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Concept images of trapezoid: Some cases from Turkey

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Abstract: The objective of this study was to determine the concept images of individuals regarding trapezoid. In order to attain this objective, this research was designed based on qualitative approach. A total of 156 students attending 5th through 8th grade middle school and 36 middle school Mathematics teacher candidates from Turkey were chosen as participants in the study. Definitions and drawings of trapezoids were asked of the participants and their responses were used to analyze their concept images. The study was conducted using semi-structured interview technique. Data were analysed through content analysis. Results were presented both in the form of frequency and actual participants' statements. According to data obtained from the participants, many of the definitions acquired were personal rather than formal. It was also determined that individuals used non-critical properties in non-formal and incorrect definitions and that they created some misconceptions by making excessive generalizations. Based on the data, some participants developed the concept image indicating that the sides and angles should be completely different. As a result, conceptual levels of the trapezoid within the framework of the obtained concept images were determined.

Keywords: Concept Image, Trapezoid, Mathematics Learning

1. Introduction

Teaching geometry is critical for multiple reasons; including comprehending information and relations about point, line, figures, space and improving spatial thinking, visual skills and geometrical reasoning [1, 2, 3].

Levels of geometric thinking put forth by van Hiele [4] have played a significant role in studies carried out on teaching of geometric concepts and the perceptions and understanding of individuals regarding geometric shapes. In addition, the "concept image" pointed out by Tall and Vinner [5], the "figural concept" for geometric shapes put forth by Fischbein [6] along with the "prototype" [7] understanding stand out as theoretical framework in such studies.

According to van Hiele's geometric thinking model, children's geometric thinking is developed at hierarchical levels [4]. These levels are defined as visualisation, analysis, informal deduction, formal deduction and rigor.

Tall and Vinner [5] define concept image as "the total cognitive structure that is associated with the concept, which includes all the mental pictures and associated properties and processes" (p. 152). They make a distinction between concept definition and concept image. They define concept definition as the words used to specify that concept. Personal concept definitions may differ from formal concept definitions.

Based on the notion of figural concept, geometric figures have conceptual properties and images [6]. For example, square is influential, due to its properties (equal sides and 90^0 angles), in the forming of a concept image in individuals' minds. Furthermore, frequently used and encountered figurative appearances of geometric shapes, in other words, their prototypes (as in van Hiele lowest level) have been observed to affect individuals in forming concept images [7].

Studies carried out revealed that the geometric concept definitions of individuals are shaped by the concept images formed in the minds of the individuals and that they can be quite different than the formal concept definitions [5, 8-10]. Additionally, various properties of geometric concepts can stand out in the formation of the concept image. Researchers have separated these properties into two groups: critical attributes and non-critical attributes. According to Hershkowitz [7], critical attributes must be present in every example of the concept, derived from the concept definition. Non-critical attributes occur only in a subset of the concept examples.

According to Hershkowitz [7], when a prototypical figure is used as a frame of reference, it leads to prototypical judgments causing individuals to certain misconceptions. These prototypical judgments are as follows [7: p.83]:

Type 1: The prototypical example is used as the frame of reference and visual judgment is applied to other instances (first van Hiele level).

Type 2: The prototypical example is used as the frame of reference, but the individual bases his/her judgment on prototypes of self-attributes and tries to impose them on other concept examples.

Type 3: Correct analytical prototypical judgment is also common. This type of reasoning is based on the concept's critical attributes.

Review of literature reveals that there are various studies about "perception of quadrilaterals" nearly in every age group. In these studies, students were directed questions about quadrilaterals, listing their properties or drawing the named quadrilaterals, differentiating and forming relations between them and making classifications.

Based on these studies, it has been observed that individuals at the primary school level had difficulties in recognizing, naming, defining the given shape and listing its properties. For example, while there were some individuals (between the 6-8 age groups) who claimed that a square turned to one side that looks like a diamond is actually not a square, there were no problems in naming the given shapes in studies carried out with older students, however, they had difficulties in defining them and understanding inclusion relations [3, 11-13]. These problems have been mostly observed in forming the parallelogram family. Studies carried out have shown that individuals define parallelogram correctly and that they can recognize it when they see one. However, the ratio to accept square, rectangle and rhombus as part of this family was not high. A similar issue was observed in the relationship between rectangle and square. Erez and Yerushalmy [14] have carried out a study using computer software and have determined that children cannot understand why they will use parallelogram key to make square, rectangle and rhombus since they cannot structure the relationship between these quadrilaterals. Similarly, Heinze and Ossietzky [9] have observed in their study that more than half of the individuals saw square as a quadrilateral with 90^0 angles and there were certain number of individuals with perceptions that a quadrilateral with all four sides equal could only be a square (one third of the participants). According to the results obtained from various studies, although some individuals can draw any given shape correctly, they fail to make a formal definition [3]. Studies conducted reported that individuals did not have any difficulty in establishing a relation between parallelogram and rhombus and that the reason for this was the fact that they had similar appearances [3].

Studies carried out on teachers and teacher candidates yielded similar results with studies on smaller age groups [3]. In these studies, it was determined that individuals generally digressed from the formal definitions of quadrilaterals within their own concept images and that they preferred to classify quadrilaterals based on the images in their own minds, thereby, making various mistakes. A study carried out on elementary school Mathematics teachers revealed that the ratio of people who identified quadrilaterals correctly was not high (about 30 %) [15]. In addition, results such as seeing parallelogram and square as a different family, the standing out of the parallelness feature of the rectangle and square due to the typical drawing of a parallelogram and seeing the square as belonging to a family different than that of the rectangle were obtained. Another study carried out on Mathematics teachers yielded similar results [16].

There is limited literature on quadrilateral perceptions that include trapezoid. One of these is an experimental study on the teaching of the definition of trapezoid [17]. The study carried out by Nakahara [18] put forth that the ratio of the correct definition of trapezoid and the structuring of its relationship with other quadrilaterals was very low. In addition, it was stated that the relationship between parallelogram and trapezoid was the hardest to grasp.

In light of the aforementioned theoretical framework along with the studies carried out in this field, the number of studies on the concept images of students regarding trapezoid is limited. The objective of this study is to determine the concept images of individuals regarding the trapezoid. In addition, conceptual levels of the trapezoid within the framework of the obtained concept images will be put forth. This study is a part of a project carried out to determine the perceptions of individuals from all age groups on quadrilaterals.

2. Methodology

The objective of this study is to put forth the concept images of middle school students and middle school Mathematics teacher candidates regarding the trapezoid. In order to reach this aim, this research was designed through qualitative approach and used semi-structured interview method. According to Patton [19], interviews allow the researcher to gain an inner perspective of outward behaviors. Semi-structured interviewing is more flexible than the other interviewing forms in terms of individual circumstances. The researcher obtains in-depth information by taking opportunities to probe, explore and expand the interviewee's responses. This kind of interviewing allows the researcher to specify and enhance issues in advance [19].

During the interview, individuals were asked to define the trapezoid and to draw one. The concept images reflecting on the definition and description of individuals were elicited.

2.1. Participants and Setting

The study group consisted of individuals from two different levels. First group consisted of 156 students attending the 5th, 6th, 7th and 8th grade middle schools in a city, in Turkey (10-13 year-old students). Second group consisted of 36 middle school Mathematics teacher candidates from a faculty of education in Izmir. Permission to conduct the study was obtained from the teachers and the parents of the participating children. The consent forms were obtained from all of the participants.

All of the participants had been taught basic information about the trapezoid which is a study topic in Mathematics courses. The Mathematics education at all levels of the participants' covers the definition of the trapezoid, its properties and relevant problem solving concerning the trapezoid. Trapezoid is defined in course books as "a quadrilateral with straight sides that has a pair of opposite parallel sides". There is no definitive judgment regarding whether the remaining two sides are parallel or not.

2.2. Data Analysis

Data collected through semi-structured interview were analysed through content-analysis. Qualitative content analysis was used for determining different kinds of concept images of trapezoid. Content analysis is the process of identifying, coding and categorizing the primary patterns in the data [19]. In the study, participants' definitions and drawings of trapezoid were both taken into account to analyze the perceptions. Data were categorized under three main titles: formal or non-formal definitions; critical attributes and non-critical attributes; and also correct or incorrect definitions. Results were presented in the form of participant statements, frequency and drawings.

3. Results and Discussion

In this section, the concept images related to trapezoid will be presented in light of the definitions and drawings provided by the participants; first, for middle school students and second, for teacher candidates.

3.1. Middle School Students' Definitions and Drawings of Trapezoid

When the participants' (5-8 grades students) definitions of trapezoid were examined, various and generally non-formal definitions were observed. It was determined that the definitions students provided were far from using the mathematical language correctly. Students were not able to give exact definitions of a trapezoid and they, instead, tried to define it by stating some of its properties. Reflections of the concept images of students concerning trapezoid may be observed in these definitions.

It was observed that some students emphasized the parallelness property of trapezoid. Various statements of the participants can be seen below.

"Trapezoid is a polygon with four sides and four angles, two sides of which are parallel to each other." (8^{th} grade student)

"It has four corners; its top and bottom are parallel." (6^{th} grade student)

Definitions of some students indicated that only two sides of a trapezoid could be parallel, whereas, the other two sides could not be parallel or that some sides should be oblique:

"Trapezoid has four corners. The right and left sides are not parallel to each other." (6^{th} grade student)

"A shape with two sides parallel and some sides oblique." (7^{th} grade student)

"A shape with longer sides parallel and shorter sides not parallel." (8^{th} grade student)

These students formed a concept image due to the effect of the prototype shape of the trapezoid, and according to this image, two sides can never be parallel. It was observed that students who expressed the paralellness property generally drew the prototype figures (Fig. 1). According to some research results, learners might be affected by prototype figures and defined the geometric shapes based on these images [9, 12].

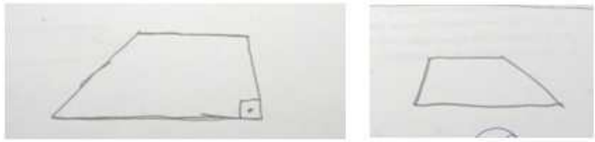

Figure 1. *Prototype trapezoid drawings.*

In some interviews carried out with middle grade students, side and angle properties became significant. In such definitions there were those who defined trapezoid as a geometric shape with irregular side lengths and different angles. The statements of these students have been given below.

"An object with four sides, each of which have different lengths." (5^{th} grade student)

"Trapezoid is a quadrilateral with four sides and these four sides are all different." (6^{th} grade student)

"A geometrical object with four corners, each of which is different." (6^{th} grade student)

"An oblique quadrilateral with non-equal sides." (5^{th} grade student)

Students who gave such answers did not think about the sides being parallel for a trapezoid. This leads us to the assessment that the concept images of students with these answers were all variations stating that it was a shape with different sides and angles. It signifies that the concept images of students concerning a trapezoid such answers could not point to a trapezoid with equal sides and angles. In this case, it can be stated that they made an excessive generalization for special cases as is defined by Vinner and Hershkowitz "prototypical judgment Type 2" [7]. In addition, it also points out that these students have not considered the paralellness property as a critical attribute. The trapezoids drawn by these students are generally prototype trapezoid drawings with two sides that are parallel, and in their images, the different angles and sides are dominant rather than its paralellness. There are some studies (those studies are not related to trapezoid) which showed that the students based their judgments on some of the properties of the prototype [9, 11, 12].

There are also drawings of isosceles trapezoid and right trapezoid. Various conditions such as some sides being of equal length or some angles being equal have been disregarded for isosceles or right trapezoids.

When the definitions of some middle school students who participated in the study were examined, it was determined that they saw trapezoids as a completely ambiguous shape with no specific features as seen in the following statements.

"Undistinguished shapes with no specific shape." (5th grade student)

"It has four corners and it looks like a curved shape."(5th grade student)

"A shape with no specific shape that does not resemble a regular polygon." (6th grade student)

"A shape has no specific shape and it has not identical to each other." (6th grade student)

Students who gave such definitions drew shapes with curved sides that did not resemble any geometrical figure. Some examples of the drawings of these students can be seen in Fig. 2. In addition, some students who gave such definitions drew other quadrilaterals (square, rectangle etc.) after which they drew their sides in a curved manner asserting that this is, indeed, a trapezoid (Fig. 2b). It can be stated that students with such drawings and definitions did not know anything about trapezoids and only drew curved shapes based on the meaning of the Turkish denomination. Because Turkish term for trapezoid means 'oblique'. It can be said that the perceptions of these students were affected the linguistics aspect, as some studies point out the influence of the language on students' understanding [9, 20, 21].

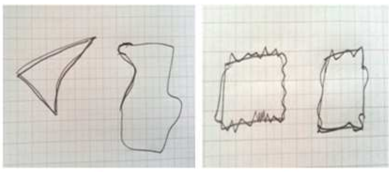

Figure 2. *Ambiguous drawings as trapezoid.*

It was determined, in addition to all these definitions, that especially students from smaller age groups resembled trapezoid to objects from their surroundings, instead of giving a definition. For example, a 5th grade student used the expression "resembles a volcano" while another one resembled it to a "vase" and another student used the expression "looks like a house". It can be stated that the conception of these students concerning the trapezoid were images in their minds as a shape, and that they were not aware of its properties. Based on the Van Hiele theory, it can be said that these students are in the first level-visualization. They are not able to identify attributes of these figures. They named a figure based on its visual appearance. The concept image of these students is limited only to a visual image and that it is not shaped by definitions learned or properties observed. It can be said that they made a wrong judgment like described by Vinner and Hershkowitz's the prototypical judgment "Type 1" [7].

The definitions of middle grade students suggested three different possibilities. First, some students emphasized that two sides of a trapezoid are parallel. Second, some students emphasized that the sides and angles are different. Third, some students stated that it is an oblique shape with no rule at all. In addition, it has been observed that a group of students perceived the trapezoid as similar to various objects in their surroundings. Within this framework, answers obtained from the students were grouped and their frequency values are given in Table 1.

Table 1. *Frequencies of 5th- through 8th-grade students' definitions of trapezoid.*

Grade	Those who emphasized that the two sides were parallel(f)	Those who emphasized that the sides or angles were different(f)	Those who emphasized that there was no specific rule (f)	Those who used various objects to define the trapezoids (f)	Those who gave no definition (f)	Total
5th	11	6	7	10	12	46
6th	0	12	6	6	8	32
7th	14	10	5	4	4	37
8th	21	14	6	0	0	41

When Table 1 is taken into account, it is observed that only 46 of the 156 participants have used the parallelness property of the trapezoid. The fact that two opposite sides or the remaining two sides are parallel is accepted as a critical property for the trapezoid. However, according to the definitions of 42 students the lengths of the sides along with the angles are different. This leads us to think that parallelness is not a critical property of trapezoids for these students. The concept image of a trapezoid was shaped by non-critical properties by these students. According to Burger and Shaughnessy [22], non-critical properties can form a basis for visual argument.

According to data in Table 1, majority of students who made a definition by resembling the trapezoid to various other objects along with those who gave no definition belonged to smaller age groups (5th and 6th grade). As was stated earlier, these students have a conception of a trapezoid only as a visual appearance and they cannot define it verbally without referring to objects from their surroundings.

3.2. Middle School Mathematics Teacher Candidates' Definitions and Drawings of Trapezoid

Various formal and non-formal definitions were observed when the trapezoid definitions of middle school Mathematics teacher candidates who participated in the study were examined. It was determined that the teacher candidates were far from using the mathematical language properly in their definitions. In addition to most candidates who were not able to give a proper definition of the trapezoid, 22 of the 36 mathematics teacher candidates (62 %) gave an erroneous definition or could not define it at all. There is little research on definitions of trapezoid and the results of them vary. For example, in one study [3], the results showed that while the Japanese teacher trainees defined trapezoid correctly at a high rate, Scottish trainees did not.

From the definitions, it was possible to determine the concept images of teacher candidates regarding trapezoid. When non-academic and personal definitions of a trapezoid were evaluated, concept images similar to those of the middle grade students were observed. Some examples of these definitions are as follows.

"A convex quadrilateral with four corners and sides independent of each other."

"No specific rule and property. We can draw the trapezoid in any number of ways".

"A shape with four sides that we can draw however we want."

"Has no relation with other quadrilaterals; only the top and bottom sides are parallel."

"Has different top and bottom side lengths."

"A shape with different sides and angles."

"A quadrilateral with oblique sides that have no specific relationship with each other."

"Only two sides are parallel, the others are oblique as the name trapezoid implies"

"Parallel lines and other lines that are not perpendicular to these."

Regarding these definitions, it can be observed that some participants have the concept image that trapezoid is a randomly drawn quadrilateral. As was the case in the statements of middle grade students, some teacher candidates also stated that the sides and angles of a trapezoid were different from each other. Some even defined it as a shape with no rules. These findings may suggest that the prototypical figure is used as a reference and they make wrong visual judgments (like Vinner & Hershkowitz's prototypical judgment 'type 1'). All these findings may depend on the concept images of candidates that are affected by the appearance of trapezoids rather than critical attributes.

It can be seen from the definitions that some teacher candidates emphasized the parallelness of the trapezoid. When the trapezoid drawings of teacher candidates were examined, it was observed that they generally drew right or isosceles trapezoids (Fig. 3). In fact, even those who stated that there was no rule drew trapezoid correctly.

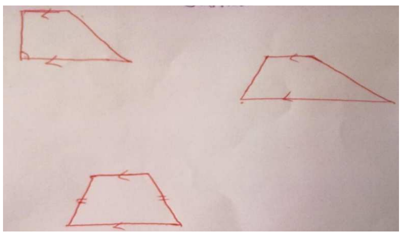

Figure 3. *Middle school Mathematics teacher candidates' drawings of trapezoid.*

The drawings of trapezoid are generally correct. This result is consistent with the previous research [3]. Fujita and Jones [3] found that teacher candidates drew some geometric shapes but, they were not able to provide their definitions. In the current study, many of the teacher candidates were not able to define trapezoid correctly and some even said that it was a quadrilateral drawn with no rules; but it is striking to see that even they drew the two sides parallel to each other. This inconsistency between definitions and drawings indicates that some of the teacher candidates who participated in the study perceived trapezoid as a whole and they placed it in their minds as such. The fact that they were not able to provide a definition or gave the wrong definition but drew the figure correctly shows that they cannot make a distinction between the properties of the shape.

4. Conclusions

The trapezoid definitions and drawings of the participants provided us with the opportunity to determine their concept images regarding trapezoid. The findings of the current study are consistent with previous literatures suggesting that many of the definitions acquired were personal rather than formal. Both critical and non-critical properties of trapezoid were used in these definitions by the participants. It was also determined in some studies that individuals used non-critical properties in non-formal and wrong definitions and that they created some misconceptions by making excessive generalizations [11, 15-17], which was observed in this study as well.

The definition of a trapezoid taken from Mathematics curriculum and course books in Turkey is 'a quadrilateral with opposite sides that are parallel to each other' and this might have caused the participants to create an inaccurate concept image regarding a trapezoid. This definition and prototype drawings might have created the perception that trapezoids should have two non-parallel sides and, indeed, they should not be parallel at all. In addition, it was also determined that some participants developed the concept image indicating that the sides and angles should be completely different. Actually, this was never a point of discussion as part of the properties and definition of a trapezoid in the course books. However, these properties were mentioned for isosceles and right trapezoids. Although angle properties are listed for special quadrilaterals such as square, rectangle and rhombus; this is not the case for trapezoids. In fact, trapezoids are generally excluded by researchers while forming inclusion relations. By the same reasoning, it can be stated that individuals created a personal perception concerning unmentioned angles and sides and that they also created an image which was affected mostly by the shape. In addition, since the Turkish word 'yamuk' meaning trapezoid also means 'oblique', it can be said that the perceptions of people are affected by the Turkish denomination. The definitions of some participants such as "all crooked" or "has no rules" are an indication of this. Although this study did not take in to account language aspects in its theoretical framework, it is said that the real life meaning of a geometric concept influences students' concept images. According to Leung and Park [20], geometric figures with names that resemble everyday life objects are meant to help students' understanding. However, in this study, the meaning of trapezoid in Turkish became a barrier to creating correct concept images.

All these results provides us with clues regarding the teaching of trapezoid concept along with the teaching of other quadrilaterals. For example, the case of the two sides should also be discussed as mentioned in the definitions of trapezoid, such as "it has two opposite sides that are parallel". Various visual examples could be used to discuss how the other two sides should be. In addition, the angles should also be examined from many different trapezoid drawings. Of course, the position of the trapezoid among other quadrilaterals should be established as well.

The fact that this study was applied on two different participant groups provided the means to determine individuals with different concept images of a trapezoid. Various levels emerge when all the definitions and drawings of the participants are taken into account. These levels are mostly in accordance with the levels of geometric thinking of van Hiele. These levels differ as per age but it was observed that even some teacher candidates had misconceptions. This conceptualization is consistent with Nakahara's [18] argument. Nakahara argued that individuals learn about concepts as a result of the teaching environment, instead of maturation with age. Learners' conceptual levels for a trapezoid can be determined as below:

1st Level: Learner does not have any knowledge of trapezoid, ha an image based on its everyday meaning.

2nd Level: Learner has an image as a geometric shape; however, the concept image is related to non-critical properties. There is excessive generalization of these properties.

3rd Level: Learner has an image as a geometric shape, has a concept image based on parallelness. However, there is excessive generalization regarding non-critical properties.

4th Level: Learner can make a correct formal definition and draw a correct shape. Naturally, learner is aware of the critical properties, does not excessively generalize non-critical properties, and can make analytical decisions.

The aforementioned levels were put forth using the data acquired from the definitions and drawings of middle grade students and middle school Mathematics teacher candidates. The conceptual levels of individuals about trapezoids can be shaped with different research environments. For example, a study on how an individual structures the inclusion relation of a trapezoid with other quadrilaterals may lead to more in-depth and detailed results.

References

[1] Baykul, Y. (1999). *İlköğretimde Matematik Öğretimi.* Ankara: Anıyayıncılık.

[2] Duatepe, A. (2000). *An investigation of the relationship between van hiele geometric level of thinking and demographic variable for pre-service elementary school teacher*. Unpublished master thesis, ODTÜ, Turkey.

[3] Fujita, T. & Jones K. (2007). Learners' understanding of the definitions and hierarchical classification of quadrilaterals: Towards a theoretical framing. *Research in Mathematics Education*. *9* (1&2): 3–20.

[4] Van Hiele, P.M. (1999). Developing geometric thinking through activities that begin with play. *Teaching Children Mathematics*. *5*(6): 310-316.

[5] Tall, D. & Vinner, S. (1981). Concept image and concept definition in mathematics with particular reference to limits and continuity. *Educational Studies in Mathematics, 12*(2): 151-16.

[6] Fischbein, E. (1993). The theory of figural concepts. *Educational Studies in Mathematics*. *24* (2): 139–162.

[7] Hershkowitz, R. (1990). Psychological aspects of learning geometry. In P. Nesher & J. Kilpatrick (Eds.), *Mathematics and cognition* (pp. 70–95). Cambridge: Cambridge University Press.

[8] Monaghan, F. (2000). What difference does it make? Children's views of the differences between some quadrilaterals. *Educational Studies in Mathematics*, *42* (2): 179-196.

[9] Heinze, A. & Ossietzky, C. (2002). "…Because a square is not a rectangle" students' knowledge of simple geometrical concepts when starting to learn proof. In A. Cockburn & E. Nardi (Eds.), *Proceedings of The 26th Conference of the International Group for the Psychology of Mathematics Education*. *3*: 81–88.

[10] Vighi, P. (2003). The triangle as a mathematical object. *European Research in Mathematics Education III Congress Proceedings. Bellaria, Italy, 28 Februrary-3 March:* 1-10.

[11] Fujita, T. (2012). Learners' level of understanding of inclusion relations of quadrilaterals and prototype phenomenon. *The Journal of Mathematical Behavior. 31:* 60–72.

[12] Fujita, T. & Jones, K. (2006). Primary trainee teachers' understanding of basic geometrical figures in Scotland. In J. Novotana, H. Moraova, K. Magdelena & N. Stehlikova (Eds.), *Proceedings of The 30th Conference of the International Group for the Psychology of Mathematics Education. 3:* 14–21.

[13] Okazaki, M. & Fujita, T. (2007). Prototype phenomena and common cognitive paths in the understanding of the ınclusion relations between quadrilaterals in Japan and Scotland. In H. Woo, K. Park & D. Seo (Eds.), *Proceedings of The 31st Conference of the Internatıonal Group for the Psychology of Mathematics Education. 4*: 41–48.

[14] Erez, M. M. & Yerushalmy, M. (2006). "If you can turn a rectangle into a square, you can turn a square into a rectangle ..." young students experience. *International Journal of Computers for Mathematical Learning. 11*: 271–299.

[15] Türnüklü, E., Alaylı, F.G. & Akkaş, E.N. (2013). Investigation of prospective primary mathematics teachers' perceptions and images for quadrilaterals. *Educational Sciences: Theory & Practice. 13*(2): 1213-1232.

[16] Türnüklü, E., Akkaş, E.N. & Alaylı, F.G. (2013). Mathematics teachers' perceptions of quadrilaterals and understanding the inclusion relations. In B.Ubuz, Ç.Haser & M.A.Mariotti (Eds.), *Proceedings of 8th Congress of the European Society for Research in Mathematics Education* Antalya, Türkiye, 6-10 February: 705–714.

[17] De Villiers, M. (1998). To Teach Definitions in Geometry or Teach to Define?. In A.Oliverve & K. Newstead (Eds.), *Proceedings of The 22nd Conference of the International Group for the Psychology of Mathematics Education.* 2: 248–255.

[18] Nakahara, T. (1995). Children's construction process of the concepts of basic quadrilaterals in Japan. In A.Oliver & K. Newstead (Eds.), *Proceedings of the 19th Conference of the International Group for the Psychology of Mathematics Education. 3:* 27–34.

[19] Patton, Q.M. (1987). *How to Use Qualitative Methods in Evaluation*. London: Sage Pub.

[20] Leung, F. & Park, K. (2009). The influence of language on the conception of geometric figures. In M. Tzekaki, M. Kaldrimidou, H. Sakonidis (Eds), *Proceedings of The 33th Conference of the International Group for the Psychology of Mathematics Education*. *1*: 81–88.

[21] Matsuo, N. (2000). States of understanding relations among concepts of geometric figures: considered from the aspect of concept image and concept definition. In T. Nakahara & M. Koyama (Eds.), *Proceedings of The 24th Conference of the International Group for the Psychology of Mathematics Education*. *3*: 271–278.

[22] Burger, W.F. & Shaughnessy, J.M. (1986). Characterizing the van Hiele levels of development in geometry. *Journal for Research in Mathematics Education. 17* (1): 31–48.