

Numerical Approximation of a Non-Newtonian Flow with Effect Inertial

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Abstract: Viscoplastic fluids are materials of great interest both in industry and in our daily lives. These applications range from food and cosmetics products to industrial applications such as plastics in the industry of polymers and drilling muds the oil industry. This class of material is characterized by having a yield stress that must be exceeded to the material starts to flow. These fluids are classically predicted by purely viscous models with yield stress. In the last decade, however, some experimental visualizations have reported that the unyielded regions exhibit elasticity inside. This work is an attempt to investigate the effect of elasticity and inertia in those materials. We will study, therefore, inertia flow of elastic-viscoplastic materials with no thixotropic behavior, according to the material equation introduced in de Souza Mendes (2011). The mechanical model is approximated by a stabilized finite element method in terms of extra stress, pressure and velocity. Due to its fine convergence feature, the method allows the use of equal-order finite elements and generates stable solutions in high advective-dominated flows. In this study is considered the geometry of a biquadratic cavity, in which the top wall moves to the right at constant velocity. In all computations is used biquadratic Lagrangian (Q1) elements. Results focus on determining the influence of elasticity and inertia on the position and shape of unyielded. These results proved to be physically meaningful, indicating a strong interlace between elasticity and inertia on determining of the topology of yield surfaces.

Keywords: Elastic-viscoplastic Materials, Finite Element, Biquadratic Cavity, Elasticity and Inertia

1. Introduction

Elasto-viscoplastic fluids are structured materials that exhibit a complex non-Newtonian behavior that is related to their structure state, which, in turn, depends on the level of stress applied to it. Below a certain stress limit, called the yield stress, the material is highly structured, with high levels of elasticity and viscosity. This region can be called a apparently unyielded regions. When subjected to stress levels above the flow value, the material undergoes a rupture leading to a fluid-like behavior, where the viscosity decays in order of magnitude and their elasticity tends to disappear and these regions are called apparently yielded regions. Recent experiments present some data showing some elastic effects on viscoplastic fluid flows [12]. This class of material is present in several important industrial sectors such as

petroleum, food products and cosmetics.

The constitutive equation used in this work is a modification of the Oldroyd-B equation, where it considers the elasticity below the yield stress and a pseudoplastic behavior above the yield stress. However, the model employed does not consider the structure of the fluid and has not been tested in simple flow. More recently, a new and more reliable constitutive equation of the Oldroyd-B type was proposed [12]. An important feature of this equation is that it is also able to predict the thixotropic behavior of fluids, a feature that may be present in many viscoplastic materials. This model is more representative than that used in the study, since it involves the determination of a structure parameter to describe the microstructure of the fluid [13]. In this work, numerical solutions of conservation and government equations were obtained using a formulation of three Galerkin least squares (GLS) fields, which takes into

account the fields of velocity, pressure and extra-stress as prime variables [2]. This formulation can be seen as an extension - for the elasto-viscoplastic case subject to shear-thinning of the relaxation and retardation times, and the viscoplastic SMD function - of the formulation proposed in the paper, for fluids of constant viscosity [11]. Thus, the numerical results of inertial flows of elasto-viscoplastic fluids are obtained within a lid-driven cavity and a discussion is presented on inertia, elastic and viscous contributions to the flow pattern.

2. The Mechanical Model

For the flow of elasto-viscoplastic material can be modeled by the following governing equations,

$$\nabla \cdot \mathbf{u} = 0 \quad (1)$$

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \text{div}(\boldsymbol{\tau}) + \rho \mathbf{g} \quad (2)$$

where \mathbf{u} is velocity vector, \mathbf{g} is the gravitational force per unit mass, p is the pressure field and $\boldsymbol{\tau}$ is the extra stress tensor.

To model the elasto-viscoplastic behavior of the material, the extra stress tensor is described by an Oldroyd equation that takes into account not only elasticity, but also viscoplasticity and thixotropy. The constitutive equation of the model adopted in this work was proposed by the study [12], which follows the following relation:

$$\boldsymbol{\tau} + \theta_1(\dot{\gamma}) \overset{\nabla}{\boldsymbol{\tau}} = 2\eta(\dot{\gamma}) \left(\mathbf{D}(\mathbf{u}) + \theta_2(\dot{\gamma}) \overset{\nabla}{\mathbf{D}}(\mathbf{u}) \right) \quad (3)$$

Where $\dot{\gamma} = \sqrt{2\text{tr}[(\mathbf{D}(\mathbf{u}))^2]}$ is the magnitude of the strain rate tensor, \mathbf{D} is the strain rate tensor and $\overset{\nabla}{\boldsymbol{\tau}}$ and $\overset{\nabla}{\mathbf{D}}$ represent the upper convected derivatives, respectively given by:

$$\overset{\nabla}{\boldsymbol{\tau}} = (\nabla \boldsymbol{\tau})\mathbf{u} - (\nabla \mathbf{u}) \cdot \boldsymbol{\tau} - \boldsymbol{\tau} \cdot (\nabla \mathbf{u})^T \quad (4)$$

$$\overset{\nabla}{\mathbf{D}} = (\nabla \mathbf{D})\mathbf{u} - (\nabla \mathbf{u}) \cdot \mathbf{D} - \mathbf{D} \cdot (\nabla \mathbf{u})^T \quad (5)$$

The differential equation of the extra stress tensor is the standard Oldroyd-B viscoelastic model, except that structural viscosity, η , relaxation time, θ_1 , and retardation time, θ_2 , are dependent parameters. The evolution of the structure parameter is governed by a kinetic equation, with its material derivative in time given by:

$$\dot{\lambda} = \mathbf{u} \cdot \left(\frac{\partial \lambda}{\partial \mathbf{x}} \right) = \left[(1 - \lambda) - (1 - \lambda_{eq}) \left(\frac{\lambda}{\lambda_{eq}} \right) \right] \quad (6)$$

where the imbalance between the building term, $\frac{1}{t_{eq}}(1 - \lambda)$, it's the break, $\frac{1}{t_{eq}} \left[(1 - \lambda_{eq}) \left(\frac{\lambda}{\lambda_{eq}} \right) \right]$, determines whether the material will undergo aging or a rejuvenation process.

The equilibrium viscosity adopted is a function of the stress when the fluid is apparently flowing characterized by a high finite viscosity at the limit where $\dot{\gamma} \rightarrow 0$, given by [11]:

$$\eta_{eq}(\dot{\gamma}) = \left[1 - \exp\left(\frac{-\eta_0 \dot{\gamma}}{\tau_y}\right) \right] \left\{ \frac{\tau_y}{\dot{\gamma}} + K \dot{\gamma}^{n-1} \right\} + \eta_\infty \quad (7)$$

Where η_0 is the low shear rate viscosity plateau, τ_y is the yield stress, K is the consistency index, n is the power-law index, and η_∞ is the high shear rate viscosity plateau.

The relaxation time and retardation, respectively used are defined:

$$\theta_1 = \left(1 - \frac{\eta_\infty}{\eta_{eq}} \right) \frac{\eta_{eq}}{G_{eq}} \quad (8)$$

$$\theta_2 = \left(1 - \frac{\eta_\infty}{\eta_{eq}} \right) \frac{\eta_\infty}{G_{eq}} \quad (9)$$

where, G_{eq} is the elastic modulus in equilibrium and follows the relation,

$$G_{eq}(\lambda_{eq}) = G_0 e^{m \left(\frac{1}{\lambda_{eq}} - 1 \right)} \quad (10)$$

This function is used to predict the elastic behavior of viscoplastic fluids only in regions where the stress level is lower than the flow stress.

As η_∞ , the low viscosity region corresponds to the retardation time is practically zero and the relaxation time is reduced to:

$$\theta_{eq} = \frac{\eta_{eq}}{G_{eq}} \quad (11)$$

And the relationship between the equilibrium structure parameter and the equilibrium viscosity is given by,

$$\lambda_{eq}(\dot{\gamma}) = \frac{\ln \eta_{eq}(\dot{\gamma}) - \ln \eta_\infty}{\ln \eta_0 - \ln \eta_\infty} \quad (12)$$

3. The Numerical Modeling

To approximate the mechanical model described above, a multi- field Galerkin least-squares formulation, in terms of velocity, pressure and extra-stress, is employed. This formulation may be viewed as a direct extension of the model introduced by the author for constant viscosity fluids, to flows of elasto-viscoplastic materials [2]. Proposed by the research for Stokes flow, and later extended by the study for Navier-Stokes flow, this formulation has been successfully applied to many engineering applications [5, 7].

This model overcomes the shortcomings present in classical Galerkin approximations for fluid problems of interest, primarily, the need to satisfy functional compatibility conditions among the finite element subspaces of its primal variables. The model produces stable and meaningful approximations for fluid problems of interest that are exempt from numerical pathologies, even employing equal-order combinations of Lagrangean finite elements (for details, and references therein) [2, 5]. Exploiting such a feature in all of the computations shown in the upcoming numerical section, an equal-order bi-linear (Q1) finite element interpolation is used.

A. Geometry and boundary conditions

The geometry considered is shown in Figure 1. It consists

of a unit cavity of length L with the upper wall moving with constant velocity ($u_1 = u_c; u_2 = 0$) and its other walls and the two points of singularity in the two upper corners of the cavity subjected to non-slip and impermeable conditions ($u_1 = u_2 = 0$). All results were obtained using Lagrangian bi-linear interpolations (Q1) for all primary variables.

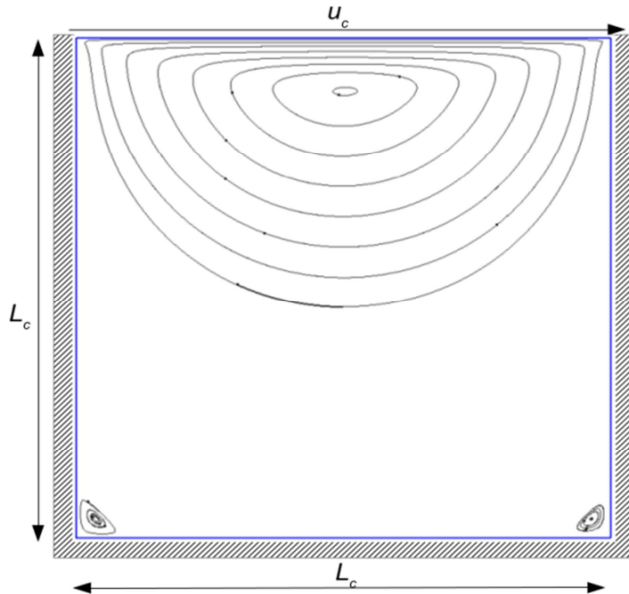


Figure 1. Geometry and boundary conditions.

In this figure, we can see three recirculation regions, one central region associated with the main flow and two secondary regions in the lower chinas of the cavity. These regions are caused by the effect of velocity on the cavity cover and will always be present in this geometry under the contour conditions discussed above.

4. Numerical Results

The results aim to study the effect of inertia on the flow pattern of viscoplastic materials subject to elasticity, by determining the morphology and position of their apparently unyielded regions, being that the evaluation of the effects caused by inertia is due to the advective term of the momentum equation.

A. Influence of flow intensity

The Figure 2 shows the influence of the flow intensity on the flow surfaces. The material parameters used were: $\theta_0^* = 100, n = 0,5, \rho^* = 500$.

In this figure, it is perceived that as the flow intensity increases, the apparently unyielded regions decrease throughout the cavity. When the flow intensity reaches a high value (high velocity), a strong displacement occurs of the apparently unyielded regions in the top of the cavity, this is due to the displacement of the central vortex, also called the main flow vortex, since with a high velocity, the inertial effects become more evident and also, with a high velocity and high inertial effects, we will have an increase of the advection in the flow.

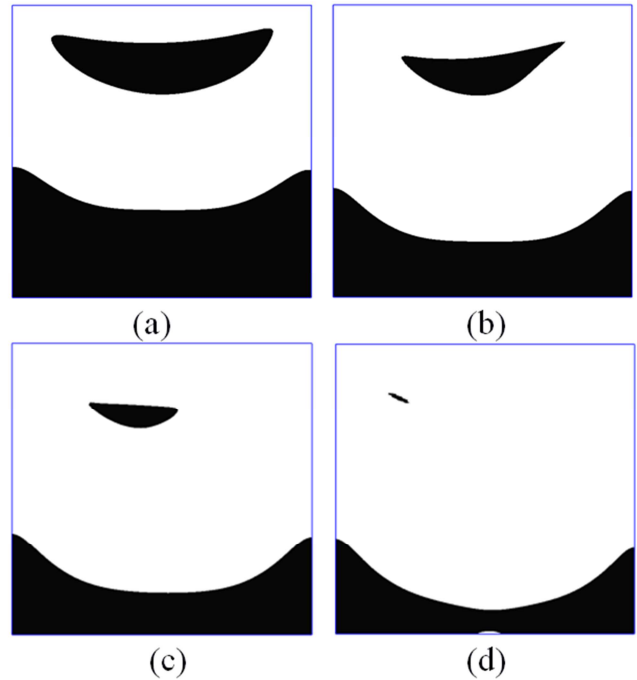


Figure 2. Flow surfaces: (a) $U^* = 0,01$, (b) $U^* = 0,05$, (c) $U^* = 0,1$, (d) $U^* = 0,2$.

5. Conclusions

In this work numerical simulations of elasto-viscoplastic flows were performed with the introduction of the inertial effects and neglecting the thixotropy, where the geometry used was a forced cavity.

The mechanical modeling was done using the mass conservation equation, the equation of the conservation principle of the momentum coupled to an elasto-viscoplastic material equation proposed in the study [12]. The mechanical model was approximated by a Finite Element method, namely the least squares Galerkin multi-field method in terms of extra stress, pressure and velocity.

Regarding the influence of the flow intensity, a marked reduction was observed in the apparently unyielded regions with increasing flow intensity. This is because the increase in U^* causes increasing levels of tension throughout the cavity, causing larger regions to exceed the flow limit of the material and begin to flow as a power-law fluid. By the variation of U^* and fixing ρ^* , increasing the advective effects in the flow the apparently yield regions suffer a strong displacement to the right in the superior part of the cavity.

The advective term of the momentum equation is the term that represents the forces of inertia in the flow, that is, the greater the inertia in the flow, the greater the advection in that flow.

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