

On the Investigation of the Methods of Parameter Estimation of the Best Probability Model for Wind Speed Data

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Abstract: The focus of this paper is to estimate parameters of the best distribution for modelling wind speed data, real-life data sets of wind speed of Maiduguri, the biggest city in the North Eastern, Nigeria were adopted for application purposes. Six (6) probability density functions, specifically, Weibull, Gamma, Lognormal, Pareto, Burr and Log-Logistic are considered for modelling the wind speed data. In selecting the model of best fit for the variability of the wind speed data, five (5) methods of estimating parameter, such as; Maximum Likelihood Estimation (MLE), Matching Quantiles Estimation (MQE), The Cramer-von Mises Minimum Distance Estimators (CvM), Anderson-Darling Minimum Distance Estimation and Kolmogorov-Smirnov Minimum Distance Estimation (K-S)) were further applied to obtain the best estimates for the best model among compared ones. We discovered in our investigation that Weibull distribution best fitted the wind data per Goodness-of-fit tests, since it has the smallest p-value for K-S (0.03179314), CvM (0.03137888) and AD (0.23725978) revealing the curve is fairly close to our data and the maximum likelihood estimators with the smallest AIC (972.7990) and BIC (980.3105) estimates for Weibull parameters, proved to be the best as compared with other methods of estimation.

Keywords: Wind Energy, Probability Distribution Models, Maximum Likelihood Estimators, Matching Quantiles Estimation, Goodness of Fit-Tests

1. Introduction

Wind energy is obtained from wind; it is now increasingly relevant and useful as a renewable source of energy. “Salameh and Nandu describe wind energy as a valuable form of renewable energy that gives clean and inexhaustible source [20]”. Amaya-Martínez *et al.*, [1] used Weibull, Rayleigh, Gamma, and Lognormal distributions to model and select the probability density function that best fit the variability of the wind speed data. “Lo Brano *et al.* established that Gamma, Weibull, Lognormal, Inverse Gaussian, Pearson type 5 and Burr probability functions best represented by wind data behaviour in Palermo urban, Italy [10]”. In the literature, Weibull, Gamma, Log-normal, Rayleigh, Inverse Gaussian and Maximum Entropy Principle have been used for practical studies related to the wind

energy modelling with positive results, for example: Li and Li [7]; Lun and Lan [13]; Toure [28]; Safari and Gasore [19].

In view of the fact that there are several probability functions of exponential family that can be used to study wind behaviour, it is, therefore, imperative to investigate the best probability function that best fit wind speed. “Amaya-Martínez *et al.* opined that more than one goodness of fit test is commonly used to guarantee the selection of the probability function that best models the wind speed in a specific place [1]”. Zhou *et al.* [31] used several Goodness-of-fit tests, such as: Chi square test, Kolmogorov-Smirnov test, Log likelihood and criteria such as Index of Agreement, Coefficient of Determination, Root Mean Square Error to select the best probability density function. When the best fitted distribution has been established it is necessary to determine the best estimation method from several available

methods.

There are various methods available in works of literature to estimate the parameters of exponential family distributions. Louzada *et al.* [11] compared different methods of estimation, such as: moments, modified moments, L-moments, maximum likelihood estimator, least square, percentile, weighted least square, the maximum product of spacing, and minimum distance estimator to determine the best fit for Weibull, Gamma and Generalized exponential distributions. Kaoga *et al.* [5] used six numerical methods, namely, Modified Maximum Likelihood Estimator, Energy Pattern Factor, Graphics, Maximum Likelihood Estimator, Moment to estimate parameters of the Weibull distribution. Yilmaz and Celik [30] compared ten probability density functions for wind data, in determining appropriate probability function, Goodness-of-fit tests and fitted graphics were applied.

The fact that countries are becoming interested in green sources of energy to boost their economy and increase the standard of living to preserve the environments, and with the view that several publications have been made on the best probability models that most appropriately fit wind data. Obanla *et al.* [16] concluded that the Weibull model fits wind data best among the three models considered. However, we have also extended our research by increasing the number of models for adequate comparison. After which, five (5) different methods of parameter estimations (maximum likelihood, quantile matching, Kolmogorov Smirnov minimum distance, Anderson Darling minimum distance and Cramer von Matching minimum distance) were adopted to best estimate the parameters of Weibull, Gamma, Lognormal, Burr, Log-logistic and Pareto distributions. Further to this, we have been able to judge the best method of estimation of the best model using three Goodness-of-fit tests. This paper is organized as follows: In Section 2, the methodology used to model the wind data is described. The data analysis and discussions are presented in Section 3. Lastly, the conclusion is presented in Section 4.

2. Methodology

In this section, consideration was given to real data sets of wind speed of Maiduguri, the biggest city in North Eastern Nigeria. Secondary data employed in this study were obtained from office of the Nigerian Meteorological Agency (NiMET) at hub heights of ten (10) meters for twenty nine (29) months period. We used six probability distributions (Weibull, Gamma, Lognormal, Burr, Log-logistic and Pareto distributions) to model the wind data, the six distributions were selected on the basis that from past researches on modeling wind data they were the most commonly applied. Also, the distributions are selected due to their easiness, preference, and acceptance in literature for frequency analysis of random occurrences Rahman *et al.* [17], Li *et al.* [8].

2.1. Probability Distribution Models

Probability functions are mathematical functions that

describe the likely behaviour of a data set. The Data set under study can correspond to the behaviour of a random variable that is continuous in time. Given that wind speed data is a continuous random variable, it is suitable to employ probability density functions to describe it.

2.1.1. Weibull Distribution

Weibull distribution is a distribution used to model the reliability of several diverse forms of physical systems. The distribution has two parameters, namely: shape and scale. Combinations of different values of the two parameters can lead to models with either.

(a) increasing failure rates over time, decreasing failure rates over time, or

(b) constant failure rates over time.

Weibull distribution is flexible and can acquire the features of other forms of probability distributions depending on the value of the shape parameter β . Li *et al.* [8].

Definition

A continuous random variable X is said to have a Weibull distribution with scale and shape parameters denoted by α and β respectively if the probability density function (pdf) of X is expressed as:

$$f(x; \alpha, \beta) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \quad (1)$$

Where, $x \geq 0, \alpha > 0, \beta > 0$, we write $X \sim \text{Weibull}(\alpha, \beta)$.

The cumulative density function Weibull distribution is given by:

$$F(x; \alpha, \beta) = 1 - \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \quad (2)$$

The Weibull distribution with two parameters can be expressed as a special case of the three parameter Weibull probability distribution where $\delta = 0$. When $\beta = 1$ Moeschberger [15]. In this research work we will be dealing with two-parameters Weibull distribution. Weibull distribution is mostly used for the analysis lifetime variables which is found to be useful in diverse field ranging from medical science to engineering Lawless [6].

2.1.2. Lognormal Distribution

There are occasions in which natural logarithm transformation are required to transform some observation to become normally distributed. The distribution attributed to natural logarithm transformation is known as the standard lognormal distribution, this distribution is used for modeling life and for reliability analysis.

Definition

Given that X is a random variable from a population with normal distribution having mean (μ) and variance (σ^2). Transformation $X = \log_e Y$, that is, $Y = e^X$ to a new variable Y is known as lognormal distribution. The probability density function of Y becomes:

$$f(y, \mu, \sigma) = \frac{1}{y\sigma\sqrt{2\pi}} e^{\left[-\frac{1}{2}\left(\frac{\log_e(y)-\mu}{\sigma}\right)^2\right]} \quad (3)$$

for

$y \geq 0, -\infty < \mu < \infty$, and $\sigma > 0$

$$F(y, \mu, \sigma) = \Phi\left(\frac{\log_e(y) - \mu}{\sigma}\right) \quad (4)$$

It is observed that the probability function of Lognormal distribution in equation (3) was obtained by replacing X in the normal distribution pdf with $\log_e Y$. The random variable y appearing in denominator of lognormal pdf is as a result of the Jacobian of transformation $J = \frac{1}{y}$.

2.1.3. The Gamma Distribution

The Gamma distribution is a two-parameter continuous probability distribution with shape and scale parameters. Let X denote a random variable taking values in the interval $0 < x < \infty$ following the Gamma distribution. Then, the probability density function of the Gamma distribution with α (Alpha) as the shape and β (Beta) as the scale and can be expressed as:

$$f(x, \alpha, \beta) = \begin{cases} \frac{x^{\alpha-1} e^{-\frac{x}{\beta}}}{\Gamma(\alpha)\beta^\alpha} \\ 0 \text{ elsewhere} \end{cases} \quad (5)$$

where

$$\alpha > 0, \beta > 0, \text{ and } \Gamma(\alpha) > 0 \rightarrow \Gamma(\alpha) = \int_0^\infty \left(\frac{x}{\beta}\right)^{\alpha-1} e^{-\frac{x}{\beta}} dx \quad (6)$$

$$F(x, \alpha, \beta) = \frac{1}{\Gamma(\alpha)} \gamma\left(\alpha, \frac{x}{\beta}\right) \quad (7)$$

2.1.4. Burr Distribution

The Burr distribution is a continuous distribution with three parameters; it is a flexible distribution which is used to express the shapes of large range of distributions. Let X denote a random variable from a Burr distribution with two shape parameters denoted α respectively including a scale parameter β . The pdf is specified by

$$f(x, k, \alpha, \beta) = \begin{cases} \frac{\alpha k \left(\frac{x}{\beta}\right)^{\alpha-1}}{\beta \left[1 + \left(\frac{x}{\beta}\right)^\alpha\right]^{k+1}} \\ 0 \text{ elsewhere} \end{cases} \quad (8)$$

Where, $k > 0, \alpha > 0$ and $\beta > 0$

Its cdf is specified by

$$F(x, k, \alpha, \beta) = 1 - \left[1 + \left(\frac{x}{\beta}\right)^\alpha\right]^{-k} \quad (9)$$

The Burr distribution includes overlaps and has limiting case as many commonly used distributions such as Gamma, Lognormal and Log-logistic.

Weibull and Pareto type I distributions are asymptotic limiting cases of the Burr distribution. Due to different values of parameters of Burr distribution, it can fit a wide range of empirical data in various fields such as hydrology, meteorology, and finance, e.t.c.

2.1.5. The Pareto Distribution

The Pareto distribution was invented by an Italian civil engineer Vilfredo Pareto, the distribution is a power law

probability distribution that is employed to characterize geophysical, insurance, social, wealth and scientific and several forms of observations Amoroso [2]. A random variable X from a Pareto distribution has two parameter, namely shape and scale parameters denoted by α and β respectively. The pdf for Pareto type II which is used in this paper is given by

$$f(x, \alpha, \beta) = \begin{cases} \frac{\alpha \beta^\alpha}{[x + \beta]^{\alpha+1}} \\ 0 \text{ elsewhere} \end{cases} \quad (10)$$

where, $\alpha > 0, \beta > 0$

$$F(x, \alpha, \beta) = 1 - \left(\frac{\beta}{x + \beta}\right)^\alpha \quad (11)$$

2.1.6. The Log-logistic Distribution

The Log-Logistic distribution is a continuous probability distribution with shape and scale parameters denoted by α and β respectively. The distribution has similar shape with the Log-normal distribution and the Weibull distribution.

Definition

Let X denote a continuous random variable from a population with Log-Logistic distribution that has two parameters α and β . Its pdf is specified by

$$f(x, \alpha, \beta) = \begin{cases} \frac{\left(\frac{\beta}{\alpha}\right)\left(\frac{x}{\alpha}\right)^{\beta-1}}{\left[1 + \left(\frac{x}{\alpha}\right)^\beta\right]^2} \\ 0 \text{ elsewhere} \end{cases} \quad (12)$$

where, $\alpha > 0, \beta > 0$,

Its cdf specified by

$$F(x, \alpha, \beta) = \frac{1}{1 + \left(\frac{x}{\alpha}\right)^\beta} \quad (13)$$

2.2. Methods of Estimation

2.2.1. The Maximum Likelihood Estimator

The Maximum likelihood estimator (MLE) $\hat{\theta}$ of a parameter θ with probability density function is the value of $\hat{\theta}$ that maximizes the likelihood function. Let $f(x, \theta)$ be the probability density function (pdf) of the random variable X where θ is the parameter to be estimated. Supposed x_1, x_2, \dots, x_n are independent observations of a random variable X , the following function

$$L(X; \theta) = f(x_1; \theta), f(x_2; \theta), f(x_3; \theta), \dots, f(x_n; \theta) \\ = \prod_{i=1}^n f_i(x_i, \theta) \text{ is the likelihood function of } X \quad (14)$$

In this research the maximum likelihood estimator is used in computing parameters of Weibull, Gamma, Log-Normal, Burr, Log-Logistic and Pareto probability density functions. According to Seguro and Lambert [22] the maximum likelihood estimator is regarded as accurate and robust estimator. Parameters of various pdfs under study were computed using R software.

2.2.2. Matching Quantiles Estimation (MQE)

The Matching Quantiles Estimation (MQE) is an effective

technique that enables the determination of the linear combination of a set of random variables, that is, equivalents to the distribution of a target random variable. MQE have similarities with the ordinary least squares estimator (OLS) in estimating parameters of regression models. However, there are basic differences between MQE and OLS, while MQE is used for matching unconditional distribution functions, the OLS for estimating conditional mean functions, Sgouropoulos *et al.* [23].

Methodology of MQE

Let Y denote a random variable and $X=(X_1 \dots \dots X_p)'$ denote a collections of p random variables. The goal is to find a linear combination of

$$\beta'X = \beta_1 X_1 + \dots \dots \dots + \beta_p X_p \quad (15)$$

such that the distribution of (15) is equivalent to the distribution of Y , to look for β such that the following integrated squared difference of the two quantile functions is minimized by

$$\int_0^1 [Q_{Y(\alpha)} - Q_{\beta'X(\alpha)}]^2 d\alpha \quad (16)$$

where, $Q_{\xi(\alpha)}$ denotes the α th quantile of random variable ξ , that is, $P\{\xi \leq Q_{\xi(\alpha)}\} = \alpha \in [0,1]$

According Sgouropoulos *et al.* [23] it is easier to match the quantiles than that of matching the probability functions directly:

Consider the random samples $\{Y_1 \dots \dots Y_n\}$ and $\{X_1 \dots \dots X_n\}$ obtained respectively from the populations of Y and X .

Given that $Y_{(1)} \leq \dots \leq Y_{(n)}$ be order statistics of $Y_1 \dots \dots Y_n$. Subsequently, $Y_{(j)}$ is the j 'nth sample quantile.

To obtain the sample equivalent of the minimizer of equation (15), we define the estimator:

$$\hat{\beta} = \arg \min_{\beta} \sum_{j=1}^n \{Y_{(j)} - \beta'X_{(j)}\}^2, \quad (17)$$

where $\beta'X_{(1)} \leq \dots \leq \beta'X_{(n)}$ are order statistics of $\beta'X_1 \dots \dots \beta'X_n$. We refer $\hat{\beta}$ to be the matching quantiles estimator (MQE), because it attempts to match the quantiles of all possible levels between 0 and 1.

2.2.3. The Cramer-Von Mises Minimum Distance Estimation

The Cramer-Von Mises Estimator (CME) is sort of minimum distance estimator, which is also known as Maximum Goodness-of-fit estimator. The estimator is based on the difference between the estimate of the cumulative distribution function and the empirical distribution function D'Agostino *et al.* [4], Luceno [12].

The choice of Cramer-Von Mises type minimum distance estimator is motivated by MacDonald [14] who provided empirical evidence that the bias of the estimator is smaller than the other minimum distance estimators. The Cramer-Von Mises estimators, α_{CME} and β_{CME} are obtained by minimizing

$$C(\alpha, \beta) = \frac{1}{12n} + \sum_{i=1}^n \left(F(y_{(i)} | \alpha, \beta) - \frac{2i-1}{2n} \right)^2 \quad (18)$$

with respect to α and β . Estimators in this equation above can be derived by solving the equations below.

$$\sum_{i=1}^n \left(F(y_{(i)} | \alpha, \beta) - \frac{2i-1}{2n} \right)^2 \Delta_1(y_{(i)} | \alpha, \beta) = 0, \quad (19)$$

$$\sum_{i=1}^n \left(F(y_{(i)} | \alpha, \beta) - \frac{2i-1}{2n} \right)^2 \Delta_2(y_{(i)} | \alpha, \beta) = 0, \quad (20)$$

Where $\Delta_1(\cdot | \alpha, \beta)$ and $\Delta_2(\cdot | \alpha, \beta)$ can be obtained from ordinary and weighted least square estimates.

2.2.4. Methods of Anderson-Darling Minimum Distance Estimation

The Minimum distance estimator is based on Anderson-Darling statistic Luceno [12], which is called Anderson-Darling estimator (ADE). The Anderson-Darling estimators, θ_{ADE} and ϕ_{ADE} of parameters γ and λ are obtained by minimizing with respect to θ and ϕ , the function

$$A(\theta, \phi) = -n \frac{1}{n} \sum_{i=1}^n (2i-1) \cdot (\log F(x_{(i)} | \theta, \phi) + \log S(x_{((n+1-i))} | \theta, \phi)) \quad (21)$$

However, estimates above can be derived by simplifying the nonlinear equations below:

$$\sum_{i=1}^n (2i-1) \left[\frac{\Delta_1(x_{(i)} | \theta, \phi)}{F(x_{(i)} | \theta, \phi)} - \frac{\Delta_1(x_{(n+1-i)} | \theta, \phi)}{S(x_{(n+1-i)} | \theta, \phi)} \right] = 0, \quad (22)$$

$$\sum_{i=1}^n (2i-1) \left[\frac{\Delta_2(x_{(i)} | \theta, \phi)}{F(x_{(i)} | \theta, \phi)} - \frac{\Delta_2(x_{(n+1-i)} | \theta, \phi)}{S(x_{(n+1-i)} | \theta, \phi)} \right] = 0 \quad (23)$$

Where $\Delta_1(\cdot | \theta, \phi)$ and $\Delta_2(\cdot | \theta, \phi)$ can be obtained from ordinary and weighted least square estimates.

2.3. Goodness-of-Fit Tests

For the determination of Goodness-of-fit test of chosen distributions for the wind speed data, the following tests were adopted:

2.3.1. Lilliefors (Kolmogorov-Smirnov) Test

The Lilliefors (Kolmogorov-Smirnov) test is an empirical distribution function (EDF) omnibus test for the composite hypothesis of normality test. Its test statistic is the maximal absolute difference between empirical and hypothetical cumulative distribution function. Lilliefors [9] proposed the Lilliefors test as an improvement on the Kolmogorov-Smirnov (K-S) test, by correcting the K-S test for small values at the tail of the probability distribution. The Lilliefors test is also known as the K-S D test. Unlike the K-S test, the Lilliefors test is employed when the population parameters are unknown, therefore, the Lilliefors test is a K-S test that enables the estimation of population parameters from the samples. It may be computed as

$$D = \max_{i=1, \dots, N} \left| CDF(x_i) - \frac{i-1}{N}, \frac{i}{N} - CDF(x_i) \right|, \quad (24)$$

where CDF represent the cumulative distribution function. If the test statistic is greater than the critical D-statistic for

sample size n , the null hypothesis is rejected. It is so because parameters of the CDF are estimated from the sample, the K-S test becomes conservative and loses power, so the Lilliefors correction is applied.

2.3.2. Anderson-Darling (AD) Test

The Anderson-Darling (AD) Test is a general test that used to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function. The AD test is employed to test a set of data from a population with specific probability distribution. Stephens [26] proposed the AD Test. The (AD) Test is a modified Kolmogorov-Smirnov Test, since more weight to the tails of the distribution is given. The Critical value of the AD Test is based on the specific distribution being tested.

The test statistic is defined as:

$$A^2 = -N - S, \quad (25)$$

where

$S = \sum_{i=1}^N \frac{2i-1}{N} [\ln F(Y_i) + \ln(1 - F(Y_{N+1-i}))]$, F is the cumulative distribution function and Y_i is the ordered sample values. A lower value of AD Test test statistic indicate a better fitted model.

2.3.3. Cramer-Von Mises Test

The Cramer-Von Mises Test is an empirical distribution function (EDF) omnibus test for the composite hypothesis of normality test. According to Stephens [26], the test statistic is defined as

$$W = \frac{1}{12} + \sum_{i=1}^n \left(p(i) - \frac{2i-1}{2n} \right) \quad (26)$$

where $p(i) = \Phi\left(\frac{[x(i) - \bar{x}]}{s}\right)$, Φ is the cumulative distribution function of the standard normal distribution, \bar{x} and s are mean and standard deviation of the data values respectively. The p-value for the test is computed from the modified statistic $Z = W\left(\frac{1.0+0.5}{n}\right)$.

3. Data Analysis and Discussions

The real-life data adopted for application in this article is a wind speed data adapted from Saporu and Esbond [21] officially obtained from the Nigerian Meteorological Agency (NiMET) Office in Maiduguri, at hub height of 10 meters. The data obtained were recorded from September, 1985 to December, 2011. However, R software was used to perform every necessary graphical and analytical procedures.

Table 1. Output for five (5) methods of estimation for unknown parameters of six (6) probability distributions considering wind data of Maiduguri.

Method of Estimation	Criteria	Probability Distribution Models					
		Gamma	Weibull	Lognormal	Pareto	Burr	Log-logistic
MLE	LL	-488.567	-484.399	-499.526	-671.260	-484.308	-500.513
	AIC	981.1347	972.799	1003.052	1346.521	974.6171	1005.027
	BIC	988.6462	980.3105	1010.563	1354.033	985.8844	1012.538
MGE (K-S)	LL	-489.706	-484.618	-502.415	-683.003	-494.098	-500.8987
	AIC	983.4127	973.2365	1008.83	1370.007	994.1967	1005.797
	BIC	990.9242	980.748	1016.342	1377.518	1005.464	1013.309
QME	LL	-493.068	-484.434	-512.234	-725.239	-709.848	-46506.84
	AIC	990.1362	972.8685	1028.47	1454.479	1425.697	93017.68
	BIC	997.6477	980.38	1035.981	1461.99	1436.964	93025.19
MGE (CvM)	LL	-489.137	-484.467	-501.402	-681.763	-484.676	-500.862
	AIC	982.2743	972.9351	1006.805	1367.526	975.3525	1005.725
	BIC	989.7857	980.4465	1014.316	1375.037	986.6197	1013.236
MGE (AD)	LL	-488.852	-484.435	-500.056	-681.021	-484.447	-500.772
	AIC	981.7053	972.8718	1004.113	1366.042	974.8943	1005.546
	BIC	989.2168	980.3833	1011.624	1373.554	986.1615	1013.057

In table 1 above, it is clearly observed that among the five (5) methods of estimations considered, ML estimates appear most appropriate, this is because when we consider maximum LL values, that of ML has the highest values of -488.5674, -484.3995, -499.526, -671.2606, -484.3086, -500.5133 across the six (6) probability distributions and as compared against the remaining four methods of parameter estimation (MGE (KS), QME, MGE (CvM), MGE (AD)). In support of the judgment that ML is best amongst other methods of parameter estimation, minimum AIC values were obtained for the five methods of estimation with that of the ML as the lowest estimates across the six probability distributions (981.1347, 972.799, 1003.052, 1346.521, 974.6171, 1005.027). Comparison of BIC values throughout the methods of estimation (ML, MQ, MG (KS), MG (AD), MG (CvM)) considered in this study show that those of MLE

are lowest across the five probability distributions (Gamma, Weibull, Log-Normal, Pareto, Burr and Log-logistic) with minimum values (988.6462, 980.3105, 1010.563, 1354.033, 985.8844, 1012.538). However, in the same table 1 above, as it is confirmed by LL, AIC and BIC estimates that ML method of estimation is best among others compared, it is further observed that when these values of ML are keenly looked across the six distributions Weibull estimates of LL, AIC and BIC best fit the wind data of Maiduguri followed by Burr distribution.

Figure 1, further showed that graphically that Gamma and log-normal are close in fitting the data too while there is a huge deviation of Pareto and Log-logistic from the fit. That is, Pareto and Log-logistic have very poor fit in fitting the sample under consideration.

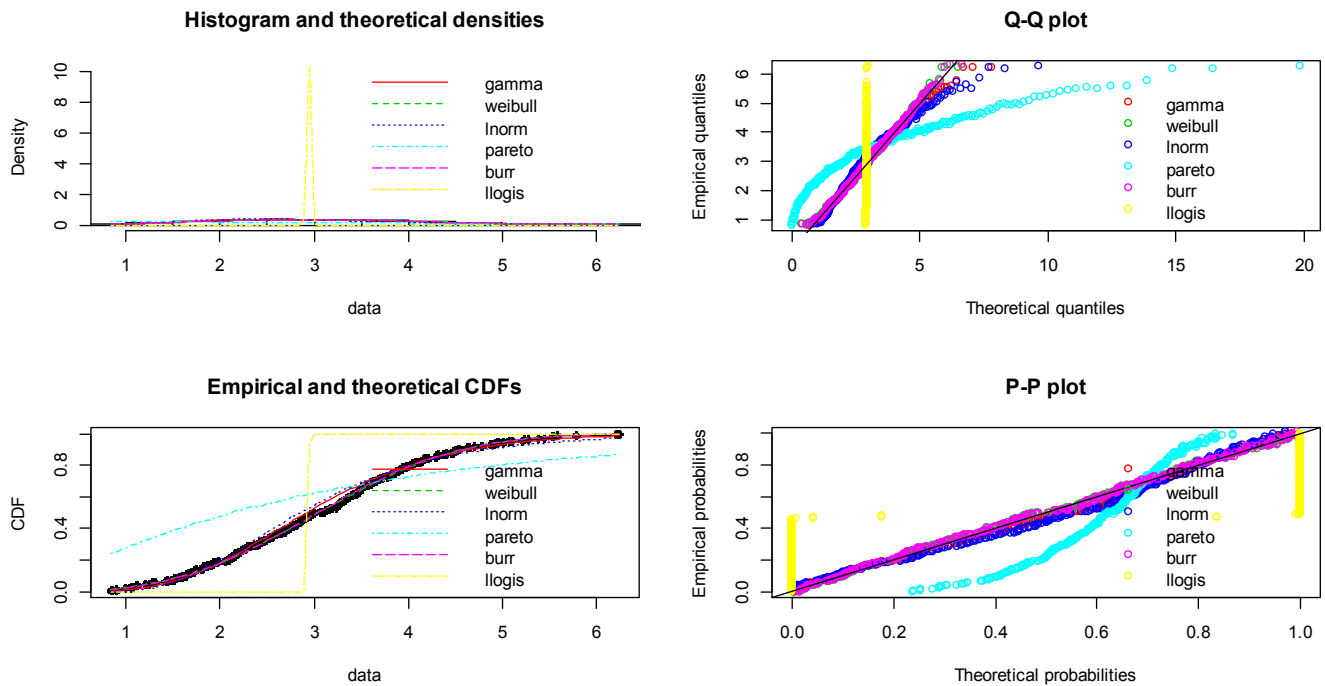


Figure 1. Graphical Descriptive Statistics showing histogram-Q plot, CDFs and P-P plots for Six different probability distributions (Gamma, Weibull, Lognormal, Pareto, Burr and log-logistic).

Table 2. Comparison of Parameter Estimation of Weibull Model using Five (5) different Methods of Estimation of Parameters.

Parameters	Methods of Estimation				
	MLE	QME	GME (KS)	GME (CvM)	GME (AD)
Shape (α)	2.949555	2.965693	2.900283	2.902054	2.915188
Rate (β)	3.452937	3.471079	3.481039	3.444601	3.444203

Table 3. Results for Comparing Weibull and Burr Distributions.

Goodness-of-Fit Test	WEIBULL	BURR
KS	0.03179314	0.03497396
CvM	0.03137888	0.03519690
AD	0.23725978	0.24121964

Table 3 above present p-values of Weibull and Burr distributions computed by using goodness of fit criteria and which reveals that Weibull distribution best fitted wind speed data per the Goodness-of-fit Tests since it has the smallest p-value for K-S, CvM and AD compared to Burr distribution, where low value for p-value indicates that the selected curve is fairly close to our data. However, in the consideration of all the distributions under study here, Burr also prove better than the rest following Weibull.

Table 4. AIC and BIC results for Weibull and Burr Distributions.

Information Criteria	Weibull	Burr
AIC	972.7990	974.6171
BIC	980.3105	985.8844

Table 4 shows results of AIC and BIC for Weibull and Burr distributions considering the wind data. It is easily seen from Table 4 that Weibull distribution is more suitable than Burr distribution because of its minimum values of

AIC and BIC compared to the Burr. It is apparent that the result above confirmed from AIC of Weibull has minimum value as compared to that of Burr model. Critically comparing the estimates obtained by the different methods of parameter estimations adopted, it is seen that in both Weibull and Burr, log likelihood values were maximized with ML estimates of -484.3086 and -484.3995 for Burr and Weibull respectively.

4. Conclusion

In this research, we made comparison among six different probability models to decide which one best fit wind speed data and further investigate the best method of parameter estimation considering six (6) different methods. In order to evaluate the available real data sets of wind speed of Maiduguri, the biggest city in the North Eastern, Nigeria. The PDFs were used to model the wind speed data. Based on the comparison of Pdfs and Goodness-of-fit Tests: we conclude that the maximum likelihood estimate has the lowest value of the estimates of the distribution parameters AIC and BIC, turns out to be the most efficient method compared to other methods.

Meanwhile, our analysis from goodness-of-fit tests showed Weibull distribution produced the best fitted model for the wind speed data under consideration.

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