_\mu [\bar{\psi}_R \gamma_\mu \psi_R + J_\mu^2] = 0$$

and

$$\partial_\mu [\bar{\psi}_L \gamma_\mu \psi_L - J_\mu^2] = 0 \qquad (5)$$

$$\partial_\mu (\bar{\psi}\gamma_\mu \gamma_5 \psi) = \partial_\mu J_\mu^5 = -2\partial_\mu J_\mu^2 \qquad (6)$$

In fact from the relations (5) and (6) it is observed that the axial vector current arises from a combination of the right-handed and left-handed chiral currents andthe divergence of this current is related to the divergence of the topological current J_μ^2. Ithas been pointed out that when we represent the chiral currents $\bar{\psi}_R \gamma_\mu \psi_R (\bar{\psi}_L \gamma_\mu \psi_L)$ as the charged current the relation (4) suggests that the topological current $J_\mu^1 (J_\mu^3)$ corresponds to a charged current which may be thought as the source of the charged gauge field [4].

In fact we may asociate the three weak-interaction gauge fields W_μ^+, W_μ^0, W_μ^- with the topological currents J_μ^1, J_μ^2 and J_μ^3

$$\vec{J}_\mu = \begin{pmatrix} J_\mu^1 \\ J_\mu^2 \\ J_\mu^3 \end{pmatrix} \rightarrow \begin{pmatrix} W_\mu^+ \\ W_\mu^0 \\ W_\mu^- \end{pmatrix} \qquad (7)$$

From the relation (4) we may associate W_μ^0 with the state $W_\mu^0 = \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}}$. This implicitlysuggests that there should be a U(1) sing let $X_\mu^0 = \frac{W_\mu^+ + W_\mu^-}{\sqrt{2}}$. The mixing effect of W_μ^0 and X_μ^0 gives rise to the gauge boson Z_μ^0 and photon.

It is noted that the Pontryagin index q as well as the monopole charge μ is related to thedivergence of the axial vector current through the relation

$$q = 2\mu = \int J_0^2 d^3x = \int \partial_\mu J_\mu^2 d^4x = -\frac{1}{2}\int \partial_\mu J_\mu^5 d^4x \quad (8)$$

From the relation $\partial_\mu J_\mu^2 = -\frac{1}{2}\partial_\mu J_\mu^5$ we can write

$$J_\mu^2 = -\frac{1}{2}(J^5 + J_\mu^V) \quad (9)$$

where J_μ^V is a conserved vector current [11-13]. So we can write the equivalent formulation of the topological current J_μ^2 as a chiral current

$$J_\mu^2 = -\frac{1}{2}(\bar{\psi}\gamma_\mu(1+\gamma_5)\psi) \quad (10)$$

where ψ is a fictitious spin or which satisfies Dirac equation. Since the Dirac spinor satisfiesthe eigenvalue equation:

$$\partial_\mu^2\psi = -m^2\psi \quad (11)$$

we find from eqns.(10) and (11)

$$WJ_\mu^2 = -\frac{1}{2}(-4m^2)J_\mu^5 = -\tilde{m}^2 J_\mu^2 \quad (12)$$

with $\tilde{m} \neq 0$. W -symbol is the D'Alembert Operator. Now as shown in topological mass generation, we write

$$W\partial_\nu J_\mu^2 = -\tilde{m}^2\partial_\nu J_\mu^2 \quad (13)$$

which implies

$$W(\partial_\nu J_\mu^2 - \partial_\mu J_\nu^2) = -\tilde{m}^2(\partial_\nu J_\mu^2 - \partial_\mu J_\nu^2) \quad (14)$$

Now noting from eqn.(1)that

$$\partial_\nu J_\mu^2 - \partial_\mu J_\nu^2 = W^* F_{\mu\nu}^2 \quad (15)$$

we have from eqn. (15)

$$WW^* F_{\mu\nu}^2 = -\tilde{m}^2 W^* F_{\mu\nu}^2 \quad (16)$$

implying

$$(W + \tilde{m}^2)^* F_{\mu\nu}^2 = 0 \quad (17)$$

This suggests that $\tilde{m}$ is the mass of the gauge boson. This implies that after mixing effect Z_μ^0 becomes massive. This analysis when generalized to $J_\mu^1(J_\mu^3)$ leads to the mass of gauge bosons. As mentioned earlier the attribution of mass to a gauge field involves the introduction ofthe longitudinal component to the spin J=1 field and the existence of MES ensures this. In fact we can associate a scalar field $\phi(x)$ representing the longitudinal component to the topological current J_μ^2 which is responsible for the topological mass generation through chiralanomaly. We can write:

$$J_\mu^2 = \varepsilon^{\mu\nu\lambda\sigma}\partial_\nu F_{\lambda\sigma}^2 = \varepsilon^{\mu\nu\lambda\sigma}\varepsilon_{\nu\chi\sigma}\phi(x) \quad (18)$$

which follows from the fact that $F_{\mu\nu}^2$ is an antisymmetric tensor. This is true for $J_\mu^1(J_\mu^3)$ also when the scalar field bears charge [14]. Thus we find that the MES for a two spin 1/2 staterepresenting a spin 1 boson bears the signature of chiral anomaly through the Berry phasefactor which corresponds to the measure of entanglement viz. concurrence and gives rise tothe mass topologically through chiral anomaly [15]. This essentially corresponds to the existenceof longitudinal component of the spin 1 boson.

In a generalized form the MES of a spin 1/2 system of two spins can be written as

$$|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha|00\rangle + \beta|01\rangle - \beta^*|10\rangle + \alpha^*|11\rangle) \quad (19)$$

where *α, β* are complex coefficients and $\alpha^*(\beta^*)$ denotes the complex conjugate. For MESthe concurrence is given by $C = |\alpha|^2 + |\beta|^2 = 1$. Milman and Mosseri [16] have pointed out that the MES is related to the double connectedness of the SO(3) group. In fact as we have $(\alpha,\beta) \in C^2$ with $C = |\alpha|^2 + |\beta|^2 = 1$ and we have the symmetry $(\alpha,\beta) \to (-\alpha,-\beta)$, the Hilbert space of all the MES can be defined as *S3/Z2=SU*(2)*/Z2=SO*(3). Now we notethat in the definition of the topological current J_μ^i (*i*=1,2,3*)* the antisymmetric tensor $e^{\mu\nu\lambda\alpha}$ suggests that the topological mass will not possess discrete symmetries of space (*P*)and time (*T*) inversion implying *Z*2 symmetry breaking. However for the two gauge fields W_μ^+ and W_μ^- with mass $+|m|$ and $-|m|$ respectively *P* and *T* conservation will be restoredwhen the coordinate inversion is associated with the field exchange. This corresponds tothe double connectedness of SO(3). In fact as long as the field exchange is not performed, the *P* and *T* inverted mass $+|m|$ and $-|m|$ for the two fields W_μ^+ and W_μ^- is attributed to the right-handed and left-handed universes. This implies that an internal helicity is inducedin the configuration of gauge bosons during mass generation. The *U*(1) character of thishelicity breaks the gauge symmetry *SU*(2) →*U*(1) [17, 18]. When there is full breaking of *SU*(2) symmetry the gauge fields reside only in a single universe whether left-handed orright-handed indicating that we are concerned with only particles or anti-particles. Thusthe signature of mass becomes irrelevant. This analysis suggests that the significant role ofMES in topological mass generation as well as electroweak symmetry breaking when spin 1bosonic state is considered as an entangled system of two spin 1/2 states.

3. Discussion

We have analyzed here the relationship between entanglement and Berry phase, suggests that entanglement is associated with chiral anomaly as the Berry phase is related to this. Indeed chiral anomaly gives rise to the Pontryagin index which is twice the monopole charge and the Berry phasefactor is related to this monopole charge. Again chiral anomaly is found to be instrumentalin topological mass generation and this suggests that entanglement has also its role in thegeneration of mass. It is shown here that when a spin 1 state is viewed as an entangled system of two spin 1/2 states the maximally entangled state corresponds to nonvanishingchiralanomaly and leads to the mass generation.

4. Conclusion

In view of this we may consider that entanglement has a specific role in topological mass generation. In this note we shall explore this and shall show that topological mass generation can indeed be achieved through quantum entanglement. Certainly the MES corresponds to the longitudinalcomponent of the spin 1 state and thus breaks the gauge symmetry. In an earlier paper [4] it has been argued that chiral anomaly leads to the topological massgeneration and gauge symmetry breaking in electroweak theory. This can be transcribedin the framework of quantum entanglement when the spin 1 state is viewed as an entangled system of two spin 1/2 states. In view of this we find that entanglement has a moregeneralized role in various aspects of quantum phenomena.

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