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Calculating Effective Wilson Coefficients for Kaon Decays in Renormalization Scale $\mu = 1\ GeV$

Khadijeh Ghasemi[1], Somayeh Mehrjoo[2]

[1]Physics Department, Islamic Azad University, Central Tehran Branch, Tehran, Iran
[2]Physics Department, Lorestan University, Lorestan, Iran

Email address:
kh.ghasemi86@gmail.com (Kh. Ghasemi), mehrjoo_ph84@yahoo.com (S. Mehrjoo)

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Abstract: The decay rates of $K\ and\ \bar{K}$ mesons, consisting of a quark-anti quark, as a weak decay in the presence of strong interactions have been studied by means of the Effective Hamiltonian Theory. One of the most important key factors for calculating Effective Hamiltonian is Wilson coefficients. In this paper, effective Wilson coefficients in renormalization scale $\mu = 1GeV$ are calculated.

Keywords: K Meson, Effective Hamiltonian, Wilson Coefficients, CKM Matrix

1. Introduction

One of the successful models in particle phenomenology is the quark model which is applied to calculate the decays of various particles with a few differences. The particles called kaons, or K mesons, were first observed in the late 1940s in cosmic-ray experiments. By today's standards, they are common, easily produced, and well understood. Over the last four decades research into how kaons decay has played a major role in the development of the Standard Model. Yet, after all this time, kaon decays may still prove to be a valuable source of new information on some of the remaining fundamental questions in particle physics.

When first observed, kaons seemed quite mysterious. Experiments showed that they were produced in reactions involving the strong force, or strong interaction—the most powerful of the four fundamental forces in nature—but that they did not decay (that is, transform into two or more less massive particles) through the strong interaction. This is because kaons have a property, ultimately labeled "strangeness," which is conserved in the strong interaction [10].

One of the most interesting and unique observed particles in the nature is kaon. There are two neutral kaons which are, in fact, strange mesons.

$$\begin{aligned} k^\circ &= d\bar{s} \quad (s=-1) \\ \bar{k}^\circ &= s\bar{d} \quad (s=+1) \end{aligned} \tag{1-1}$$

s is the Eigenvalue of the strange state. Since each kaon under CP effect turns into another kaon, neither of these kaons have determined CP number. k° and $\bar{k}^\circ$ are not eigenstate of CP. However, when CP acts on them, they are conjugate of each other.

$$\begin{aligned} CP\left|k^\circ\right\rangle &= -\left|\bar{k}^\circ\right\rangle \\ CP\left|\bar{k}^\circ\right\rangle &= -\left|k^\circ\right\rangle \end{aligned} \tag{1-2}$$

But theorists can make a pair kaon with determined CP from combination of wave function k° and $\bar{k}^\circ$.According to Quantum Mechanics rules, these combinations corresponding with real particles and have a mass and determined lifetime. Therefore normalized eigenstate CP are [3, 9]:

$$\begin{aligned} \left|k_1\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|k^\circ\right\rangle - \left|\bar{k}^\circ\right\rangle\right) \\ \left|k_2\right\rangle &= \frac{1}{\sqrt{2}}\left(\left|k^\circ\right\rangle + \left|\bar{k}^\circ\right\rangle\right) \end{aligned} \tag{1-3}$$

So,

$$\begin{aligned} CP\left|k_1\right\rangle &= \left|k_1\right\rangle \quad (CP=+1) \\ CP\left|k_2\right\rangle &= -\left|k_2\right\rangle \quad (CP=-1) \end{aligned} \tag{1-4}$$

k_1 just can decays to $CP=+1$ state, while k_2 should go to $CP=-1$ state. Neutral kaons usually decay to two or three pions. Arrangement of two pions has +1 parity and three pions system has -1 parity and both of them have a $C=+1$. As a result, k_1 decays to two pions and k_2 decays to three pions [11].

$$\begin{aligned} k_1 &\to 2\pi \\ k_2 &\to 3\pi \end{aligned} \tag{1-5}$$

Since a kaon has hardly enough mass to produce three pions, two pion decays are fast but three pion decays are longer. Observed lifetimes are about 10^{-10}s and 10^{-7}s , respectively [3, 12].

K mesons decay as a weak decay in the presence of strong interactions requires a special approach. The main tool to investigate these decays is the effective Hamiltonian theory. Beginning of any phenomenological weak decay of hadrons is the effective weak Hamiltonian that its structure is as follows [4, 6]:

$$H_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) Q_i \tag{1-6}$$

Where G_F is the Fermi constant that in terms of the g_w weak coupling constant and W boson mass is defined as follows:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2} \tag{1-7}$$

And Q_i are the local operators that decays discussed in turn controlled. V^i_{CKM} Cabibbo – Kobayashi – Maskawa factors and C_i Wilson Coefficients are described the force with which an operator enters the Hamiltonian. In fact, the effective point-like vertices are represented by local operators can correct picture of the decay of hadrons with a mass of the order of $O(m_b, m_c)$ a better way to provide. C_i The Wilson coefficients to be used as coupling constants (depending on scale) corresponding to the vertices are considered. Select the μ scale is optional, but it is customary that to choice μ the order of the mass of hadrons decay, eg for B and D mesons decays, the value of μ are respectively the order of m_b and m_c . For kaon decays the common choice of μ is the order of $1-2$ Gev instead of m_K order [1].

2. Theoretical Framework

In this paper, Wilson coefficients of s quark and $\bar{s}$ antiquark decays are calculated [2]. General framework of how to calculate Wilson coefficients is based on that (1-6) equation which has already been mentioned in the introduction. Effective Hamiltonian of the $k \to \pi\pi$ transition is defined as follows [2].

$$H_{eff}(\Delta S=1) = \frac{G_F}{\sqrt{2}} V_{ud} V^*_{us} \left(\sum_{i=1}^{10} C_i(\mu) Q_i(\mu) \right) + \text{h.c.} \tag{2-1}$$

In this equation, G_F is the Fermi constant and Q_i is the local operator which controls the decay. C_i coefficients are showed Wilson coefficients. The overall structure of the Wilson coefficients is as follow:

$$C_i(\mu) = Z_i(\mu) + \tau\, y_i(\mu) \tag{2-2}$$

In this equation τ is defined as follows:

$$\tau = \frac{-V_{td} V^*_{ts}}{V_{ud} V^*_{us}} \tag{2-3}$$

In the τ equation V_{td}, V_{us}, V_{ts} and V_{ud} are the elements of the Cabibbo – Kobayashi – Maskawa matrix. Cabibbo – Kobayashi – Maskawa matrix is a unitary matrix which contains information on the strength of flavor changing weak decays. Technically, it specifies the mismatch of quantum states of quarks when they propagate freely and when they take part in the weak interactions [3, 13].

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2-4}$$

CKM matrix is the 3×3 matrix, since there are three generations of quarks, which Kobayashi and Maskawa in 1973 stated that the third generation of quarks to the matrix, mixed phases that, if not zero, it is symmetry breaking. If this phase is virtually zero, to explain the CP failure must seek something beyond the standard model.

Several methods have been proposed for CKM matrix parameterization which among them to discuss the introduction of standard parameterization.

$$V_{CKM} = \begin{pmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta} & c_{23}\, c_{13} \end{pmatrix} \tag{2-5}$$

In which, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$ for $i, j = 1, 2, 3$. δ is the phase which is in the range of $\circ \le \delta \le 2\pi$. Matrix elements are calculated by using the following data [14]:

$$\theta_{12} = 0.221$$
$$\theta_{13} = 0.0035$$
$$\theta_{23} = 0.041 \tag{2-6}$$
$$\delta = \frac{\pi}{2}$$

$$V_{CKM} = \begin{pmatrix} 0.9756 & 0.2192 & -0.0034i \\ -0.2190 - 0.000135i & 0.9747 - 0.000030i & 0.4089 \\ 0.0089 - 0.003410i & -0.0399 - 0.000760i & 0.9990 \end{pmatrix} \tag{2-7}$$

To obtain s quark decay rate, we need the effective Wilson coefficients of the tree and penguin decay. The effective Wilson coefficients can be defined as follows [3]:

$$C_i^{eff}(\mu) = Z_i^{eff}(\mu) + \tau\, y_i^{eff}(\mu) \tag{2-8}$$

Here $Z_i^{eff}(\mu)$ are defined $as\ follows$ [2]:

$$Z_6^{eff}(\mu) = Z_6(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{6i} Z_i(\mu) + \frac{\alpha_s}{8\pi}\left(C_t + C_p\right)$$
$$Z_7^{eff}(\mu) = Z_7(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{7i} Z_i(\mu) + \frac{\alpha}{8\pi}C_e$$
$$Z_8^{eff}(\mu) = Z_8(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{8i} Z_i(\mu) \tag{2-9}$$
$$Z_9^{eff}(\mu) = Z_9(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{9i} Z_i(\mu) + \frac{\alpha}{8\pi}C_e$$
$$Z_{10}^{eff}(\mu) = Z_{10}(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{10i} Z_i(\mu)$$

$$Z_1^{eff}(\mu) = Z_1(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{1i} Z_i(\mu)$$
$$Z_2^{eff}(\mu) = Z_2(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{2i} Z_i(\mu)$$
$$Z_3^{eff}(\mu) = Z_3(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{3i} Z_i(\mu) - \frac{\alpha_s}{24\pi}\left(C_t + C_p\right)$$
$$Z_4^{eff}(\mu) = Z_4(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{4i} Z_i(\mu) + \frac{\alpha_s}{8\pi}\left(C_t + C_p\right)$$
$$Z_5^{eff}(\mu) = Z_5(\mu) + \frac{\alpha_s}{4\pi}\left(\gamma_V^{(0)T}\ln\frac{\mu_f}{\mu} + \hat{r}_V^T\right)_{5i} Z_i(\mu) - \frac{\alpha_s}{24\pi}\left(C_t + C_p\right)$$

In these equations, T is matrix transpose operator. Matrix with variable dimensions $\gamma_V^{(0)}$ and constant matrix $\hat{r}_V$ are obtained from correction of $Q_1 - Q_{10}$ operators' vertex. Also, the values of C_e, C_p and C_t are extracted from QCD Penguin diagram, $Q_{1,2}$ operators and QCD Penguin diagram, $Q_3 - Q_6$ operators and electroweak penguin diagrams $Q_7 - Q_{10}$, respectively.

$$\begin{aligned} C_t &= \tilde{G}(m_u) Z_1 \\ C_p &= \left[\tilde{G}(m_s) + \tilde{G}(m_d)\right] Z_3 + \sum_{i=u,d,s} \tilde{G}(m_i)(Z_4 + Z_6) \\ C_e &= \frac{8}{9}\tilde{G}(m_u)(Z_1 + 3Z_2) \\ \tilde{G}(m_q) &= \frac{2}{3}\kappa - G(m_q, k, \mu) \end{aligned} \tag{2-10}$$

In which, κ is parameter that in dimensional regulation, describes dependency γ_5 model. For example, in γ_5 model we have Naïve Dimensional Regularization (NDR) and Hooft-Veltman (HV):

$$\kappa = \begin{cases} 1 & \text{NDR} \\ 0 & \text{HV} \end{cases} \tag{2-11}$$

Function $G(m,k,\mu)$ in (2-6) equations is defined as follows:

$$G(m,k,\mu) = -4\int_0^1 dx\, x(1-x)\ln\left(\frac{m^2 - k^2 x(1-x)}{\mu^2}\right) \tag{2-12}$$

In which k^2 is the square of the momentum carried by virtual gluons. $\hat{r}$ Matrix in (2-9) equations gives constant terms; they are independent of the momentum that is based on the γ_5 behavior in the dimensional regularization. For the kaon decays, there was no heavy quark mass scale between m_c and m_K. Therefore, logarithmic term arising from corrections four quarks operations vertex to $\ln \mu_f/\mu$ form will be like (2-9) equation. We will assume that $\mu_f = 1\ GeV$ as a reliable estimate obtained of destruction effects the effective Wilson coefficients. As a result, we have [2, 4]:

$$\gamma^{(0)} = \begin{pmatrix} -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -2 & 6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 6 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -16 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix} \tag{2-13}$$

$$\hat{r}_{NDR} = \begin{pmatrix} 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & -9 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -9 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & -9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -9 & 3 \end{pmatrix} \quad (2\text{-}14)$$

$$\hat{r}_{HV} = \begin{pmatrix} 7/3 & -7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -7 & 7/3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 7/3 & -7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -7 & 7/3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1/3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -3 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7/3 & -7 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -7 & 7/3 \end{pmatrix} \quad (2\text{-}15)$$

3. Conclusion

Table 1. *Z_i^{eff} in renormalization scale $\mu = 1GeV$.*

	NDR	HV	$Z_i^{eff}(NDR)$	$Z_i^{eff}(HV)$
Z_1	1.278	1.371	1.718	1.713
Z_2	−0.509	−0.640	−1.113	−1.110
Z_3	0.013	0.007	0.032	0.032
Z_4	−0.035	−0.017	−0.081	−0.084
Z_5	0.008	0.004	0.024	0.025
Z_6	−0.035	−0.014	−0.086	−0.086

Table 2. *y_i^{eff} in renormalization scale $\mu = 1GeV$.*

	NDR	HV	$y_i^{eff}(NDR)$	$y_i^{eff}(HV)$
y_1	0	0	0	0
y_2	0	0	0	0
y_3	0.031	0.036	0.050	0.049
y_4	−0.056	−0.059	−0.053	−0.053
y_5	−0.001	0.016	0.003	0.002
y_6	−0.109	−0.096	−0.160	−0.138

Table 3. *The effective Wilson coefficients in renormalization scale μ=1GeV.*

	$C_i^{eff}(NDR)$	$C_i^{eff}(HV)$
C_1^{eff}	1.718	1.713
C_2^{eff}	−1.113	−1.110
C_3^{eff}	$0.0320834 - 0.0000335139i$	$0.0320817 - 0.0000328436i$
C_4^{eff}	$-0.0810884 + 0.0000355247i$	$-0.0840884 + 0.0000355247i$
C_5^{eff}	$0.024005 - 2.01083\times10^{-6}i$	$0.0250033 - 1.34056\times10^{-6}i$
C_6^{eff}	$-0.0862669 + 0.000107244i$	$-0.0862302 + 0.0000924984i$

By using the effective Lagrangian density of the weak interaction, we can calculate decay rate in tree level. Furthermore, decay rates of S quark –anti quark can be calculated in tree and penguin level by the use of the effective Hamiltonian Theory. This is possible by the means of Effective Wilson coefficients.

In this paper, Effective Wilson coefficients are calculated. In table 1 Z_i^{eff} values were calculated. Moreover, the numerical values of y_i^{eff} are showed in the table 2 [5]. In conclusion, Table 3 shows calculated values for the effective Wilson coefficients for the decay of s quark and $\bar{s}$ antiquark in renormalization scale μ=1GeV.

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