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On Efficient Memory-Type Control Charts for Monitoring out of Control Signals in a Process Using Diabetic Data

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Abstract: Control chart is a useful technique which helps in detecting out of control signal in a process and it can either be a memory-type or memory-less control chart. This work is focused on evaluating the monthly incidence of diabetic disease using four univariate memory-type control charts. In this study we evaluated the average run length (ARL) properties of the memory-type control charts by adjusting the ARL value's determinant parameters in each control charts, and the ARL value was set to be 500. The output of the analysis of the data set indicates that the exponential weighted moving average (EWMA) control chart is better in detecting the out of control signal faster in small shifts than its counter parts including CUSUM, MECH and MEC charts. This will help in having adequate plans and prevent the increase in diabetics in the country.

Keywords: Cusum Chart, Ewma Chart, Average Run length, Mec Chart, Mech Chart, Univariate Control Charts

1. Introduction

Control chart is a useful technique that helps in detecting out of control signal in a process. Control charting can either be univariate or multivariate. Univariate chart is useful if only one variable is taking into consideration. The univariate control chart can either be memory-type or memory-less control charts.

A memory-type control chart is useful in detecting small and moderate shift in a process, this includes; Exponential Weighted Moving Average (EWMA) control chart, Cumulative Sum (CUSUM) chart, Mixed EWMA-CUSUM (MEC) chart, Mixed EWMA-CUSUM with Head-start (MECH) chart and some other recent modifications of these charts. The memory-type control charts were designed in a way that it takes the past and present information into consideration. Recent authors have worked on memory-type control charts. Abbas et al. (2013) proposed progressive mean control chart for monitoring process location parameter; Ajadi et al. (2016) proposed mixed multivariate Ewma-Cusum for improved process monitoring; Zaman et al. (2014) introduced mixed Cusum-Ewma control charts as an efficient way of monitoring process location and also Abbas (2015) show that progressive mean is a special case of Exponentially Weighted Moving Average.

Memory-less control chart is useful in detecting large shift in a process, it takes only the present but not the past information into consideration. This includes Shewhart chart of the pioneer work of Dr. W. Shewhart in 1931. In this article, we are interested in the univariate control charts; we are aiming at monitoring process out of control signal of diabetic patients using the EWMA and CUSUM charts and some recent modifications of these two charts. The reason for this is to determine which of this univariate charts detect out of control signal faster, as this will help in taking adequate measure on the management of diabetic patients in the study area.

The Average Run Length (ARL) properties of these charts will be set at 500, by adjusting the parameters of each of the charts.

2. Exponential Weighted Moving Average (EWMA) Control Charts

The EWMA control chart was proposed by Robert (1959). It uses both the present and past information. The control scheme for EWMA is given by the statistic:

$$Z_i = \lambda X_i + (1-\lambda) Z_{i-1} \quad (1)$$

Where λ is the sensitivity parameter and it has the value between 0 and 1 ($0 < \lambda \le 1$). The Z_{i-1} is the past information of the EWMA statistics, Z_i is the present information and X_i is the set of observations of the data set.

The upper and lower control limit of the EWMA statistics is given in equation (2) and (3) below.

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (2)$$

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (3)$$

The EWMA statistic Z_i is plotted against the LCL_i and UCL_i, the process will be said to be in a control state when $Z_i > LCL_i$ and $Z_i < UCL_i$ i.e. $LCL_i < Z_i < UCL_i$.

2.1. Cumulative Sum (CUSUM) Charts

The CUSUM control chart was proposed by Page (1954). It uses both the present and past information. The control scheme for the upper and lower CUSUM is given in equation (4) and (5) below by the statistic

$$C_i^+ = \max\left[0, (X_i - \mu_0) - k + C_{i-1}^+\right] \quad (4)$$

$$C_i^- = \max\left[0, -(X_i - \mu_0) - k + C_{i-1}^-\right] \quad (5)$$

C_i^+ and C_i^- are the present information and they are plotted against the control limits, H. While C_{i-1}^+ and C_{i-1}^- are the past information. The k is referred to as the reference value, which is taken to be half way of the shifts in the process, that is, $k = \frac{1}{2}\delta\sigma$.

2.2. Mixed Exponentially Weighted Moving Average-Cumulative Sum (MEC) Charts

The EWMA control chart was proposed by Abbas et al. (2013). It also uses both the present and past information. The authors combined EWMA and CUSUM charts, and the control scheme for the MEC chart is given as upper MEC and lower MEC in equation (6) and (7) by the statistic

$$M_i^+ = \max\left[0, (Z_i - \mu_0) - a_i + M_{i-1}^+\right] \quad (6)$$

$$M_i^- = \max\left[0, -(Z_i - \mu_0) - a_i + M_{i-1}^-\right] \quad (7)$$

Where a_i is a time-varying reference value for the MEC charting structure and it is given as

$$a_i = a^*\sqrt{\mathrm{var}(Z_i)} \quad \text{Where} \quad \mathrm{var}(Z_i) = \sigma^2\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)$$

and a^* is just like k in classical CUSUM; we set $a^* = 0.5$. The control limit for this chart is given as,

$$b_i = b^*\sqrt{\mathrm{var}(Z_i)} = b_i = b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (8)$$

Where b^* is a constant like h in classical CUSUM which can be obtained through the simulation and both M_i^+ and M_i^- are plotted against the control limit b_i.

2.3. Mixed EWMA-CUSUM with Head-Start (MECH)

Ajadi et al. (2016) proposed a new memory type control chart named mixed EWMA-CUSUM with headstart chart. In order to improve the sensitivity of the mixed EWMA-CUSUM at process start-up, the authors suggested a headstart based structure, namely MECH, by assigning the initial values of M_0^+ and M_0^- to be 50% of the first value of the control limit (b_1). i.e. $M_0^+ = M_0^- = 0.5b_1$. The headstart of the mixed EWMA-CUSUM may be expressed as:

$$M_0^+ = M_0^- = 0.5b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^2\right)}$$

$$= 0.5b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-2\lambda+\lambda^2\right)\right)}$$

$$= 0.5b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-1+2\lambda-\lambda^2\right)}$$

$$M_0^+ = M_0^- = 0.5b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(2\lambda-\lambda^2\right)} \quad (9)$$

And simplifying it further.

$$M_0^+ = M_0^- = 0.5\sigma b^*\lambda \quad (10)$$

3. Results

The dataset used for the illustrative example of this research work was adopted from Edokpa et al. (2009). The dataset is on incidence of diabetic disease in Ibadan town of Oyo State, the data set was collected from the University teaching hospital, Ibadan, for the period of 146 months, from Jan 1974 to Feb 1986.

Table 1. *Monthly incidence of Diabetic Disease in Oyo State.*

YEAR	JAN	FEB	MAR	APRIL	MAY	JUN	JUL	AUG	SEP	OCT	NOV	DEC
1974	20	23	16	19	23	16	22	12	9	17	20	18
1975	2	8	8	23	14	25	16	25	7	2	3	13
1976	20	18	30	17	23	21	14	22	18	18	13	27
1977	25	20	31	22	15	26	21	23	14	13	58	15
1978	29	27	25	10	17	17	30	22	14	15	14	14
1979	24	14	19	9	11	7	19	8	19	22	11	22
1980	25	22	19	23	17	17	10	23	24	15	41	16
1981	15	7	10	26	9	17	23	22	30	32	22	27
1982	25	20	35	17	19	19	27	29	11	23	25	16
1983	24	20	19	12	16	10	9	16	7	9	18	9
1984	18	17	14	14	19	23	12	20	7	17	9	14
1985	18	2	6	18	14	17	22	12	18	13	6	18
1986	21	17										

4. Determining the Average Run Length (ARL) Value

We estimated the Average Run Length (ARL) values of the memory-type control charts, the ARL values was set to be 500; this was done by adjusting the parameters of each of the chart used in the study area.

In EWMA control chart, the determinants of the ARL value is the L and λ. The L was set to be 3, while λ was adjusted to be 0.25 so that it can be sensitive to small shifts. If we set λ>0.5, then the process will be sensitive to large shifts, but since we are interested in small shifts, we use L=3 and λ=0.25. In CUSUM chart, *h* and *k* are the two parameters used in determining the ARL value. In order to set ARL=500, we adjusted the values of *h*=5.071 and *k*=0.5, where *h* is the control statistic and *k* is our reference value. To determine the ARL value of MEC control chart, the determinants of the ARL value are a^*, b^* and λ. a^* was set to be 0.5, b^*=20.18 while λ was adjusted to be 0.25 so that it can be sensitive to small shifts. If we set λ>0.5, then the process will be sensitive to large shifts, but since we are interested in small shifts, we use a^*=0.5, b^*=20.18 and λ=0.25.

To determine the ARL value of MECH control chart, the determinants of the ARL value were M_0^+, M_0^- a^*, b^* and λ. a^* was set to be 0.5, b^*=20.49 while λ was adjusted to be 0.25 so that it can be sensitive to small shifts. The initial values M_0^+ and M_0^- were set to be 2.561.

5. Discussion of Results

The analysis of this research work was carried out using R package, and the graphical outputs were presented in figure 1-4 below, while the values of the outputs were presented in table 2 (See Appendix).

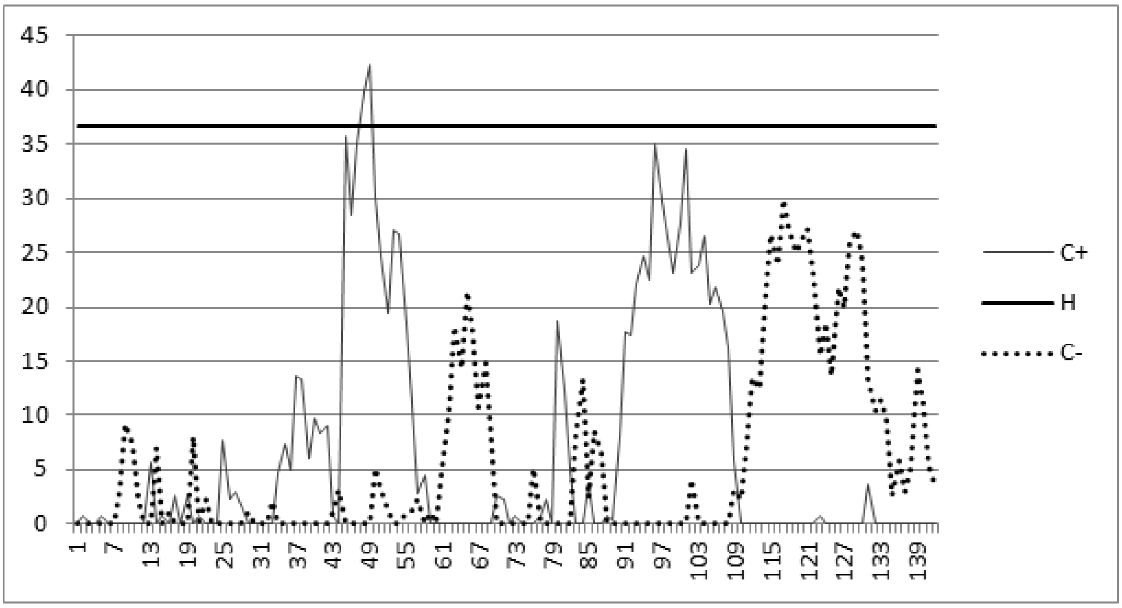

Figure 1. *CUSUM Chart when h=5.071σ, k=0.5σ.*

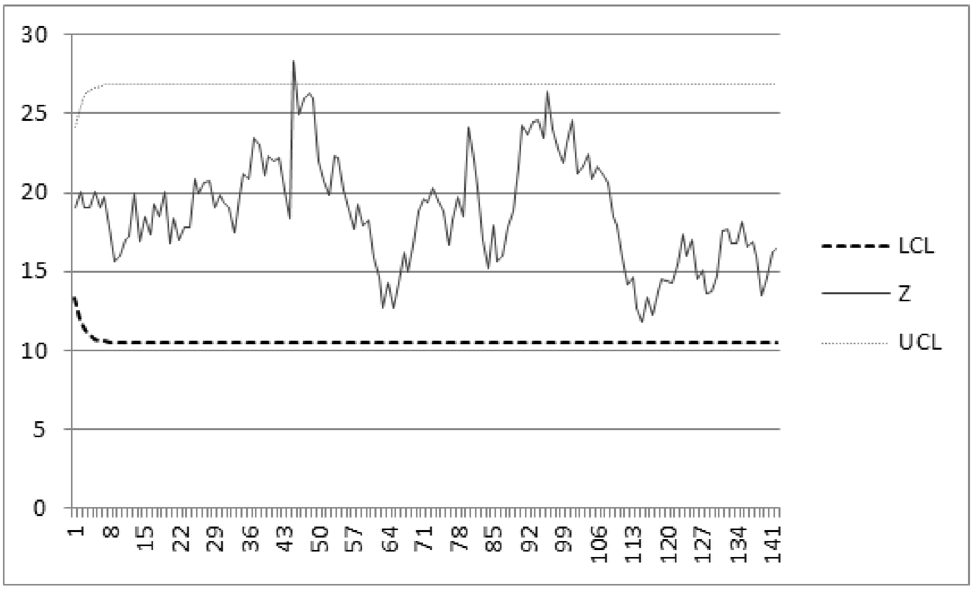

Figure 2. *EWMA Chart when λ =0.25, L= 3.*

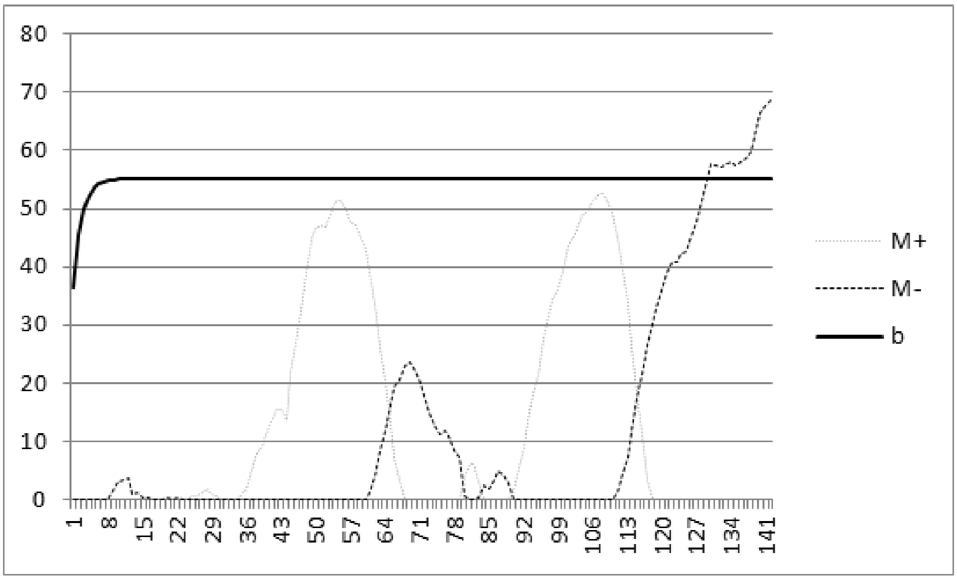

Figure 3. *MEC Chart when λ =0.25, a^* = 0.5 and b^* = 20.18.*

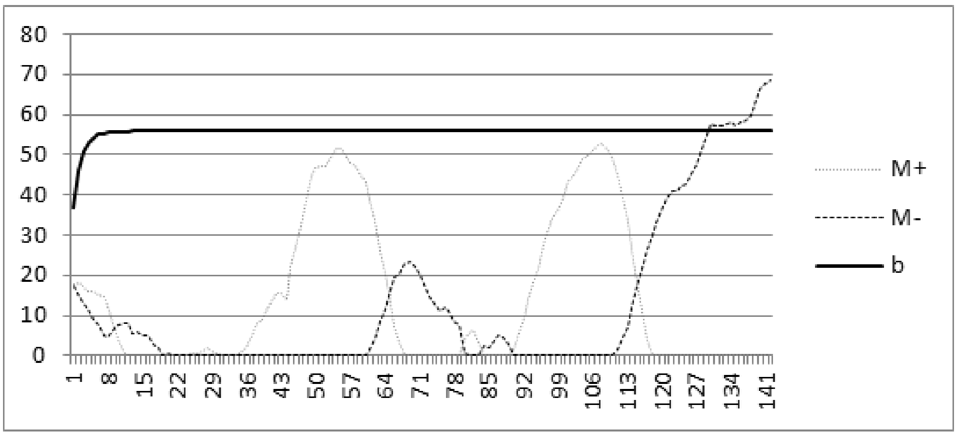

Figure 4. *MECH Chart when (M_0^+=M_0^-=2.561), λ =0.25, a^* = 0.5 and b^* = 20.49.*

Figure 1-4 shows the graphical output of the monthly incidence of diabetic disease in Oyo State. In Figure 1, the output shows that there is an out of control signal at sample point 48 in the upper CUSUM, this means that there is an increase in the incidence of diabetic disease. The output of EWMA chart in Figure 2 reveals that the EWMA statistic Z cut across the upper control limit (UCL), this shows that there is an out of control signal also at sample point 45. The outputs of MEC and MECH charts in Figure 3 and 4 respectively reveal that the MEC and MECH statistic has a cut of point at sample point 131, this indicates that there is an out of control signal in the monthly incidence of diabetics. (See the numbers in bold in Appendix for support)

6. Conclusion

The average run length (ARL) values of several memory-type control charts were investigated, and we set it to be 500, by adjusting the parameters of the four charts taking into consideration in this study. The aim is to monitor the monthly incidence of the diabetics in Oyo State and detect an out of control signal in the process. The findings show that the EWMA chart detected an out of control signal at sample point 45, which is the month of December 1977. CUSUM at sample point 48, MEC and MECH at sample point 131. Since the EWMA chart detected an out of control signal faster than its counterparts in a small shift, this indicates that the EWMA chart is more sensitive to small shift compared with its counterparts in this study. Hence we strongly recommend the use of EWMA chart in monitoring the incidence of diabetic disease. This will help the government and the management of hospitals to have adequate plans and prevent the increase in diabetics in the country.

Appendix

Table 2. *Output of the memory-type control charts at ARL_0=500.*

S/N	CUSUM Chart			EWMA Chart			MEC Chart			MECH Chart		
	C+	H	C-	LCL	Z	UCL	M+	M-	b	M+	M-	b
1	0	36.6352	0	13.2929	19.033	24.1296	0	0	36.4473	17.9227	17.2784	37.0072
2	0.6765	36.6352	0	11.9384	20.025	25.48419	0.185	0	45.5592	18.1077	14.8357	46.259
3	0	36.6352	0	11.2842	19.019	26.13835	0	0	49.9595	17.1774	13.2903	50.727
4	0	36.6352	0	10.9404	19.014	26.48216	0	0	52.2722	16.1851	11.6924	53.0752
5	0.6765	36.6352	0	10.7535	20.011	26.66903	0	0	53.5292	16.1582	9.06674	54.3515
6	0	36.6352	0	10.6503	19.008	26.77223	0	0	54.2234	15.1113	7.42657	55.0564
7	0	36.6352	0	10.5928	19.756	26.82971	0	0	54.6101	14.803	5.02881	55.449
8	0	36.6352	3.09904	10.5607	17.817	26.86187	0	0	54.8264	12.5502	4.56468	55.6686
9	0	36.6352	9.19809	10.5426	15.613	26.8799	0	1.73711	54.9476	8.09023	6.30179	55.7917
10	0	36.6352	7.29713	10.5325	15.96	26.89002	0	3.12571	55.0158	3.97538	7.69039	55.8609
11	0	36.6352	2.39618	10.5268	16.97	26.89571	0	3.50324	55.054	0.8697	8.06792	55.8998
12	0	36.6352	0	10.5236	17.227	26.89891	0	3.62266	55.0755	0	8.18734	55.9216
13	5.6765	36.6352	0	10.5218	19.92	26.90071	0	1.04859	55.0876	0	5.61327	55.9339
14	0	36.6352	7.09904	10.5208	16.94	26.90172	0	1.45446	55.0944	0	6.01914	55.9408
15	0.6765	36.6352	0	10.5203	18.455	26.90229	0	0.34531	55.0983	0	4.90999	55.9447
16	0	36.6352	1.09904	10.5199	17.341	26.90261	0	0.34992	55.1004	0	4.9146	55.9469
17	2.6765	36.6352	0	10.5198	19.256	26.90279	0	0	55.1016	0	3.00454	55.9481
18	0	36.6352	0	10.5196	18.442	26.90289	0	0	55.1023	0	1.90849	55.9488
19	2.6765	36.6352	0	10.5196	20.082	26.90295	0.005	0	55.1027	0.00499	0	55.9492
20	0	36.6352	8.09904	10.5196	16.811	26.90298	0	0.53483	55.1029	0	0.53483	55.9494
21	0.6765	36.6352	0.19809	10.5195	18.358	26.903	0	0	55.103	0	0	55.9495
22	0	36.6352	2.29713	10.5195	17.019	26.90301	0	0.3272	55.1031	0	0.3272	55.9496
23	0	36.6352	0	10.5195	17.764	26.90301	0	0	55.1031	0	0	55.9496
24	0	36.6352	0	10.5195	17.823	26.90302	0	0	55.1032	0	0	55.9497
25	7.6765	36.6352	0	10.5195	20.867	26.90302	0.7907	0	55.1032	0.79074	0	55.9497
26	2.353	36.6352	0	10.5195	19.9	26.90302	0.6146	0	55.1032	0.61465	0	55.9497
27	3.0295	36.6352	0	10.5195	20.675	26.90302	1.2134	0	55.1032	1.21344	0	55.9497
28	1.706	36.6352	0	10.5195	20.757	26.90302	1.8934	0	55.1032	1.8934	0	55.9497
29	0	36.6352	1.09904	10.5195	19.067	26.90302	0.8842	0	55.1032	0.88423	0	55.9497
30	0	36.6352	0	10.5195	19.801	26.90302	0.6082	0	55.1032	0.60821	0	55.9497
31	0	36.6352	0	10.5195	19.35	26.90302	0	0	55.1032	0	0	55.9497
32	0	36.6352	0	10.5195	19.013	26.90302	0	0	55.1032	0	0	55.9497
33	0	36.6352	2.09904	10.5195	17.51	26.90302	0	0	55.1032	0	0	55.9497
34	4.6765	36.6352	0	10.5195	19.882	26.90302	0	0	55.1032	0	0	55.9497
35	7.353	36.6352	0	10.5195	21.162	26.90302	1.0851	0	55.1032	1.08509	0	55.9497
36	5.0295	36.6352	0	10.5195	20.871	26.90302	1.8798	0	55.1032	1.87977	0	55.9497
37	13.706	36.6352	0	10.5195	23.403	26.90302	5.2066	0	55.1032	5.20664	0	55.9497
38	13.383	36.6352	0	10.5195	23.053	26.90302	8.1827	0	55.1032	8.18265	0	55.9497
39	6.0591	36.6352	0.09904	10.5195	21.039	26.90302	9.1455	0	55.1032	9.14552	0	55.9497
40	9.7356	36.6352	0	10.5195	22.28	26.90302	11.349	0	55.1032	11.3485	0	55.9497

S/N	CUSUM Chart			EWMA Chart			MEC Chart			MECH Chart		
	C+	H	C-	LCL	Z	UCL	M+	M-	b	M+	M-	b
41	8.4121	36.6352	0	10.5195	21.96	26.90302	13.232	0	55.1032	13.2317	0	55.9497
42	9.0886	36.6352	0	10.5195	22.22	26.90302	15.375	0	55.1032	15.3749	0	55.9497
43	0.7651	36.6352	1.09904	10.5195	20.165	26.90302	15.463	0	55.1032	15.4631	0	55.9497
44	0	36.6352	3.19809	10.5195	18.374	26.90302	13.76	0	55.1032	13.7602	0	55.9497
45	35.677	36.6352	0	10.5195	28.28	26.90302	21.964	0	55.1032	21.9638	0	55.9497
46	28.353	36.6352	0.09904	10.5195	24.96	26.90302	26.847	0	55.1032	26.8474	0	55.9497
47	35.03	36.6352	0	10.5195	25.97	26.90302	32.741	0	55.1032	32.741	0	55.9497
48	39.706	36.6352	0	10.5195	26.228	26.90302	38.892	0	55.1032	38.892	0	55.9497
49	42.383	36.6352	0	10.5195	25.921	26.90302	44.736	0	55.1032	44.7361	0	55.9497
50	30.059	36.6352	5.09904	10.5195	21.941	26.90302	46.6	0	55.1032	46.6001	0	55.9497
51	24.736	36.6352	3.19809	10.5195	20.705	26.90302	47.229	0	55.1032	47.2289	0	55.9497
52	19.412	36.6352	1.29713	10.5195	19.779	26.90302	46.931	0	55.1032	46.9314	0	55.9497
53	27.089	36.6352	0	10.5195	22.334	26.90302	49.189	0	55.1032	49.1891	0	55.9497
54	26.765	36.6352	0	10.5195	22.251	26.90302	51.363	0	55.1032	51.3633	0	55.9497
55	18.442	36.6352	1.09904	10.5195	20.188	26.90302	51.475	0	55.1032	51.4748	0	55.9497
56	11.118	36.6352	1.19809	10.5195	18.891	26.90302	50.289	0	55.1032	50.2892	0	55.9497
57	2.7946	36.6352	2.29713	10.5195	17.668	26.90302	47.881	0	55.1032	47.8809	0	55.9497
58	4.4711	36.6352	0	10.5195	19.251	26.90302	47.056	0	55.1032	47.0556	0	55.9497
59	0	36.6352	1.09904	10.5195	17.938	26.90302	44.917	0	55.1032	44.9174	0	55.9497
60	0	36.6352	0	10.5195	18.204	26.90302	43.045	0	55.1032	43.0447	0	55.9497
61	0	36.6352	6.09904	10.5195	15.903	26.90302	38.871	1.44313	55.1032	38.8709	1.44313	55.9497
62	0	36.6352	10.1981	10.5195	14.677	26.90302	33.472	4.11196	55.1032	33.4715	4.11196	55.9497
63	0	36.6352	18.2971	10.5195	12.758	26.90302	26.153	8.70008	55.1032	26.1528	8.70008	55.9497
64	0	36.6352	14.3962	10.5195	14.318	26.90302	20.395	11.7277	55.1032	20.3946	11.7277	55.9497
65	0	36.6352	21.4952	10.5195	12.739	26.90302	13.057	16.3349	55.1032	13.0569	16.3349	55.9497
66	0	36.6352	17.5943	10.5195	14.304	26.90302	7.2844	19.3767	55.1032	7.28441	19.3767	55.9497
67	0	36.6352	10.6933	10.5195	16.228	26.90302	3.4359	20.4946	55.1032	3.43592	20.4946	55.9497
68	0	36.6352	14.7924	10.5195	14.921	26.90302	0	22.9196	55.1032	0	22.9196	55.9497
69	0	36.6352	7.8914	10.5195	16.691	26.90302	0	23.5748	55.1032	0	23.5748	55.9497
70	2.6765	36.6352	0	10.5195	18.768	26.90302	0	22.1526	55.1032	0	22.1526	55.9497
71	2.353	36.6352	0	10.5195	19.576	26.90302	0	19.9225	55.1032	0	19.9225	55.9497
72	0	36.6352	0	10.5195	19.432	26.90302	0	17.8365	55.1032	0	17.8365	55.9497
73	0.6765	36.6352	0	10.5195	20.324	26.90302	0.2475	14.8584	55.1032	0.24748	14.8584	55.9497
74	0	36.6352	0	10.5195	19.493	26.90302	0	12.7113	55.1032	0	12.7113	55.9497
75	0	36.6352	0	10.5195	18.87	26.90302	0	11.1875	55.1032	0	11.1875	55.9497
76	0	36.6352	5.09904	10.5195	16.652	26.90302	0	11.8812	55.1032	0	11.8812	55.9497
77	0.6765	36.6352	0	10.5195	18.239	26.90302	0	10.9879	55.1032	0	10.9879	55.9497
78	2.353	36.6352	0	10.5195	19.679	26.90302	0	8.65446	55.1032	0	8.65446	55.9497
79	0	36.6352	0.09904	10.5195	18.51	26.90302	0	7.49086	55.1032	0	7.49086	55.9497
80	18.677	36.6352	0	10.5195	24.132	26.90302	4.0556	0.70465	55.1032	4.05562	0.70465	55.9497
81	12.353	36.6352	0	10.5195	22.099	26.90302	6.0782	0	55.1032	6.0782	0	55.9497
82	5.0295	36.6352	0.09904	10.5195	20.324	26.90302	6.326	0	55.1032	6.32599	0	55.9497
83	0	36.6352	8.19809	10.5195	16.993	26.90302	3.2427	0.35271	55.1032	3.2427	0.35271	55.9497
84	0	36.6352	13.2971	10.5195	15.245	26.90302	0	2.45374	55.1032	0	2.45374	55.9497
85	3.6765	36.6352	2.39618	10.5195	17.934	26.90302	0	1.866	55.1032	0	1.866	55.9497
86	0	36.6352	8.49522	10.5195	15.7	26.90302	0	3.51169	55.1032	0	3.51169	55.9497
87	0	36.6352	6.59426	10.5195	16.025	26.90302	0	4.83246	55.1032	0	4.83246	55.9497
88	0.6765	36.6352	0	10.5195	17.769	26.90302	0	4.40952	55.1032	0	4.40952	55.9497
89	0.353	36.6352	0	10.5195	18.827	26.90302	0	2.92882	55.1032	0	2.92882	55.9497
90	8.0295	36.6352	0	10.5195	21.62	26.90302	1.5435	0	55.1032	1.54345	0	55.9497
91	17.706	36.6352	0	10.5195	24.215	26.90302	5.6819	0	55.1032	5.6819	0	55.9497
92	17.383	36.6352	0	10.5195	23.661	26.90302	9.2666	0	55.1032	9.2666	0	55.9497
93	22.059	36.6352	0	10.5195	24.496	26.90302	13.686	0	55.1032	13.686	0	55.9497
94	24.736	36.6352	0	10.5195	24.622	26.90302	18.231	0	55.1032	18.2314	0	55.9497
95	22.412	36.6352	0	10.5195	23.466	26.90302	21.621	0	55.1032	21.6213	0	55.9497
96	35.089	36.6352	0	10.5195	26.35	26.90302	27.895	0	55.1032	27.8946	0	55.9497
97	29.765	36.6352	0	10.5195	24.012	26.90302	31.83	0	55.1032	31.8304	0	55.9497
98	26.442	36.6352	0	10.5195	22.759	26.90302	34.513	0	55.1032	34.5131	0	55.9497
99	23.118	36.6352	0	10.5195	21.819	26.90302	36.256	0	55.1032	36.256	0	55.9497
100	27.795	36.6352	0	10.5195	23.115	26.90302	39.294	0	55.1032	39.2941	0	55.9497
101	34.471	36.6352	0	10.5195	24.586	26.90302	43.803	0	55.1032	43.8035	0	55.9497
102	23.148	36.6352	4.09904	10.5195	21.189	26.90302	44.916	0	55.1032	44.9164	0	55.9497
103	23.824	36.6352	0	10.5195	21.642	26.90302	46.482	0	55.1032	46.4819	0	55.9497
104	26.501	36.6352	0	10.5195	22.482	26.90302	48.887	0	55.1032	48.8869	0	55.9497
105	20.177	36.6352	0	10.5195	20.861	26.90302	49.672	0	55.1032	49.6716	0	55.9497
106	21.854	36.6352	0	10.5195	21.646	26.90302	51.241	0	55.1032	51.2409	0	55.9497

S/N	CUSUM Chart			EWMA Chart			MEC Chart			MECH Chart		
	C+	H	C-	LCL	Z	UCL	M+	M-	b	M+	M-	b
107	19.53	36.6352	0	10.5195	21.234	26.90302	52.399	0	55.1032	52.3987	0	55.9497
108	16.207	36.6352	0	10.5195	20.676	26.90302	52.998	0	55.1032	52.998	0	55.9497
109	5.8832	36.6352	3.09904	10.5195	18.507	26.90302	51.428	0	55.1032	51.4283	0	55.9497
110	0	36.6352	2.19809	10.5195	17.88	26.90302	49.232	0	55.1032	49.2319	0	55.9497
111	0	36.6352	7.29713	10.5195	15.91	26.90302	45.065	1.43587	55.1032	45.0654	1.43587	55.9497
112	0	36.6352	13.3962	10.5195	14.183	26.90302	39.171	4.59926	55.1032	39.1714	4.59926	55.9497
113	0	36.6352	12.4952	10.5195	14.637	26.90302	33.732	7.3083	55.1032	33.7318	7.3083	55.9497
114	0	36.6352	20.5943	10.5195	12.728	26.90302	26.383	11.9266	55.1032	26.383	11.9266	55.9497
115	0	36.6352	26.6933	10.5195	11.796	26.90302	18.102	17.4768	55.1032	18.1022	17.4768	55.9497
116	0	36.6352	23.7924	10.5195	13.347	26.90302	11.372	21.4759	55.1032	11.3724	21.4759	55.9497
117	0	36.6352	29.8914	10.5195	12.26	26.90302	3.556	26.5618	55.1032	3.556	26.5618	55.9497
118	0	36.6352	26.9904	10.5195	13.695	26.90302	0	30.2127	55.1032	0	30.2127	55.9497
119	0	36.6352	25.0895	10.5195	14.521	26.90302	0	33.0373	55.1032	0	33.0373	55.9497
120	0	36.6352	26.1885	10.5195	14.391	26.90302	0	35.9923	55.1032	0	35.9923	55.9497
121	0	36.6352	27.2876	10.5195	14.293	26.90302	0	39.045	55.1032	0	39.045	55.9497
122	0	36.6352	23.3866	10.5195	15.47	26.90302	0	40.9211	55.1032	0	40.9211	55.9497
123	0.6765	36.6352	15.4857	10.5195	17.352	26.90302	0	40.9146	55.1032	0	40.9146	55.9497
124	0	36.6352	18.5847	10.5195	16.014	26.90302	0	42.2462	55.1032	0	42.2462	55.9497
125	0	36.6352	13.6838	10.5195	17.011	26.90302	0	42.5815	55.1032	0	42.5815	55.9497
126	0	36.6352	21.7828	10.5195	14.508	26.90302	0	45.4194	55.1032	0	45.4194	55.9497
127	0	36.6352	19.8818	10.5195	15.131	26.90302	0	47.6343	55.1032	0	47.6343	55.9497
128	0	36.6352	25.9809	10.5195	13.598	26.90302	0	51.382	55.1032	0	51.382	55.9497
129	0	36.6352	27.0799	10.5195	13.699	26.90302	0	55.0293	55.1032	0	55.0293	55.9497
130	0	36.6352	24.179	10.5195	14.774	26.90302	0	57.6012	55.1032	0	57.6012	55.9497
131	3.6765	36.6352	13.278	10.5195	17.581	26.90302	0	57.3666	55.1032	0	57.3666	55.9497
132	0	36.6352	10.3771	10.5195	17.685	26.90302	0	57.0272	55.1032	0	57.0272	55.9497
133	0	36.6352	11.4761	10.5195	16.764	26.90302	0	57.6091	55.1032	0	57.6091	55.9497
134	0	36.6352	9.57514	10.5195	16.823	26.90302	0	58.1321	55.1032	0	58.1321	55.9497
135	0	36.6352	2.67419	10.5195	18.117	26.90302	0	57.3608	55.1032	0	57.3608	55.9497
136	0	36.6352	5.77323	10.5195	16.588	26.90302	0	58.1188	55.1032	0	58.1188	55.9497
137	0	36.6352	2.87228	10.5195	16.941	26.90302	0	58.5238	55.1032	0	58.5238	55.9497
138	0	36.6352	4.97132	10.5195	15.956	26.90302	0	59.9141	55.1032	0	59.9141	55.9497
139	0	36.6352	14.0704	10.5195	13.467	26.90302	0	63.7932	55.1032	0	63.7932	55.9497
140	0	36.6352	11.1694	10.5195	14.6	26.90302	0	66.5391	55.1032	0	66.5391	55.9497
141	0	36.6352	5.26845	10.5195	16.2	26.90302	0	67.685	55.1032	0	67.685	55.9497
142	0	36.6352	3.3675	10.5195	16.4	9	0	68.6309	55.1032	0	68.6309	55.9497

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