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Recent Results on Sliding Collapse for Masonry Structures Under Static Load Test

Fernando Magdalena[1], Antonio Aznar[2], Juan F. de la Torre[2], José I. Hernando[2]

[1]Department of Building Construction, Edification School, Technical University, Madrid, Spain

[2]Department of Building Structures, Architecture School, Technical University, Madrid, Spain

Email address:

joseignacio.hernando@upm.es (J. I. Hernando)

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Abstract: This paper presents experimental test on sliding collapse. An array of up to fifty three tests on dry masonry specimens has been performed. Each specimen is subjected only to self-weight and to a horizontal load, whose position is chosen from a predefined set of three different locations. For the rest of properties, all specimens are totally equal. For each of the three locations, two sub-arrays of ten specimens and one of thirty-three have been tested. For each specimen, pieces layout is randomly performed so that imperfections randomly spread throughout the specimen as well. The main aim of this work is the comparison of these static tests with the results obtained from several commonly used numerical methods, especially with the ones retrieved under the non-Standard Limit Analysis. This paper shows that when the contribution of mortar to the strength of the structure cannot be taken into account and collapse by sliding occurs, the solution for collapse load and mechanism can be multiple. Hence, and since the solution is not necessarily unique, we should carefully consider the limits under which all methods finding a unique solution can be used.

Keywords: Dry Stone Masonry Walls, Sliding Friction, Static Tests, Numerical Methods, Non-associative Limit Analysis

1. Introduction

Safety assessment of historic masonry structures is still a controversial matter nowadays [1]. In the steps prior to collapse, the field for the actual stress state cannot easily be obtained and, in most cases, actual boundary conditions are far to be known, at least not in an accurate enough description to be posed as certain. As a general property for masonry, its mechanical behaviour is quite heterogeneous and anisotropic, with high compressive strength. On the opposite, low capacity and brittle cracking appear when it acts in tension. As a matter of fact, it is certain that global failure for masonry will occur long after cracking has started at any region of the body. Furthermore, global failure is often due to instability when yield conditions appear at a certain number of points, which means that the body is no longer a structure but has turned out into a mechanism, even when compressive stresses could still be under their limit value. What described before assumes that no sliding movements between parts of the body develop, thus leading to what several authors [2-5] have proposed as an excellent simplified tool for these cases, which is the Limit Analysis theory, that has in fact proved efficient enough for such purposes. Other authors [6-8] during the first steps of plastic theory, established the equivalence of Limit Analysis theory with Linear Programming, and later some others [9, 10] have used such procedures for dry masonry, which allows obtain the actual ultimate load factor at the very start of collapse.

But when collapse develops under sliding movements [11-13] standard Limit Analysis theorems are no longer valid and the load factor for the collapse onset is no longer linked to a compulsory uniqueness. Hence many researchers have tried to find the solution for the minimal load factor related to the onset of collapse. This is a very difficult problem to be solved and in its simplest formulation is presented under a Linear Complementarity Problem one [14] and as for its computational complexity it is NP-hard complex [15-17]. In any case, there is no method, for the time being, that could guarantee the obtaining of such a minimal load factor or prove the inexistence of it with full certainty.

Developing a different approach, some researchers have pointed out that this minimum load factor solution is perhaps too conservative [4, 18], and some results recently obtained by means of the Monte Carlo simulation [19] seem to make possible the conjecture that whenever a solution can be numerous, minimum and maximum values of it are not the most likely ones. Therefore, in order to bring some clarity on this question it has been considered as necessary the retrieval of some test results.

2. Tests and Specimens Description

Our will is to study the effect of sliding in the collapse of masonry, which means that our tests will be carried out on specimens only subjected to self-weight composed with a relatively high horizontal external force, as a simulation of the mechanical behaviour of upper parts of buttresses, the upper bands of masonry shear walls under thrust of roofs or transverse walls, or the anchorage zones for tension ties.

Quite a few tests have been performed on arch shaped structures [20-22], with or without mortar in the inner joints. Specimens were both scaled and full size ones, and even in a few cases, some real arch bridges were led to collapse.

As for shear walls [23], some tests have been developed including shear resistance of mortar in the test results, which is an approach suitable to new structures. Tests on dry masonry walls are more scarce [24-26]. However, "The testing of dry stone masonry has been used in the past as a way to derive useful references for the study of historical masonry. In fact, many approaches that aim to analyze the lateral strength capacity of historical unreinforced masonry walls recognize that the tensile strength along the joints between the stone elements is negligible either because the mortar is absent or because its tensile strength and bond is usually low and cannot be quantified in a reliable way. It is also quite common the case of pre-existing cracks...", [27]. In any case, neglecting mortar strength is a safe hypothesis.

Almost all aforementioned tests analyze global effects by placing the specimens on an inclined plane, making therefore collapse develop dynamically. In our case, the test procedure provides a quasi-static collapse, which is of main importance to fulfil our will to especially look at the local effects of in-plane loads.

Our test is designed so that collapse of the dry masonry specimen is reached by pure sliding in a quasi-static manner. The reason for this is a twofold one. On the one hand this behaviour is the simplest one and the easiest one to be interpreted. Whenever there appears a rocking and sliding mixed collapse configuration, experimental results are much more difficult to be interpreted [28], apart from being a dynamic collapse configuration, which is out of our current scope. On the second hand, when considering mixed rocking and sliding collapse, results show that the minimum value of them is always higher than in our case, since sliding is the only cause for their variation.

Being given that our test has been designed to reach collapse by pure sliding, the corresponding collapse mechanism is verified by means of assembled brick groups that slide the yield line long. Such a line is defined by contact surfaces on which the friction yield constraint has reached its limit (Figure 1).

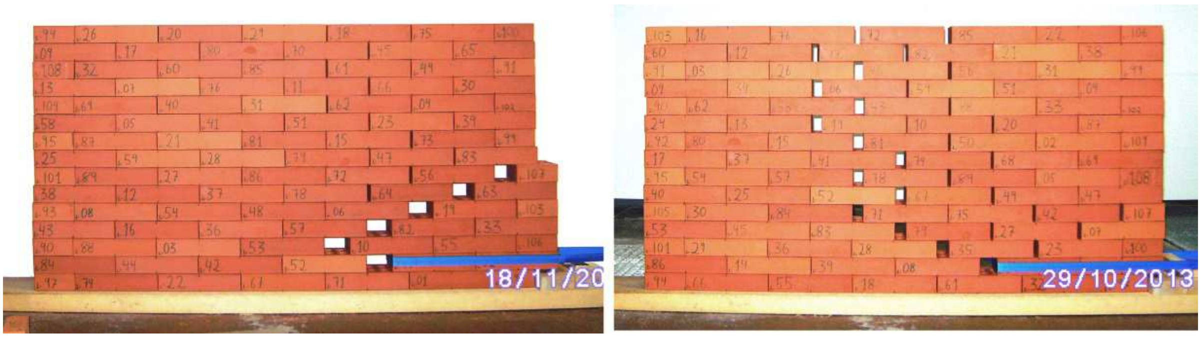

Figure 1. *Two different yield lines for a same horizontal load location.*

3. Experimental Tests

An experimental sliding tests campaign has been carried out in order to compare experimental results retrieved from it with the theoretical results that by means of several numerical methods (FEM, Limit Analysis,...) have been calculated. All tests have been performed at the Building Structures Department laboratory, ETSAM, Technical University of Madrid.

3.1. Materials

An array of 53 contact sliding tests has been carried out on dry brickwork masonry specimens. Brick pieces were chosen as base material due to its geometrical consistency, the best to be expected for this type of assemblies. Brick pieces are ceramic clay based ones, HD R-20 type, whose nominal dimensions are 240 mm in length, 115 mm in width and 50 mm in height. Its geometrical features can be found on Table 1, where %Min stands for the difference in percentage between minimum and mean values. Min stands for the minimum value, Max for its maximum, SD for the standard deviation, CV for the coefficient of variation (standard deviation / mean) and %Max for the difference in percentage between the maximum and mean values.

Table 1. *Features of the brick HD R-20 used in experimental tests.*

	%Min	Min	Mean	Max	%Max	SD	CV%
length mm.	0.42	239	240	24.1	0.42	0.50	0.20
width mm.	0.35	115	115.4	116.0	0.51	0.51	0.43
height mm.	1.05	49.8	50.3	50.8	0.94	0.21	0.41
weight N	0.63	16.42	16.52	16.76	1.43	0.07	0.42
friction coefficient	8.17	0.5	0.54	0.58	6.64	0.03	5.56

Friction coefficients have been obtained by means of the same methods and apparatuses used in the tests described below, and under those same conditions.

Brickwork specimens are 15 rows in height for a total number of 90 pieces (82 entire bricks and 8 of them split in two halves. All bricks are numbered and their layout has been performed by following the Fisher-Yates-Durstenfeld shuffle [29] method to ensure a random distribution of them in each specimen. Figure 2 shows one of these specimens.

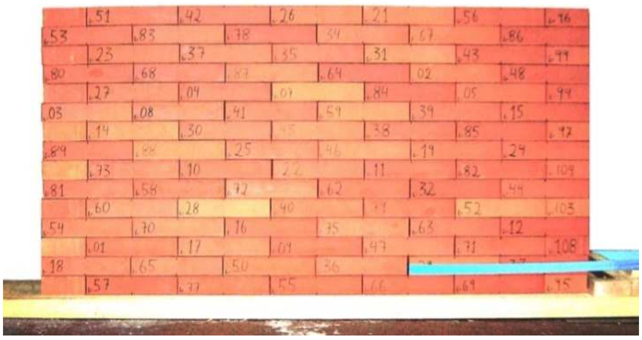

Figure 2. *Photograph of one tested specimen,*

Time for every specimen erection (recorded from the very beginning of assembly to the test start) has been controlled to make sure that the bricks' surfaces offer similar conditions in all tests.

3.2. Test Description

All tests have been performed under controlled environmental conditions both for temperature (20°C+-2) and for relative humidity (R. H. 40%+-10). Furthermore, before the manufacturing process, relative humidity and temperature of each brick have been measured by means of a Hydromette HT-85-T hygrometer. Both values have been recorded together with each test result.

As explained before, tests have been performed for three different locations of the horizontal force. These three different configurations are represented on Figure 3. As it can be found on it, configuration "A" corresponds to a position for the horizontal load far from the constrained edge of the specimen, "B" to a position of the force at the mid-width of the specimen, and "C-D" to a load applied close to the constrained edge.

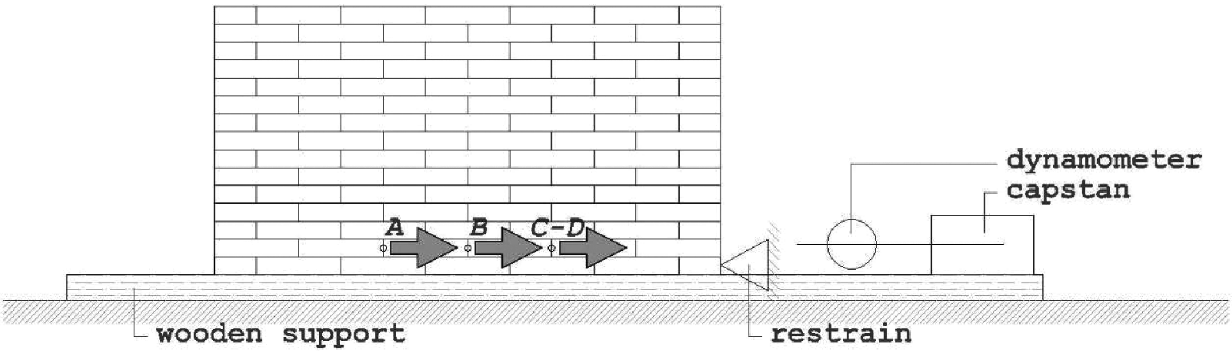

Figure 3. *Test layout.*

Of all tests, thirty three of them have been carried out for the "C-D" configuration. For "A" and "B" configurations ten been performed for each. The reason for a bigger number of "C-D" tests stands on the fact that in this case edge conditions have brought more interesting results and a higher variability for them.

The test procedure consists in applying an increasing horizontal load at the second lower row of the specimen, being displacements of the first row restricted at the edge. Load is introduced on the specimen by means of a Imnasa-350 capstan, and with the help of a Sauter FA-500 dynamometer load data have been measured. Once load starts to be applied, as soon as sliding opposition intensity decreases the horizontal load is forced to vanish. Then the gap appearing on the point of the brick on which load was applied is measured and recorded. Finally, this same process is repeated several times, and the yield line progression data are retrieved and recorded.

4. Test Results

For the sake of simplicity and legibility results are shown on tables and graphics.

As a start, ten results have been retrieved from each of three different tests, corresponding to the onset of collapse in cases A, B and C. Values, in N, are shown on Table 2.

Table 2. *Results of tests A, B and C (force in N).*

A	713.8	718.5	723.6	723.7	728.6	738.7	742.8	755.5	755.6	783.7
B	459.6	549.3	569.3	579.4	597.7	604.5	649.4	654.5	679.4	689.4
C	309.5	354.3	389.3	398.9	404.3	404.3	469.2	479.0	508.6	509.1

Then, results on case C have been extended up to thirty three different results, being these last labelled as D. Results in N are shown on Table 3. Bold characters represent values for case C.

Table 3. *Results of test D: C+23 (force in N).*

309.5	324.1	329.1	334.4	339.1	339.4	354.3	359.1	364.4	379.0	384.2
388.9	389.0	389.0	389.3	398.9	398.9	404.3	404.3	418.9	424.2	424.2
433.8	433.8	453.8	459.2	463.8	469.2	474.2	479.0	508.6	509.1	533.6

Using the same abbreviations as shown on Table 1, some descriptive statistics for values from the four cases are shown on Table 4.

Table 4. *Statistics of tests A, B, C and D.*

	%Min	Min	Mean	Max	%Max	SD	CV%
A (10)	3.34	713.8	738.5	783.7	6.13	21.58	2.77
B (10)	23.81	459.6	603.3	689.4	14.28	69.34	10.91
C (10)	26.77	309.5	422.7	509.1	20.45	66.69	14.97
D (33)	24.20	309.5	408.3	533.6	30.68	58.03	13.99

On Figure 4 frequency distribution and best-fit normal distribution for cases A, B and D are shown.

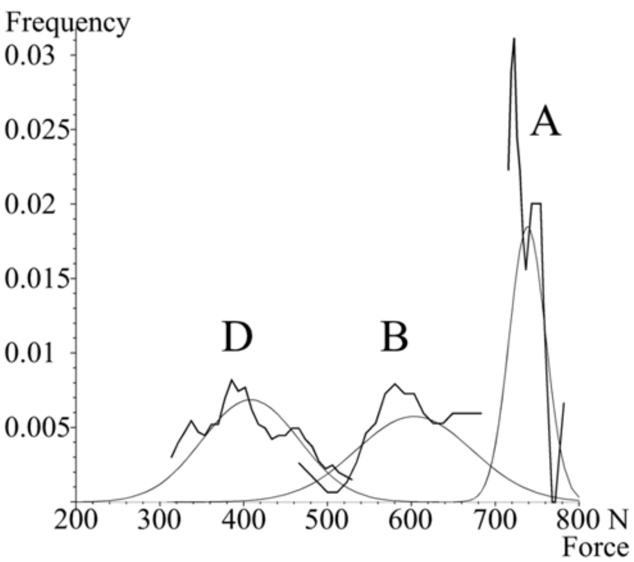

Figure 4. *Frequencies and the best-fit normal distributions.*

Because the sample size is too small, the implementation of several goodness of fit tests and normality tests does not allow to conclude which is the best-fit distribution for results on cases A, B and C. For case D, a normal distribution (gaussian) seems to be the best-fit option.

However, despite the underlying frequencies distribution is unknown beforehand and the sample size is quite restricted, a most likely range of values for the most relevant statistic parameters has to be found. Hence, as a conclusion, confidence intervals are obtained only for case D, for the 5% percentile and other parameters, for a 99% confidence level after using resampling methods [30-33] applied on the best-fit normal distribution (parametric bootstrap). Results for case D are shown on Table 5.

Table 5. *Confidence intervals for case D.*

Confidence intervals at 99% confidence level	5% percentile	Mean	95% percentile
Case D	285-347	383-435	460-538

Shown values are approximate since, in any case, bootstrap is a simulation method.

5. Comparison with Some Numerical Methods

For better understanding results retrieved from some numerical methods and from our tests are compared as shown on two different tables. The numerical methods here used are:

Min nSLA: minimum value of non-Standard Limit Analysis (without dilatancy), as described in [12-14].

Min USD: minimum value of Uniform Stress Distribution, following Rankine theory about frictional tenacity [31].

FEM: Finite Element Method, as described below.

Max SLA: maximum value of Standard Limit Analysis, obtained by Limit Analysis by Linear Programming [9,10].

This previous selection does not involve any preliminary judgement on the suitability these methods may bring to our purposes.

5.1. Brief Remarks on the Chosen Methods

Although the simplest formulation for the non-standard Limit Analysis is posed as a Linear Complementarity Problem and, therefore, it stands as a great difficulty one, in our case, once a yield line has been selected (Figure 5 a) and the corresponding limit conditions are substituted in the original problem, it turns out a linear problem and the global minimum can be obtained by means of linear programming. Figure 5 a and Figure 6 b refer to a couple of yield line plus collapse mechanism for the global minimum of the horizontal load. Figure 6 a refers to the equilibrium of the remains of the specimen once the slid bricks have been removed and, hence, lacking any contact force linked to the previous yield line. Figure 6 c refer to a collapse mechanism near maximum.

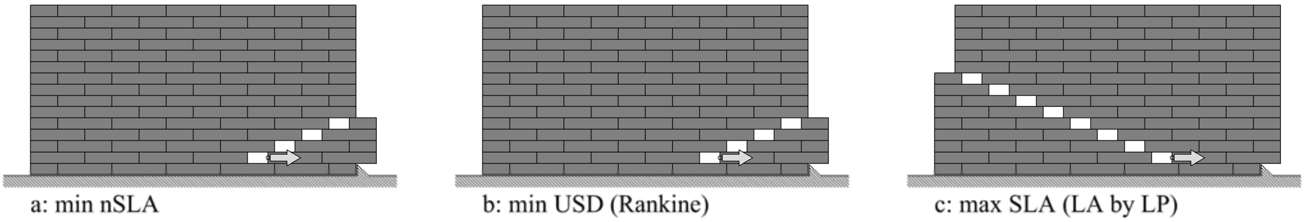

Figure 5. *Yield lines and sliding mechanisms corresponding to the three first methods.*

In the Rankine model for frictional tenacity, stresses only depend on the depth the point presents within the wall. Thus, the minimum load factor can easily be obtained by selecting the same yield line (Figure 5 b) and one uniform stress distribution along every horizontal joint.

Figure 6. *a- Post-collapse equilibrium, b- Mechanism of minimum, c- Mechanism near maximum.*

The maximum value for the load or for the load factor corresponding to Standard Limit Analysis can be obtained by means of the Limit Analysis problem formulation as a Linear Program. Such an approach guarantees that the value thus obtained is an optimum that is unique. Nevertheless, there can be different solutions related to this unique value and, therefore, we can only assure that the yield line shown is only one among the possible ones (Figure 5 c). Although apparently unlikely, yield lines running backwards, like the one shown, have actually occurred in our tests.

None of the three previous methods uses in an explicit manner the displacements that appear prior to collapse.

A wider explanation for the Finite Element Method (FEM) is required. FEM is widely accepted as a standard tool for analysis in continuous bodies, but it must be remarked that discontinuity in the current case is remarkable. Specimens are modelled as an assembly of continuous bodies under one-sided contact. Analysis has been carried out on ANSYS commercial software. Bricks have been modelled as PLANE42 elements, and contact conditions by means of TARGE169 and CONTA171 elements. The actual contact surface is only 20% out of the total contact surface due to several factors: bricks are multi-hollow extruded ones, being the void surface guaranteed by the manufacturer 40-45% of the total one. In addition, their layout is interlaced. Two different analyses have been carried out to get the aforementioned percentage: one has taken 0'20x115 mm as the actual width of the brick and the other has considered the total width of it, 115 mm. Stresses appear different in both analyses but the maxima of the maximum horizontal force are equal. As a first approach, these results make unnecessary a more complex analysis. The rest of mechanical properties used are: Young's modulus E=20000 N/mm2, Poisson´s ratio ν=0.25 (both values brought by brick manufacturer) and the friction coefficient μ=0.54. Numerical results are shown on Table 6.

Table 6. *Maximum force in N obtained for the different FEM models. Results for cases A, B and C, for two different elements' widths and three different discretizations of them are shown.*

	Thickness=115mm			**Thickness=115*0.2mm**			
		Element size			Element size		Rankine
	40mm	10mm	2.5mm	40mm	10mm	2.5mm	
A	705.21	705.91	705.93	705.21	705.91	705.93	705
B	656.16	657.18	658.94	656.22	656.65	658.72	656
C	454.68	452.77	454.22	454.68	452.77	454.22	455

Loads have been applied in two steps. First, self-weight is the only acting load. In a second step the horizontal load is increased by successive load substeps until the algorithm fails convergence. These results should be cautiously taken, for displacements are elastic and absolutely different from the post-collapse (inelastic) ones obtained for gaps in the experimental tests. Figure 7 shows the displacements and the principal stresses for a substep previous to convergence failure, showing a numerical instability when collapse is about to start.

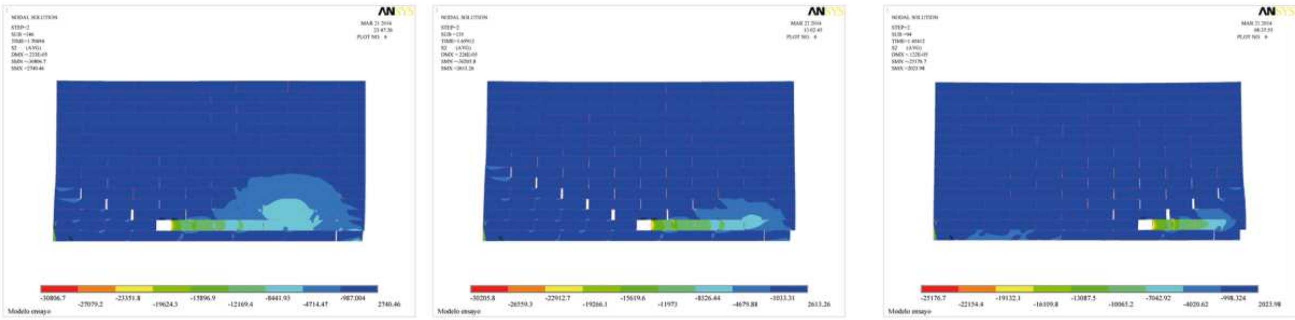

Figure 7. *FEM results in cases A, B and C-D.*

It should be noted that the maximum stresses occur at the horizontal force location point and, even for this point, stresses are much lower than the minimum strength the manufacturer guarantees for bricks. In addition, elastic displacements are much lower than geometrical imperfections of the base material. As a result, its behaviour in the elastic rank does not seem the main factor to be considered.

5.2. Comparison of Results

Table 7 and Table 8 show the results for the four different numerical methods used to estimate the minimum load to happen at the collapse onset, both in absolute values and in percentage related to the minimum value retrieved from tests results.

Table 7. *Results from numerical methods and test statistics (force in N).*

Collapse load (N)	Min nSLA	Min USD	FEM	Max SLA	Min test	Mean test	Max test
A	294	705	706.3	705	713.8	738.5	783.7
B	94	656	658.6	656	459.6	603.3	689.4
D	45	455	454.8	589	309.5	408.3	533.6

Table 8. *Results compared in percentage to the minima of tests.*

% respect to min. test	Min nSLA	Min USD	FEM	Max SLA	Min test	Mean test	Max test
A	41.19	98.77	98.95	98.77	100	103.46	109.79
B	20.45	142.73	143.30	142.73	100	131.27	150
D	14.54	147.01	146.95	190.31	100	131.92	172.41

Results overestimating the minimum collapse load are shown in bold characters and those underestimating it in *italic* ones.

On Table 9, for case D, results are referred to lower and upper bounds for the 5% percentile confidence interval respectively. The latter is best shown on Figure 8.

Table 9. *Results from D compared to test statistics and confidence intervals.*

	Min nSLA	Conf. Int. 5%	Min test	Mean test	Conf. Int. Mean	Min USD	FEM	Conf. Int. 95%	Max test	Max SLA
D (N)	45	285-347	310	408	383-435	455	455	460-538	534	589
D (%)	13		89	118		131	131		154	170
	16		109	143		160	160		187	207

On the left side of Figure 8 values and functions for tests having a zero initial gap are shown: a histogram scaled to 0'50, the frequency distribution, the best-fit normal distribution, maximum and minimum values retrieved from tests and the 5% percentile confidence interval of those values for a 99% confidence level. All values are compared with those retrieved from numerical methods. On the right side of that same graphic initial gap vs force points are represented. These points correspond to such a couple of values obtained for all tests. Those belonging to the same test are linked by straight lines.

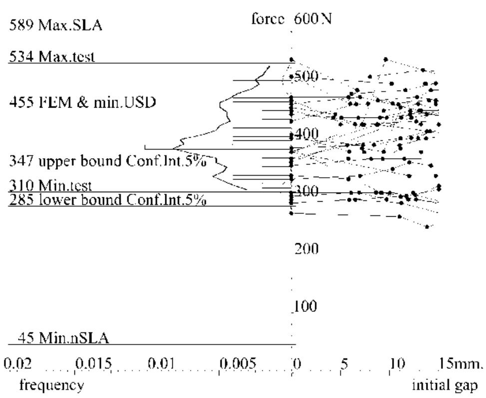

Figure 8. *Comparison of numerical and experimental results.*

6. Discussion

Obviously, a short number of tests does not allow to give an affirmative response to the questions posed, but it does allow a negative one, because a few counterexamples are enough to underline the limits for a model or a theory.

The solution for the collapse onset, and hence its load factor, is not always unique.

Dispersion in results sometimes cannot be explained through dispersion in data. The coefficient of variation is only 5'56% for friction, and 0'42% in weight, whereas, for collapse load we get 2.77% (A), 10.91% (B), 14.97% (C).

The cases in which behaviour could be qualified as more local, B and especially C-D, three of the four methods overestimate severely the collapse load, and the fourth one underestimates it extremely.

The maximum load factor obtained by applying Limit Analysis by Linear Programming does not guarantee in all cases to obtain the actual load factor for the collapse onset or at least a close value. Same results are found when the solution is obtained by applying the Rankine theory or the non-linear solution by FEM with contact conditions. Furthermore, sometimes these methods yield unsafe solutions.

On the other hand, the minimum load factor obtained by Non-Standard Limit Analysis, is not always the actual load factor of the collapse onset. In fact it never happened for the current tests, but results bring always a safe solution. Perhaps this last is excessively safe, especially when taking into account that the current case of pure-sliding collapse is much less favourable that the cases of mixed rocking-sliding collapse.

7. Conclusions

The main conclusion of this paper is that when the contribution of mortar to the strength of the structure cannot be taken into account and collapse by sliding occurs, the solution for collapse load and mechanism can be multiple. Hence, and since the solution is not necessarily unique, we should carefully consider the limits under which all methods finding a unique solution can be used.

Results obtained by some commonly used methods, like SLA and FEM, are sometimes not very close to the actual collapse loads. In addition, they do not always find safe solutions, at least in cases like the tested ones. Apparently, this is more important for the cases running under local sliding and further efforts on identifying such cases should be developed.

Answering about the collapse loads for cases presented in this paper, the search for the global minimum by non-Standard Limit Analysis does not seem to prove efficient either in terms of computational costs or for its accuracy in approximating the actual minimum. Furthermore, in cases including rocking as well, the likely experimental minimum should be placed even much farther from that global minimum.

To conclude, although sliding instability is not the main mode for collapse in masonry structures, a larger research in this field is required, both from the theoretical and the experimental points of view.

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