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# On the Variable Acceleration Motion of Falling Sphere in a Fluid Medium

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**Abstract:** Falling motion of a sphere in the stationary medium is an important basic research and has a wide range of engineering application prospects. This paper studies the free fall motion of spheres in a stationary medium considering air drag. This paper calculates the acceleration, velocity and displacement of the falling process of the sphere by Excel, and discusses the influence of the density, diameter and Re on the motion. Some analysis and fitting of displacement over time are performed. The results show that the varied-acceleration motion of the falling spheres is similar for different sphere materials, namely, the falling acceleration, the velocity and falling displacement all can be divided into three segmentations and fitted with polynomial functions. The relationship between displacement *h* and time *t* during falling with varied- acceleration is also greatly affected by the density of the sphere, the greater the density of the sphere, the closer the relationship between *h* and *t* during the fall process is closer to no drag free fall, the smaller the density of the sphere is, the closer the relationship between *h* and *t* is nearer to linear relationship. The displacement data of different solid material sphere is located between the fall of water and the free drop without drag, but does not meet power function or the multi-relationship function. Some results are compared with the literature experiments, and are consistent with the experiments.

**Keywords:** Fall of Sphere, Settlement, Kinematics, Varied Acceleration Motion, Nonlinear Motion

## 1. Introduction

The natural falling motion of sphere, or the flow around the sphere and the cylinder are the most extensive basic problem encountered in fluid mechanics, mechanical engineering, aerospace and aviation, military and other fields. For example, the movement of river sediment, various settlements [1], the migration of drilling cuttings [2], the minimization of erosion of large turbine droplets, the prediction of atmospheric pollutant motion and PIV measurement, pneumatic transport and other sphere falling motion [3-8], all these problems require an understanding of particle kinematics determined by particle dynamics. Therefore, this is the main reason why the falling movement of the sphere or the flow around sphere have been continuously paid attention and extensively studied over the past few centuries. Because the falling movement of the sphere in the medium is a nonlinear motion of variable acceleration, its motion is basic and complex, and it is the most basic topic in the field of fluid mechanics and related engineering research, hundreds of years passed, you can still find the latest research content from the literatures. Professor M. M. Zdravkovich gives a perfect analogy "The study of flows around cylinder is like putting together a complex and wonderful puzzle, although there are a large number of discrete blocks, but the puzzle is still not completed, and new blocks are still being discovered" [9]. These word are also fit to the study of falling motion of sphere with viewpoints on sphere falling motion. We are convinced that researchers in this field also want to see the whole picture of the puzzle in the motion laws on sphere falling with force caused by flow around sphere in the fluid medium, but unfortunately, a complete law on kinematics of falling sphere in the varied acceleration has not appeared, but people are still working tirelessly to uncover the theory and solution of the nonlinear problem.

From the existing literature, the study of the vertical variable acceleration movement of sediment particles is less involved [10]. The general view of sediment settlement is that sediment begins to settle and approaches a constant sedimentation velocity is a relatively fast process, which is approached in power form [10] According to references, for small particles, their speed varies very fast from 0 to the constant settling velocity. For large size particles, such as stones, it takes more time for their speed to become close to the settling velocity than small particles. These conclusions are drawn with the drag coefficient is restricted by Stokes equation.

This paper studies the free fall motion of spheres in a stationary medium considering air drag. Starting from the forces analysis of a sphere in a fluid medium, we derived the formulas of variable acceleration motion, not assuming the velocity distribution and then finding the motion rules, but sets out from the forces banlance of acceleration, through dimensional analysis and comparison, the expression of the acceleration of the falling sphere and the expression of the velocity and displacement of the sphere when the Stokes formula is used as drag coefficient are obtained. The derivation method is not opened in this paper.

In addition, many literatures on the falling motion of the sphere only consider the Stokes formula with small Re, in order to break this limit, to study the kinematics of the sphere with extensive resistance for the flow around the sphere, this paper uses Excel table to compile the calculation program of the falling sphere motion in the fluid medium within the range of $0<Re<3X10^5$, the nonlinear acceleration, velocity and displacement of the sphere falling in the medium are calculated out and the results are displayed, also, the influence of the density of the sphere, the diameter of the sphere and Re on the falling kinematics are studied.

## 2. The Basic Equations of the Motion of a Sphere Falling in a Fluid Medium

For a spherical sphere with a mass of M, general equation of motion in a fluid medium can be expressed as follows (Coordinate direction is up)

$$\sum \vec{F} = M\frac{d\overline{u}}{dt} \tag{1}$$

Where $\overline{u}$ is the instantaneous motion speed of the sphere, $t$ is time, and $\sum \vec{F}$ is the sum of the external forces experienced by the particles, which can be expressed by the following formula [10]:

$\sum \vec{F}$ =Gravity + buoyancy + drag + additional mass force + Basset force+ Magnus force+ Saffman force (2)

The last two terms on the right side of the above equation are collectively referred to as lift, which is the resistance caused by the rotation of the object when moving in the medium, It is usually in small order and can be ignored in general. The other forces are calculated as follows, where $\rho_a$, $\mu$, $v$ are the density, dynamic viscosity and kinematic viscosity of the fluid respectively, $g$ is the acceleration of gravity, $d$ is the diameter of the sphere, $\rho_s$ is the density of sphere, and $u$, $u_a$ are the vertical motion velocity of the sphere and the fluid respectively.

$$F_{drag} = \frac{1}{2}\rho_a u^2 C_d A \tag{3}$$

where $C_d$ is the drag coefficient, and in the case of small Reynolds numbers ( Re<1 ), the Stokes resistance formula holds. The resistance term is expressed using the following formula:

$$F_{drag} = -3\pi d\mu u \tag{4}$$

The additional mass force is

$$F_{add} = -\frac{1}{2}m\varepsilon \tag{5}$$

where relative acceleration is defined as

$$\varepsilon = \frac{du}{dt} - \frac{du_a}{dt} \tag{6}$$

$m$ is the fluid mass with the same volume of a sphere, the additional mass force expresses the resistance of the fluid to obstruct the unsteady motion of the object, and its effect is expressed as the increase of the apparent mass of the sphere.

Due to the viscous nature of the fluid, when the sphere has relative acceleration to change velocity, the flow field around the sphere cannot be stabilized immediately. Therefore, the force of the fluid on the sphere depends not only on the acceleration of the sphere, but also on the history of acceleration, if resistance takes Stokes equation, this part is called Basset force,

$$F_{Ba} = -\frac{3}{2}d\rho\sqrt{\pi v}\int_0^t \frac{\varepsilon(t')}{\sqrt{t-t'}}dt' \tag{7}$$

Comparing it to gravity, there are relative values

$$\phi = \frac{F_{Ba}}{mg} = -\frac{3}{2mg}d\rho\sqrt{\pi v}\int_0^t \frac{\varepsilon(t')}{\sqrt{t-t'}}dt' \tag{8}$$

If the relative acceleration $\varepsilon$ is constant, $\varepsilon = \frac{u-u_a}{t-t_0}$, then

$$\int_0^t \frac{\varepsilon(t')}{\sqrt{t-t'}}dt' \approx \frac{u-u_a}{t-t_0}\int_0^t \frac{1}{\sqrt{t-t'}}dt' = 2\frac{u-u_a}{\sqrt{t-t_0}} \tag{9}$$

Only in the early stage of accelerated motion, when the Basset force is about 10% of gravity, the Basset force is important, otherwise it can be ignored [10]. For example [10], for the particles $d = 0.1mm$, take the falling velocity of a single particle $u = 0.54cm/s$ in water with T = 15°C, then the Basset force can be ignored when $t - t_0 > 3.2\times10^{-3}s$.

$$M\frac{du}{dt} = (M-m)g - \frac{1}{2}\rho_a u^2 \mathrm{C_d A} - \frac{1}{2}m(\frac{du}{dt} - \frac{du_a}{dt}) \quad (10)$$

The first term on the right side of the equation is the effective weight of the sphere in the fluid medium; The second considers the resistance exerted by the fluid to the sphere after upward reflux; The third term takes into account the additional mass force of the acceleration change after upward reflux.

## 3. Dimensional Analysis for Varied Acceleration Motion of Sphere

During the fall of the sphere, the acceleration $a_f$ generated by the force exerted by the fluid on the sphere is related to the fluid density $\rho_\mathrm{a}$, fluid kinematic viscosity $\nu$, sphere density $\rho_\mathrm{s}$, sphere diameter $d$, time $t$, and gravitational acceleration $g$, so it can be expressed as

$$f(\rho_\mathrm{a}, \rho_\mathrm{s}, \nu, d, t, g, a_f) = 0$$

Select $\rho_\mathrm{a}$, $d$, $t$ as the basic quantity, and perform dimensional analysis on the remaining quantities, there are dimensionless numbers

$$\pi_1 = \frac{\rho_\mathrm{s}}{\rho_\mathrm{a}};\ \pi_2 = \frac{\nu t}{d^2};\ \pi_3 = \frac{gt^2}{d}$$

Therefore, the acceleration generated by the force of the fluid acting on the sphere is expressed as

$$a_f = \frac{d}{t^2} f(\frac{\rho_a}{\rho_s}, \frac{\nu t}{d^2}, \frac{gt^2}{d}) \text{ or } a_f = f(\frac{\rho_s}{\rho_a d}t^2, \frac{d}{\nu}t, \mathrm{g}) \quad (11)$$

It can be seen that when the sphere and the fluid density are certain and the sphere diameter is constant, the acceleration of the fluid acting on the sphere has a functional relationship with the dimensionless number $\frac{\nu t}{d^2}$, $\frac{gt^2}{d}$, and is a function of time and gravitational acceleration, that is, $a_f = f(e_1 t^2, e_2 t, \mathrm{g})$, $e_1$, $e_2$ are the coefficients.

In the falling process of a sphere, the motion first meets the formula (10). Generally, the kinematics does not consider the resistance of fluid. In this paper, the air resistance is related to the falling speed, and the speed changes with the falling time. The motion is with variable acceleration. Suppose the acceleration applied by the fluid to the sphere is $a_f$, the acceleration of the sphere in the fluid medium is $a$, $g - a = a_f$, as can be seen from the previous text, the acceleration generated by the fluid acting on the sphere is a function of time t. Through theoretical research such as dimensional analysis and comparison, the coefficient formula can be obtained if the resistance coefficient meets the Stokes formula. The theoretical method for obtaining the coefficient will not be opened in this article for the time being, and only the conclusion will be here. After deduction, when the resistance coefficient meets the Stokes formula, the quadratic term of time can be ignored, then there are

$$a = c_1 t^2 + c_2 t + c_3$$

$$c_2 = \frac{18\rho_a \nu g}{\rho_\mathrm{s} d^2};\ c_3 = \frac{\rho_a}{\rho_\mathrm{s}} g$$

For variable acceleration system, jerk is $X = \frac{\partial a}{\partial t}$

$$X = \frac{\partial a}{\partial t}$$

$$a = a_0 + Xt \quad (12)$$

$$u = u_0 + a_0 t + \frac{1}{2}Xt^2$$

$$h = u_0 t + \frac{1}{2}a_0 t^2 + \frac{1}{6}Xt^3$$

So there are

$$\begin{aligned} a(t) &= (1 - \frac{\rho_a}{\rho_\mathrm{s}})g - \frac{18\rho_a \nu g}{\rho_\mathrm{s} d^2}t \\ u(t) &= (1 - \frac{\rho_a}{\rho_\mathrm{s}})gt - \frac{9\rho_a \nu g}{\rho_\mathrm{s} d^2}t^2 \\ h(t) &= (1 - \frac{\rho_a}{\rho_\mathrm{s}})\frac{1}{2}gt^2 - \frac{3\rho_a \nu g}{\rho_\mathrm{s} d^2}t^3 \end{aligned} \quad (13)$$

The above formula is consistent with the conclusion of the dimensional analysis theorem, but the dimensional theorem cannot give a specific formula. Analyze the physical meaning of the acceleration formula, we know the acceleration can be divided into three parts, including gravity acceleration, buoyancy acceleration, and resistance acceleration related to the viscosity of the fluid medium and boundary separation drag of sphere. The physical meaning $\frac{18\rho_a \nu g}{\rho_\mathrm{s} d^2}$ is the change rate of acceleration with time, that is, jerk. The combined action of three kinds of accelerations produces nonlinear changes of velocity and motion displacement in fluid. The nonlinear kinematics of the sphere is a function of the density ratio of the fluid to the material of the sphere, the kinematic viscosity of the fluid, and the acceleration of gravity, and is proportional to the - 1/2 power of the diameter of the sphere.

The experimental data of distance of the falling body in the literature [11] are used to prove the correctness of the theoretical formula.

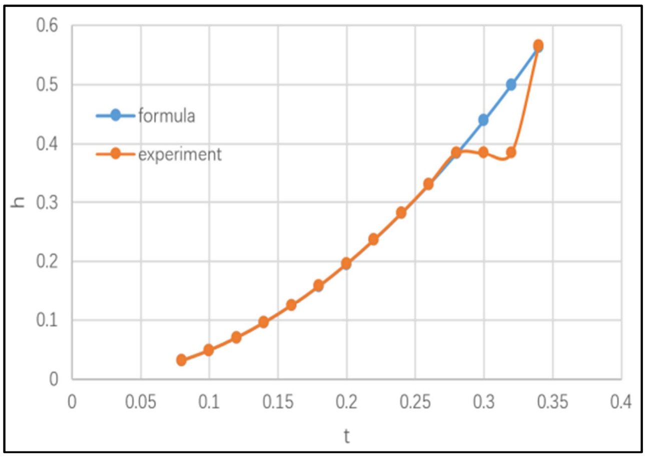


***Figure 1.*** *The formula is agree with experimental data.*

It can be seen from the figure that the distance calculated by the derived formula are consistent with the experimental data, except that the two points of the experimental data are obviously error.

## 4. Study the Falling Accelerating Motion of the Sphere Under 0<Re<3X10$^5$ with Excel Programming

The basic equation of Excel programming is still equation (10). The gravity, buoyancy and drag resistance of the sphere are reserved for the calculation of variable acceleration. The fluid medium is air. The sphere material includes stone, iron, copper and water. The air drag coefficient of the ball movement in the calculation is calculated according to the figure below [1].

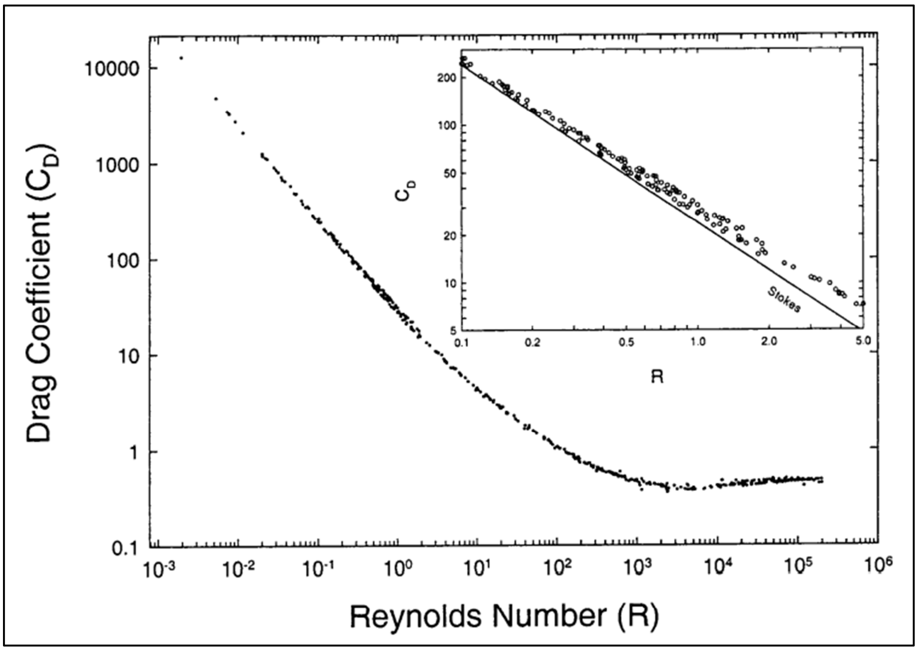


***Figure 2.*** *Drag coefficient changes with Re.*

The drag coefficient also can be calculated according to the formulas [2].

$$
\begin{aligned}
&C_d = \frac{24}{\mathrm{Re}} && (\mathrm{Re} < 1) \\
&C_d = \frac{24}{\mathrm{Re}}(1+\frac{3}{16}\mathrm{Re}) && (\mathrm{Re} < 2) \\
&C_d = 11\sqrt{\mathrm{Re}} && (30 < \mathrm{Re} < 300) \\
&C_d = 0.4 \sim 0.44 && (\mathrm{Re} > 300)
\end{aligned}
\tag{14}
$$

## 5. Falling Kinematics and the Effect of Sphere Density on It

During the falling process of the sphere, the influence of the sphere density can be known through dimensional analysis that the acceleration of the fluid acting on the sphere is related to the relative density, namely $a_f = \frac{d}{t^2} f(\frac{\rho_a}{\rho_s}, \frac{\nu t}{d^2}, \frac{g t^2}{d})$ ; When Stokes formula is taken as the drag coefficient, this paper deduces that the acceleration of the ball falling in the fluid, also velocity and displacement formula. According to formula 13, the effect of density is reflected in buoyancy acceleration $\frac{\rho_a}{\rho_s} g$ and on the drag acceleration term $\frac{18\rho_a \nu g}{\rho_s d^2}$ . When the drag coefficient is taken as the extensive range with Re increasing from 0 to 3X10$^5$, the influence of density on kinematics is reflected by the following Excel calculation figures.

While the acceleration, velocity and falling height of the sphere change with time, they also vary with the density of the sphere. The kinematic parameters of falling are not a single function of a certain parameter. For example, the falling distance is not a single function of density or diameter. It is different from Aristotle's view that the speed of falling of an object is determined by its weight. The heavier the object, the faster it falls; Galileo believed that the speed of falling objects had nothing to do with their weight. Heavy objects fell as fast as light objects. Galileo, from Aristotle's point of view, pointed out that if the big stone fell fast, the small stone fell slowly. then when the two were tied together, the big stone would be slowed down by the small one, so the overall speed would be smaller than the big stone; But when the whole thing is tied together, its total weight becomes larger, and the speed should be greater than the speed of the big stone before. This contradictory conclusion shows that Aristotle's view is wrong.

Galileo believed that there was only one possibility: "Heavy and light objects should fall with the same velocity". How was this idea established and what Galileo did to prove its reliability were not introduced in the textbooks. The physics textbook for senior high school of the People's Education Press pointed out that Galileo initially recognized that the velocity of falling body should be uniform, and speculated that the velocity might change uniformly with time or displacement. Later, it was found that if the uniform change of velocity with displacement was true, it would produce complex results, and then denied this view [12].

Through the calculation in this paper, the variable acceleration motion of spheres with different densities can be explained. The falling of spheres (stone, iron, copper) with the same diameter and different density is calculated. At first, the acceleration is gravity acceleration and buoyancy acceleration. With the drag increases during the falling process, the fluid begins to affect acceleration on the sphere, causing the total acceleration gradually decrease from gravity acceleration to zero, resulting the sphere moves at a uniform speed. From the figures of calculation, it can be seen that during the falling process of spheres with different density, the acceleration, velocity and displacement are relatively close to laws of motion without resistance in the initial time of falling. This is because at the beginning of falling, the difference in velocity is very small, and the influence of drag is negligible, but after a short time (about 1 second), the difference in the kinematics caused by the different density is obvious. Spheres of various materials have similar laws of variable acceleration motion with different magnitude. The sphere with large density as copper and iron has high acceleration, high speed and large falling distance than the lower density sphere at the same falling time. The kinematics of variable acceleration can be roughly divided into three stages, namely, the initial stage with negligible resistance (t<1s), the variable-acceleration stage with rapid acceleration change (t<4s) and the third stage gradually tend to uniform velocity (t>4s). The lower the density of the sphere, the faster it reaches uniform velocity, and the shorter the time of the three stages of motion from variable-acceleration to uniform velocity. When moving to the second stage, the drag of the fluid to the sphere and density effect on kinematics must be considered. The kinematic parameters of the second and third stages are very different from those without resistance.

Moreover, during the whole fall, the acceleration and velocity cannot be expressed as a function of time, but must be divided into three stages to add three trend line functions to fit the data. As shown in Figure 3 to Figure 6, the acceleration is close to each other at the initial stage of movement, $a = kt + c$, when it enters the second stage of variable acceleration, and the acceleration can be expressed as second-order polynomial $a = k_1t^2 + k_2t + c$. At the third stage of slow acceleration change, it can be expressed approximately by third-order polynomial $a = k_1t^3 + k_2t^2 + k_3t + c$. In short, after segmentation, acceleration can be approximated by polynomial function. The velocity is obtained by integrating the acceleration, and cannot be directly approximated by a function. After segmentation, it can be expressed by polynomial function.

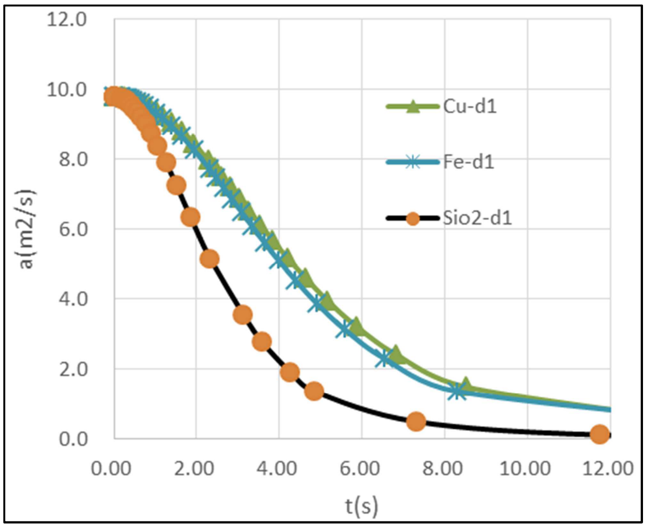


***Figure 3.*** *Acceleration of different spheres change with time.*

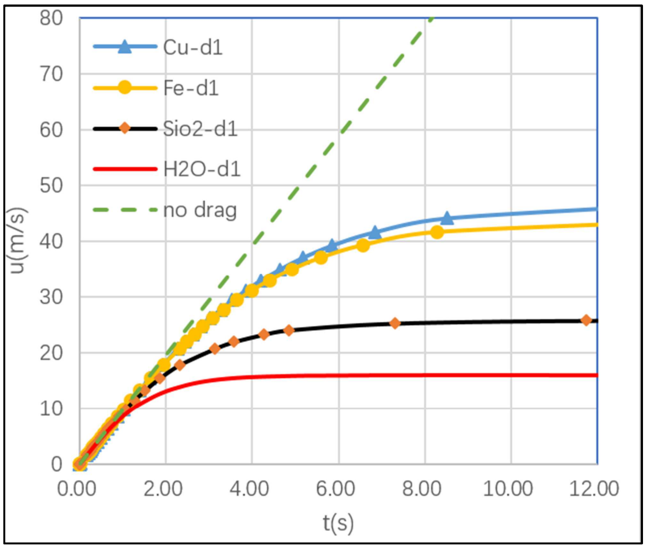


***Figure 4.*** *Velocity of spheres in different density vary with time.*

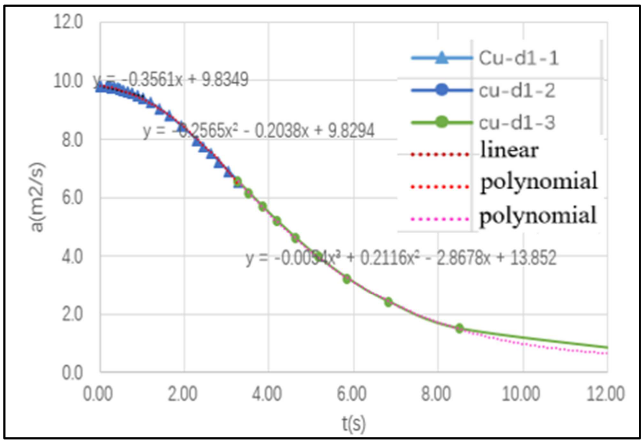


***Figure 5.*** *Polynomial fitting of acceleration in three segments.*

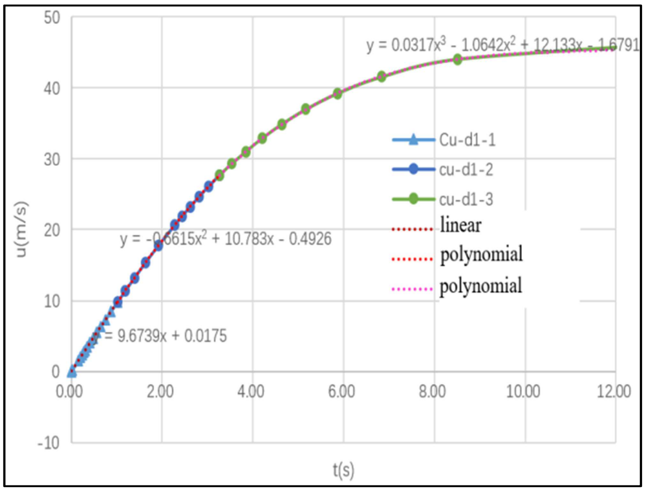


***Figure 6.*** *Polynomial fitting of velocity in three segments.*

Figure 7 showed the speed measurement spectrum made in the literature [14], and Figure 8 [14] shows the falling speed (solid line) of 1mm steel ball measured in the experiment, the calculated speed without falling resistance (dotted line) and the calculated falling speed (dotted line) with resistance coefficient of Stokes formula. The measured limit Re is 430.

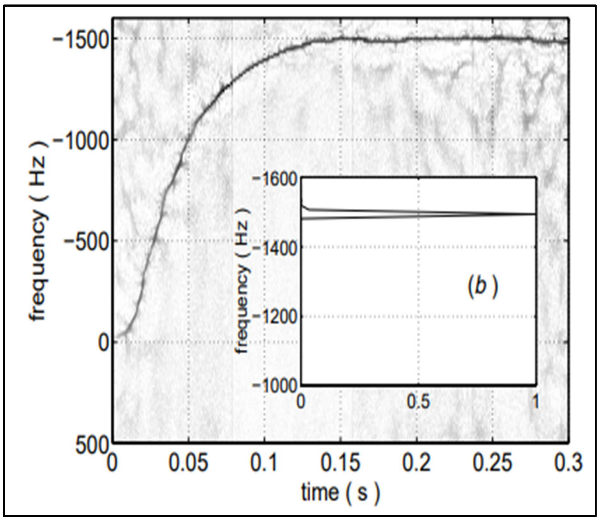


***Figure 7.*** *Frequency spectrum of velocity measurement.*

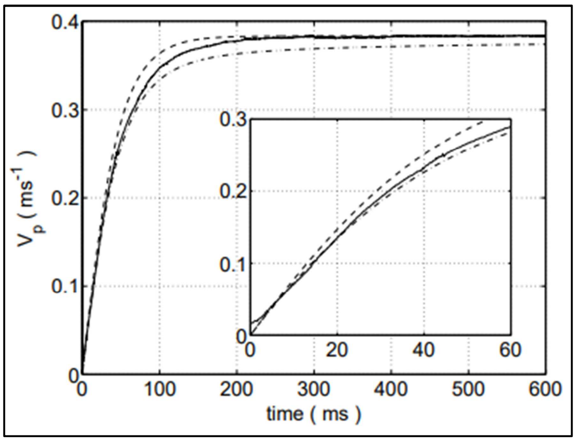


***Figure 8.*** *Experimental velocity of steel ball falling and comparison with simulation.*

Comparing the falling velocity of the sphere calculated in this paper with that measured in the literature [13] (The measured limit of Re is 430), it can be seen that the calculation in this paper is consistent with the measurement law in the literature.

Figure 9 shows the relationship between the falling velocity and displacement of spheres. We can see that the falling displacement of spheres, with the same diameter, increase with density. The larger the density is, the greater speed of the sphere arrives The dotted line is the case without drag resistance. The falling curves of solid materials is between water and non-resistance falling. At the beginning of the fall, the speed of all material is close to speed calculated without drag, but soon the speed of various spheres will be different. To reach the same speed, the minimum density requires the maximum fall distance. When sphere material changes from water to other high density, the influence of the sphere material is small in height less than 1m. After that, the influence of the sphere material is obvious. The relationship of falling distance and the speed of the sphere is not a simple power function, as shown in Figures 11 and 12 When the sphere material is water, in the second stage of variable acceleration, the falling distance is about 15 meters, and the movement speed of the sphere is higher than the power trend line $u = 3.32\sqrt{h}$ , but lower than the case without resistance, However, after the falling distance is more than 15 meters, the falling speed is lower than $u = 3.32\sqrt{h}$ , when it enters the third stage of movement. It can be seen that in the variable acceleration stage, $h$ cannot be described as one function of $u$. The function relationship between the falling distance $h$ and the speed of the sphere $u$ is between the parabola $h = 0.08u^2$ and the non-resistance function, but it is not satisfied with power function, nor exponential relations, and can not directly give a fitting formula, and it must be studied in sections to give the functional relationship of polynomials. Figure 12 shows the relation of $u$ and the $h$ of water is far away power function or logarithm relations.

Figure 10 shows the experimental data of liquid droplets with diameters of 2.96, 2.80, 2.19 and 1.78 mm at drop distance of 1~9 m [14]. The dotted line in the figure shows the situation without drag. The experimental results are consistent with the calculation in this paper.

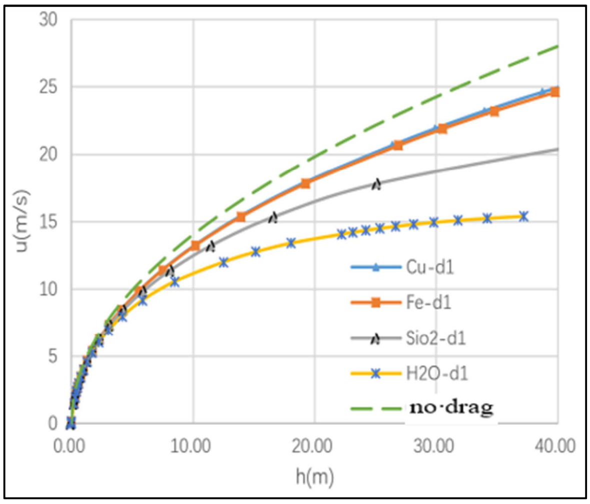


***Figure 9.*** *Relation between falling velocity and displacement of different spheres.*

The calculation of falling displacement $h$ in this paper is generated by integration $h = \int u dt$ . Figure 13 gives the relationship between $h$ and $t$ in the process of falling. It can be seen from the figure that the greater the density of the sphere is, the closer the relationship between $h$ and $t$ in the process of falling is near to $h = \frac{1}{2} g t^2$, the smaller the density of the sphere is, the closer the relationship between $h$ and $t$ is near to $h = kt + c$, and the displacement of the falling solid sphere meet the range between $h = kt + c$ for water and $h = \frac{1}{2} g t^2$ for non-resistance falling, but the relationship between distance and time is not polynomial.

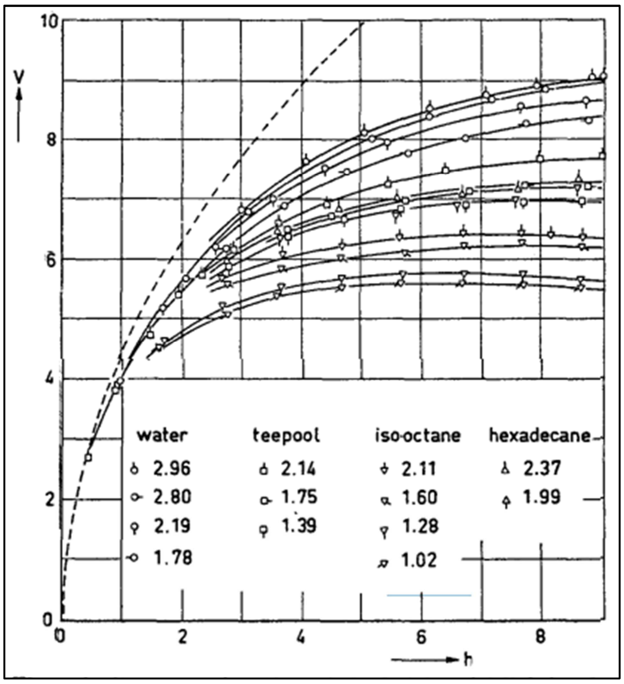


***Figure 10.*** *Experimental data of falling velocity and displacement of liquid sphere.*

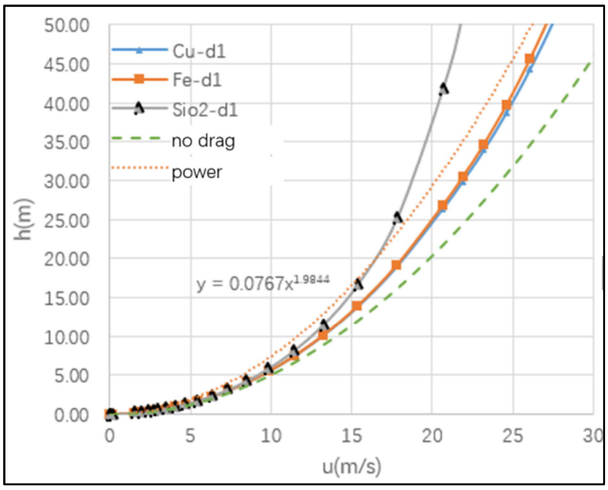


***Figure 11.*** *Relationship between falling displacement and velocity of spheres.*

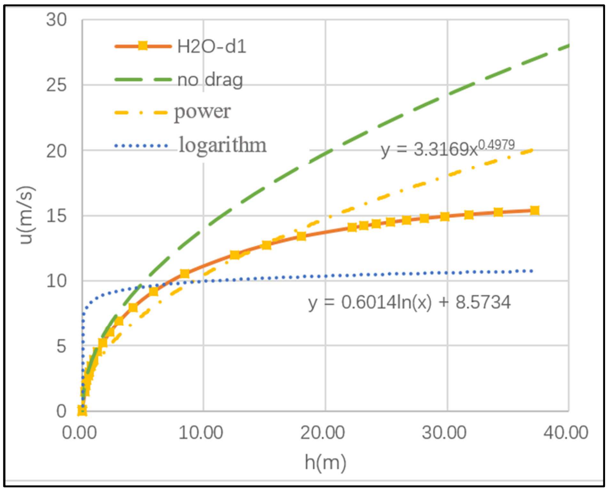


***Figure 12.*** *Relationship between falling velocity and displacement of water droplets.*

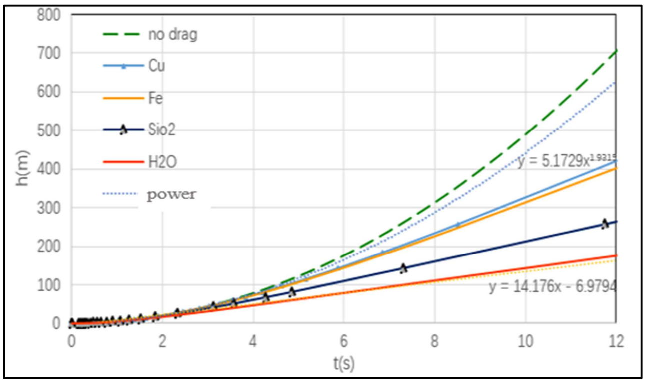


***Figure 13.*** *The relation between the falling displacement of the sphere and time.*

## 6. Effect of Re on Falling of Sphere

The comprehensive parameter Re is investigated while the dimensionless number Re change with time in the falling process. Re is defined as $\mathrm{Re} = \frac{ud}{\nu}$, where $u$ is the falling speed of the sphere, $d$ is the diameter of the sphere, and $\nu$ is the kinematic viscosity of the ambient medium. In this paper, it refers to the viscosity of air.

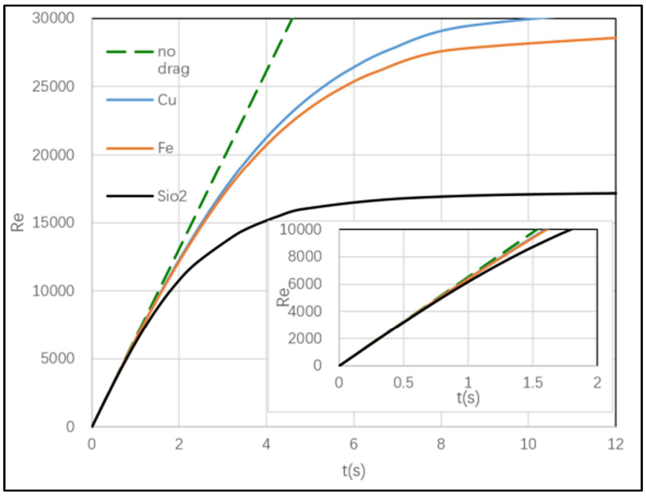


***Figure 14.*** *The change of Re with time in the falling process.*

We calculate the sphere motion in a same diameter for Re in 0~30000, from Figure 14 we can see that all spheres motion are similar, that is, at the beginning of the movement, Re coincides approximately in about 1 second, as Re increases, spheres of different materials experience variable acceleration movement, for silica spheres, from about 1s to 6 seconds is the second varied-acceleration stage, for Fe and Cu spheres, about 1s to 9 seconds is the second stage, and as Re continues to increase, in the third stage the sphere slowly approaches the limit speed for a limited time. At Re<5000, the material of the sphere has little influence on the kinematics of the sphere, and the kinematic curves of various materials approximately coincide, which indicates that the fluid is in the laminar, viscous drag and separation drag have no significant effect on the sphere, when the flow enters turbulence, due to the flow around the sphere will produce a tail vortex affecting the drag, producing a negative acceleration on the sphere.

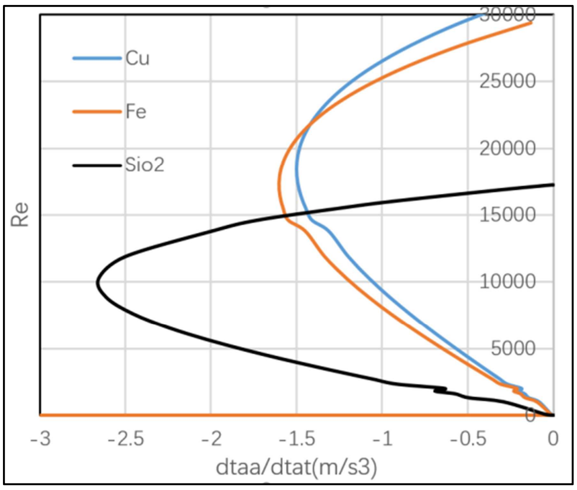


***Figure 15.*** *Jerk changes with Re.*

Figure 15 shows the relationship between Re and the derivative of acceleration, i.e. jerk. The jerk change with Re for three different sphere materials showed in the figure. The law is also similar, the jerk increase with Re firstly, then decreases to the inflection point and then increases, acting like a parabolic change, after reaching the limit speed, the jerk is zero. When Re is about 2000, jerk oscillates, because the fluid is transferred from laminar flow to turbulence, and the appearance of this phenomenon has nothing to do with the sphere material.

# 7. The Influence of the Diameter of the Sphere on Falling

The nonlinear kinematics in the falling process are also related to the diameter of the sphere. The fall of different size sphere shows that the diameter has a significant effect on the change of acceleration. For the large-diameter sphere, acceleration change is slow than those spheres in small size, because the larger size sphere fall quickly in the large Re movement, more likely to be turbulent, more easy to produce a tail vortex behind the sphere, the drag acting on the sphere is large, such motion is kept without obvious change during the whole falling process, so that the acceleration acting on the sphere is close to linear change.

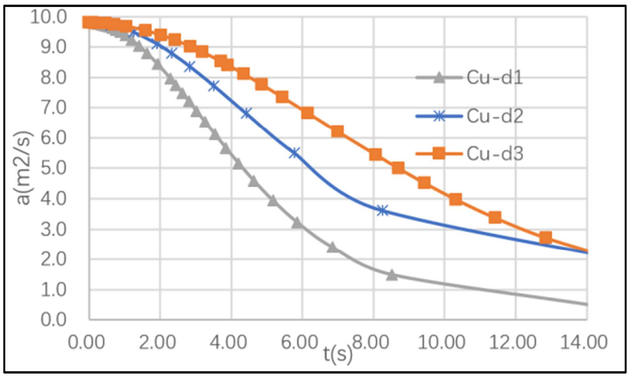


***Figure 16.*** *Acceleration varies with time in different sphere diameter.*

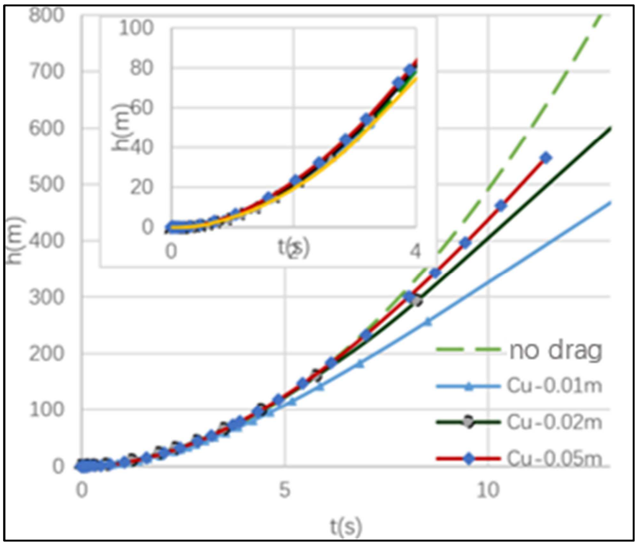


***Figure 17.*** *Displacement varies with time in different sphere diameter.*

When the falling time is longer than 4 second, the influence of the diameter of the sphere on the kinematics is obvious, the larger the diameter, the faster it falls. Compared with the previous, it can be seen that the influence of diameter is less than the influence of density on the kinematic parameters of falling.

# 8. Conclusion

In this paper, the sphere falling in the presence of wake-induced resistance in a stationary fluid medium is studied. The expression of acceleration, velocity and distance of the falling sphere with the drag coefficient of Stokes' formula is derived, and the consistency of the theoretical formula and the experiment is verified by the experimental data, and the formula shows that the nonlinear falling kinematics is a function of the density ratio of the fluid to the sphere material, the kinematic viscosity of the fluid, and the acceleration of gravity, and proportional to the -1/2 power of the diameter of the sphere.

In addition, this paper uses Excel to finish a program to calculate the kinematics of a falling sphere with the drag in $0<Re<3X10^5$, and studies the influence of the density of the sphere material, the motion Re and the diameter of the sphere on the nonlinear kinematics of the sphere, and the results show that the variable acceleration motions of various materials sphere are similar. The acceleration, velocity and falling distance are proportional to density. The variable acceleration motion can be roughly divided into three stages with different characteristics.

In the fall of the sphere with air drag, acceleration, velocity and displacement can not be expressed as a function of time, but must be divided into three stages to be fit with functions. Linear formula $a = kt + c$ can express the initial motion, the acceleration is satisfied with $a = k_1t^2 + k_2t + c$ in the second stage, and $a = k_1t^3 + k_2t^2 + k_3t + c$ is fitted in the last stage, in short, after piecewise processing, the acceleration and velocity can be approximated by a polynomial function.

The relationship between displacement *h* and time *t* during varied-acceleration fall is also controlled by the density of the sphere, the greater the density of the sphere is, the closer the relationship between *h* and *t* in the falling process is near to no-resistance formula, $h = \frac{1}{2}gt^2$, the smaller the density of the sphere is, the closer the relationship between *h* and *t* is near to linear express $h = kt + c$ , the displacement of the solid fall is located between the formula $h = kt + c$ for water and the formula without resistance $h = \frac{1}{2}gt^2$, but does not satisfy the multi-relationship $h = kt^n + c$ .

Re is also an important control parameter for kinematics. Moreover, the jerk changes with Re acting like parabolic curve. Jerk oscillates when Re is near to 2000, and the appearance has nothing to do with the sphere material.

The diameter also has a significant effect on the change of acceleration. The larger the diameter of the sphere is, the slower acceleration changes during the falling process.

# References


[1] Phillip P. Brown1 and Desmond F. Lawler2. Sphere Drag and Settling Velocity Revisited. 2003, 129 (3). 222-230.

[2] Dou Xiurong, Study on the working characteristics analysis and process parameter selection of air drilling. Master's Thesis. 2004, China Academy of Petroleum Exploration and Development.

[3] Hagbart S. Alsos, Odd M. Faltinsen, 3D motion dynamics of axisymmetric bodies falling through water. Ocean Engineering 2018, 169, 442-456.

[4] J. Gylys, R. Skvorcinskiene, L. Paukstaitis, M. Gylys, A. Adomavicius. Water temperature influence on the spherical body’s falling velocity, International Journal of Heat and Mass Transfer, 89 (2015), 913-919.

[5] LIU Lei, YANG Jian-mina, LYU Hai-ninga, TIAN Xin-lianga, PENG Tao, Effects of Degrees of Motion Freedom on Free-Fall of A Sphere in Fluid. China Ocean Eng., 2018, Vol. 32, No. 5, P. 605–613.

[6] A. Iranmanesh M. Passandideh-Fard, Two-dimensional modelling of free-surface flows in presence of a spherical object using the Modifed Volume of Fluid technique. Marine Systems & Ocean Technology (2023) 17: 123–134.

[7] Hadi Ahmadi ·Saman Farzi Sizkow, Numerical analysis of ground improvement effects on dynamic settlement of uniform sand using DEM, SN Applied Sciences (2020) 2: 689-677.

[8] E. Tempelman & M. M. S. Dwaikat & C. Spitás, Experimental and Analytical Study of Free-Fall Drop Impact Testing of Portable Products, Experimental Mechanics (2012) 52: 1385–1395.

[9] M. M. Zdravkovich, Flow around circular cylinder. 2018. China Machine Press.

[10] Wu Hualin, Shen Huanting, Li Zhongwei, Acceleration Motion of Sediment Particle, Ocean Engineering, 2000. 18 (1): 44-49.

[11] Xu Falin, Excel Scatter Chart Processing of Free fall motion to explore experimental data, Middle School Physics Teaching Reference 2019, 48 (5), 66-67.

[12] Chen Xianying, Three supplements to Galileo's research on free falling motion, Physics Teaching Discussion, 2012. Vol 30 (444), 13-15.

[13] N. Mordant and J. F. Pinton, Velocity measurement of a settling sphere, Eur. Phys. J. B 18, 343-352 (2000).

[14] P. V. D. Leeden Liem Djan Nw and P. C. Suratman, The velocity of free falling droplets. Appl. sci. Res 1955. Section A, Vol. 5, 342-347.