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# Spin Dynamics in the Ferromagnetic Resonance

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**Abstract:** The LLG equation including the spin-transfer torque term, and the frequency spectrum analysis method are used to study the dynamic process of ferromagnetic resonance. The effects of damping factor  $\alpha$ , internal anisotropic field, magnetic field inclination, and spin-transfer torque caused by the spin current are studied. The following results are found as follows. The ferromagnetic resonance spectra as functions of the frequency  $\omega$  for fixed magnetic field, and functions of magnetic field for fixed frequency are obtained, and it is found that the internal magnetic field also has contribution to the resonance field or frequency, and we know that the resonant frequency  $\omega_0 \approx h_e + h_1$  (in unit of  $\gamma H_0$ ). In addition, when the damping factor increases from 0.01 to 0.03, the resonance frequencies increases slightly, and the resonance strength decreases. And the oscillatory waves of  $m_x$  and  $m_y$  reach their stable values more quickly. Furthermore, the internal field perpendicular to the external field  $h_0$  as well as it parallel to  $h_0$  also has the effect to the resonant frequency. The positive and negative internal field will have reversed effects to the resonance field or frequency. And in the end when the spin current becomes larger the STT effect becomes stronger, even exceeds the ferromagnetic resonance effect, makes  $m_z$  reversed, and  $m_x$  and  $m_y$  decreased.

**Keywords:** Ferromagnetic Resonance, Frequency Spectrum, Spin-transfer Torque, Internal Anisotropic Field

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## 1. Introduction

Ferromagnetic resonance has been used to study the magnetic properties and magnetization dynamics of ferromagnetic material [1-10], such as the Landé  $g$  factor, the Gilbert damping parameter  $\alpha$ , and magnetic anisotropic properties, etc.

Beaujour et al. studied the ferromagnetic resonance of the  $\text{Fe}_{1-x}\text{V}_x$  alloy thin films [11]. The Landé  $g$  factor, Gilbert damping parameter  $\alpha$  are obtained as functions of the  $V$  component  $x$ . When  $x$  increases the Landé  $g$  factor increases from 2.11 for  $x=0$  to 2.17 for  $x=0.6$ . And the damping parameter  $\alpha$  also increases from 0.008 for  $x=0$  to 0.015 for  $x=0.5$ . The films exhibit an out-of-plane anisotropy, and the anisotropy constant  $K_{\perp}$  decreases with  $x$  increasing, from 3.4 erg/cm<sup>2</sup> for  $x=0$  decreases to 0.8 erg/cm<sup>2</sup> for  $x=0.66$ . Wu et al. studied the ferromagnetic resonance in a CoFe/PtMn/CoFe multilayer film [12]. In experiments the sample plane was rotated with respect to the direction of the magnetic field. The  $g$  factor and the effective magnetic anisotropy parameters of

the magnetic film were obtained from the angle dependence of the resonance peaks as:  $g=2.01$ ,  $2K_A/M \sim 0.1$  T,  $4\pi M - 2K_U/M \sim 1.9$  T. Kakazei et al studied the ferromagnetic resonance of ultrathin Co/Ag superlattices on Si (111) [13]. FMR spectra have been recorded at various polar angles between the sample plane and the magnetic field. From the angle dependence of the resonance field the fitting parameters are obtained:  $g=2.07$ , the anisotropic field  $H_{\text{eff}}=7.83$  kOe for  $5 \times [\text{Co} (4 \text{ \AA})/\text{Ag} (4.5 \text{ \AA})]$  SL sample. Urban et al. studied the Gilbert damping in single and multilayer ultrathin Fe films: role of interface in nonlocal spin dynamics [14]. They found that the FMR linewidth for the Fe films in the double layer structures was larger than the FMR linewidth in the single Fe films having the same thickness. The additional FMR linewidth scaled inversely with the film thickness, and increases linearly with increasing microwave frequency. These results demonstrate that a transfer of electron angular momentum between the magnetic layers leads to additional relaxation torques.

## 2. LLG Equation

The LLG equation has been successfully applied to study the spin reversion by the current driven spin torque. In that case the energy and the absolute value of the spin angular momentum are conserved. In the case of ferromagnetic resonance the energy is not conserved, especially at resonance the magnetic moment increases rapidly. In this paper we will

$$\frac{1}{\gamma_0} \frac{dm}{dt} = -m \times H - \alpha m \times (m \times H) + a_J m \times (m \times n_s) - \alpha a_J (m \times n_s) \quad (1)$$

Where

$$\gamma_0 = \frac{\gamma}{1+\alpha^2} \quad (2)$$

$\gamma \approx 2\mu_B/\hbar$  is the gyromagnetic ratio,  $\alpha$  is the Gilbert damping constant,  $H$  is the total magnetic field, including external magnetic field  $H_e$  and internal local anisotropic magnetic field  $H_{eff}$ ,  $n_s$  is the unit vector of the magnetization in the fixed layer.  $a_J$  is a torque constant relative to the spin-polarized current.

$$a_J = \left(\frac{\hbar}{2e}\right) \eta \left(\frac{I}{\mu_0 S d M_s}\right) \quad (3)$$

$$(1 + \alpha^2) \frac{dm}{d\tau} = -m \times h - \alpha m \times (m \times h) - a_{JH} m \times (m \times n_s) + \alpha a_{JH} (m \times n_s) \quad (4)$$

Where  $a_{JH} = a_J/H_0$ .

$H$  represents the sum of the internal and external fields on the magnet. The internal magnetic field is responsible for keeping the magnetization pointing along the easy axis. For example, a thin-film magnet oriented in the x-y plane with easy axis along z-axis is characterized by.

$$H_{eff} = H_z m_z \hat{z} + H_x m_x \hat{x} \quad (5)$$

Representing the internal "uniaxial anisotropy" effective field. It is noticed that the internal field is dependent on the magnetic moment  $m$ .

$$(1 + \alpha^2) \frac{dm_x}{d\tau} = -m_y m_z h_1 - \alpha(m_x^3 h_2 - m^2 m_x h_2 + m_x m_z^2 h_1) - m_y h_0 + m_z h_{3y} - \alpha[(m \cdot h_e) m_x - m^2 h_{3x}]$$

$$(1 + \alpha^2) \frac{dm_y}{d\tau} = m_x m_z h_1 - m_x m_z h_2 - \alpha(m_y m_x^2 h_2 + m_y m_z^2 h_1) - m_z h_{3x} + m_x h_0 - \alpha[(m \cdot h_e) m_y - m^2 h_{3y}]$$

$$(1 + \alpha^2) \frac{dm_z}{d\tau} = m_x m_y h_2 - \alpha(m_z m_x^2 h_2 + m_z^3 h_1 - m_z m^2 h_1) - m_x h_0 + m_z h_{3x} - \alpha[(m \cdot h_e) m_z - m^2 h_0] \quad (7)$$

Where

$$m \cdot h_e = m_x h_{3x} + m_y h_{3y} + m_z h_0 \quad (8)$$

The detail of the calculation see [15, 16].

## 3. Ferromagnetic Resonance Properties

### 3.1. Variation of Magnetic Moments at Different Alternating Field Frequencies

Figure 1-3 show the variation of the magnetic moment components with time at  $\omega=1.06, 1.07, 1.08$ , respectively. In the calculation we take the dimensionless quantities:  $\alpha=0.02$ ,

use the LLG equation to study the ferromagnetic resonance. The LLG equation is written in Eq. (1), where  $m$  is the unit vector of the macro magnetic moment, thus  $m^2=1$ . In studying ferromagnetic resonance we assume that before applying alternating microwave field  $m^2=1$ , after applying alternating microwave field  $m^2 \neq 1$ .

The LLG equation can be written as.

where  $\eta$  is the spin polarization of electrons,  $I$  is the current,  $M_s$  is the saturated magnetization,  $S$  and  $d$  are the area and width of the free layer, respectively. Temporally we don't consider the term related to current  $a_J$ .

In order to transfer Eq. (1) into the dimensionless form, we take the unit of the magnetic field as  $H_0$ ,  $H=hH_0$ . The time unit is taken as  $\tau_0=1/\gamma H_0$ ,  $t=\tau\tau_0$ . In this paper we took  $H_0=10^4$  A/m  $\sim 1.257 \times 10^{-2}$  T,  $\gamma H_0=176$  GHz/T  $\times 1.257 \times 10^{-2}$  T = 2.21 GHz.  $\tau_0=0.45$  ns. With  $\tau$  Eq. (1) becomes.

We take the dimensionless magnetic field,

$$h_e = h_0 \hat{z} + h_{3x} \hat{x} + h_{3y} \hat{y} + h_1 m_z \hat{z} + h_2 m_x \hat{x} \quad (6)$$

Where the first three terms are external magnetic field,  $h_0$  is the constant field in the z direction,  $h_3$  is the alternating field  $h_{3x}=h_3 \cos \omega\tau$ ,  $h_{3y}=h_3 \sin \omega\tau$ . The last two terms are the internal effective anisotropic field, dependent on the magnetic moment component. Then we write the component form of the LLG equation without the current  $a_{JH}$  terms.

$h_0=1, h_1=0.1, h_2=0, h_3=0.02$ . The initial conditions are:  $m_x=0.01, m_y=0, m_z=1$ . From Figure 1-3 we see that the magnetic moment components oscillate with the alternation field frequency, the period is  $\tau_0=2\pi/\omega$  but the amplitudes are different. At a definite frequency  $\omega_0$  the resonance occurs, the amplitude is largest, the magnetic moment components  $m_x$  and  $m_y$  increase rapidly with time, that is ferromagnetic resonance. At the same time, the  $m_z$  decreases with time, it decreases largest at resonance. Because except the external constant field  $h_0=1$ , there is also internal anisotropic field  $h_1=0.1$  in the z direction, thus the resonant frequency is approximately determined by the sum of the two magnetic fields, though the properties of the two fields are different, the letter is  $m_z$

dependent. The resonant frequency  $\omega_0 \approx h_0 + h_1 = 1.1$  (in unit of  $\gamma H_0$ ).

From Figure 1 we can see that the amplitudes of  $m_x$  and  $m_y$

increase initially, after definite time they reaches stable, and the  $m_z$  decrease.

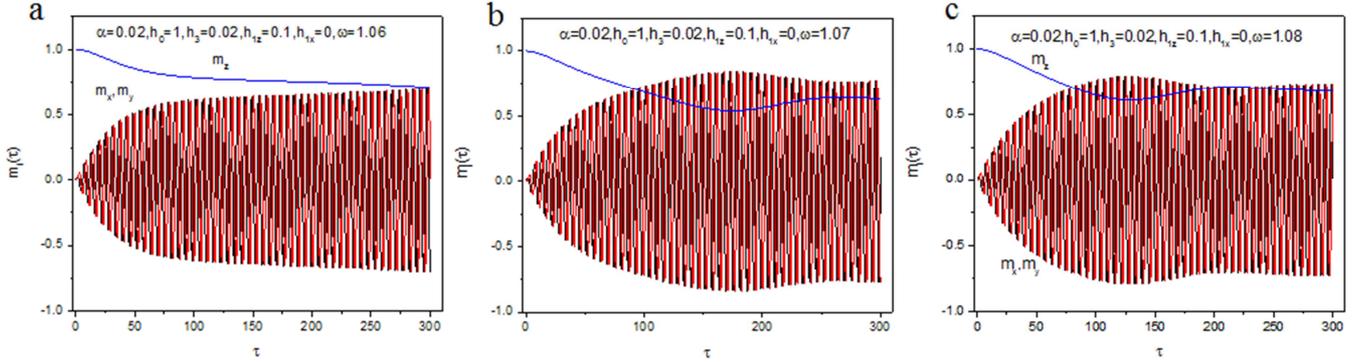


Figure 1. Variation of the magnetic moment components with time for  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\alpha=0.02$ . (a)  $\omega=1.06$ . (b)  $\omega=1.07$ . (c)  $\omega=1.08$ , respectively.

### 3.2. Ferromagnetic Resonance Spectrum

We should transfer the ferromagnetic oscillations  $m_i(t)$  ( $i=x, y$ ) in the last section Figure 1, into the frequency spectrum  $m_i(\omega)$ , i.e. the ferromagnetic resonance spectra. By use of the sine Fourier transform for the  $m_x$  oscillation,

$$F(\omega) = \lim_{a \rightarrow \infty} \frac{4}{\pi a} \left[ \int_0^a f(t) \sin \omega t dt \right]^2 \quad (9)$$

Where  $a$  is a large number respective to the oscillating period. For example,  $f(t) = \sin \omega_0 t$ , then.

$$\int_0^a \sin \omega_0 t \sin \omega t dt = \frac{1}{2} [\cos(\omega_0 - \omega)t - \cos(\omega_0 + \omega)t] = \frac{1}{2} \left[ \frac{\sin(\omega_0 - \omega)a}{(\omega_0 - \omega)} - \frac{\sin(\omega_0 + \omega)a}{(\omega_0 + \omega)} \right] \quad (10)$$

From the definition of the  $\delta$  function,

$$\delta(x) = \lim_{a \rightarrow \infty} \frac{1}{\pi} \frac{\sin^2 ax}{ax^2} \quad (11)$$

We obtain,

$$F(\omega) = \lim_{a \rightarrow \infty} \frac{4}{\pi a} \frac{\sin^2(\omega_0 - \omega)a}{4(\omega_0 - \omega)^2} = \delta(\omega_0 - \omega) \quad (12)$$

Similarly we use the cosine Fourier transform for the  $m_y$  oscillation.

Because the  $m_i(\tau)$  is given by the numerical method at the discrete points of dimensionless time points  $\tau_n$ , we integrate the equation (9) by the fixed step Simpson integrating method. Figure 2 is the frequency spectrum for  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\alpha=0.02$ , at  $\omega=1.096$  (resonance frequency), where the units of time and frequency are dimensionless  $\tau$  and  $2\pi\tau$ .

In figure 2, there are two curves of  $m_x$  and  $m_y$ , they are not distinguished as desired. From Figure 2 we see that there are three main resonance peaks for  $m_x$  and  $m_y$ , respectively. The position of the main peak is around  $\omega_0=1.096$ . There are also two other harmonic waves for each component, i.e. ferromagnetic resonance waves, but the frequencies are different from the main frequency, and the amplitudes are smaller.

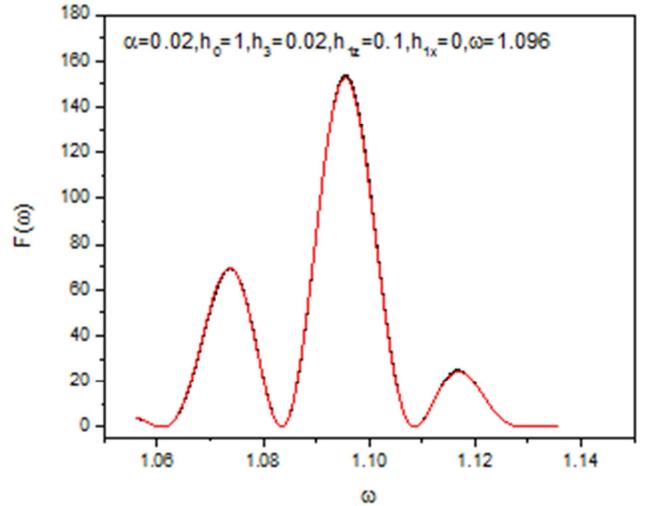


Figure 2. Frequency spectra of  $m_x$  and  $m_y$  at resonance frequency  $\omega_0=1.096$ .

### 3.3. Ferromagnetic Resonance Spectrum

Figure 3 are the ferromagnetic resonance spectrum for  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\alpha=0.02$ , (a)  $h_0=1$ ,  $\omega$  is the variable quantity, (b)  $\omega=1$ ,  $h_0$  is the variable quantity, respectively.

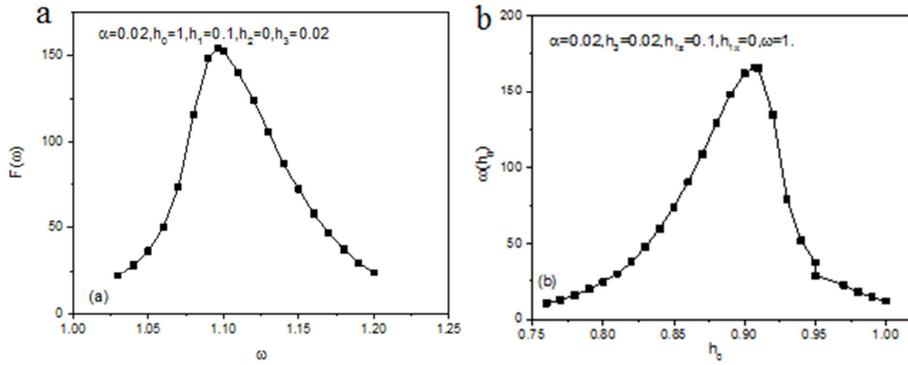


Figure 3. Ferromagnetic resonance spectrum for  $h_1=0.1, h_2=0, h_3=0.02, \alpha=0.02$ , (a)  $h_0=1, \omega$  is the variable quantity. (b)  $\omega=1, h_0$  is the variable quantity.

Comparing Figure 3 (a) and (b) we found that the shape of resonance peak is more symmetric for  $\omega$  as a variable, while it is non-symmetric for  $h_0$  as a variable. In Figure 3 (a)  $h_0=1, h_1=0.1$  in the  $z$  direction, the resonance frequency  $\omega_0 = 1.096 \approx h_0 + h_1$ . In Figure 3 (b)  $\omega=1$ , the resonance magnetic field  $h_0 = 0.907 \approx \omega - h_1$ . It is noticed that here we use the dimensionless quantities so the values of  $\omega$  and  $h$  are the same.

3.4. Effect of Damping Factor  $\alpha$

In Figure 4 the ferromagnetic resonance spectrum for  $h_0=1, h_1=0.1, h_2=0, h_3=0.02$ , and  $\alpha=0.01, 0.02, 0.03$  are shown as functions of  $\omega$ . From Figure 4 we see that the resonance strength is smaller when the damping factor  $\alpha$  is larger, and the resonance frequency blue shifts slightly, equal 1.09, 1.096, and 1.11, respectively.

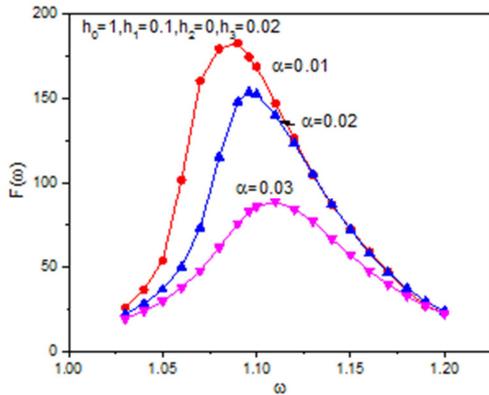


Figure 4. Ferromagnetic resonance spectrum for  $h_0=1, h_1=0.1, h_2=0, h_3=0.02$ , and  $\alpha=0.01, 0.02, 0.03$  as functions of  $\omega$

3.5. Effect of Internal Anisotropic Field  $h_1$  and  $h_2$

In this paper we assume that the external magnetic  $h_0$  is in the  $z$  direction, the internal anisotropic field  $h_1$  is also in the  $z$  direction, while the internal anisotropic field  $h_2$  is in the  $x$  direction, perpendicular to  $h_0$ . Besides, the internal fields are magnetic moment dependent, see Eq. (6).

Figure 5 (a) and (b) shows the frequency spectra and  $m_x$  and  $m_y$  for  $\alpha=0.02, h_0=1, h_1=0, h_3=0.02, \omega=1$ , and  $h_2=\pm 0.1$ , respectively. From Figure 5 we see that due to breaking of the axial symmetry in the  $z$  direction the curves of  $m_x$  and  $m_y$  are different. For  $h_2=+0.1, m_x > m_y$ , for  $h_2=-0.1, m_x < m_y$ .

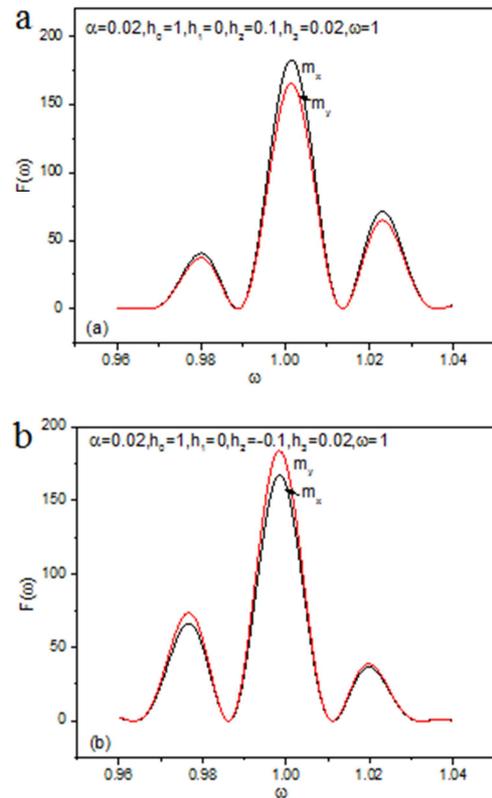


Figure 5. Frequency spectra and  $m_x$  and  $m_y$  for  $\alpha=0.02, h_0=1, h_1=0, h_3=0.02, \omega=1$ , and (a)  $h_2=+0.1$  (b)  $h_2=-0.1$ .

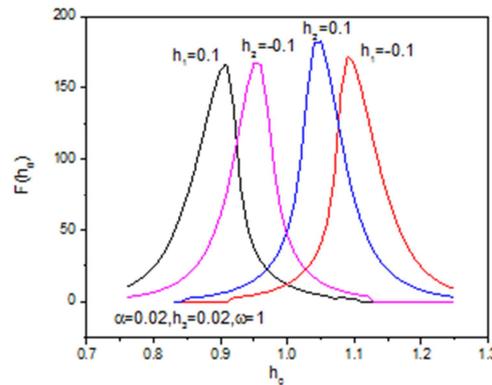


Figure 6. Ferromagnetic resonance spectra as functions of  $h_0$  for  $\alpha=0.02, h_3=0.02, \omega=1$ , and  $h_1=\pm 0.1, h_2=0$ , or  $h_1=0, h_2=\pm 0.1$ , respectively.

Figure 6 shows the ferromagnetic resonance spectra (from  $m_x$ ) as functions of  $h_0$  for  $\alpha=0.02$ ,  $h_3=0.02$ ,  $\omega=1$ , and  $h_1=\pm 0.1$ ,  $h_2=0$ , or  $h_1=0$ ,  $h_2=\pm 0.1$ , respectively. As shown in Sec. 3.3 because the internal field  $h_1$  is in the  $z$  direction, it will influence the resonance fields, which are 0.907 and 1.09 for  $h_1=+0.1$  and  $h_1=-0.1$ , respectively. While the internal field  $h_2$  is in the  $x$  direction, it also influence the resonance fields, which are 0.95 and 1.05 for  $h_2=-0.1$  and  $h_2=0.1$ , respectively. When the absolute value of the internal field increases the resonance field will extend to both sides as in Figure 6.

### 3.6. Effect of Magnetic Field Inclination

In the previous calculations we assumed that the external magnetic field  $H_0$  is always in the  $z$  direction, here we consider the effect of the magnetic field inclination. Assume that the inclination angle between  $H_0$  and the  $z$  axis is  $\theta$ , then the external magnetic field

$$H_{0z} = H_0 \cos \theta, \quad H_{0x} = H_0 \sin \theta.$$

Figure 7 shows the frequency spectrum of  $m_x$  and  $m_y$  for  $\theta=15^\circ$ ,  $\alpha=0.02$ ,  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\omega=1.09$ . Comparing with Figure 2, where  $\theta=0^\circ$ , the curves of  $m_x$  and  $m_y$  are not duplicated again due to the axial symmetry breaking.

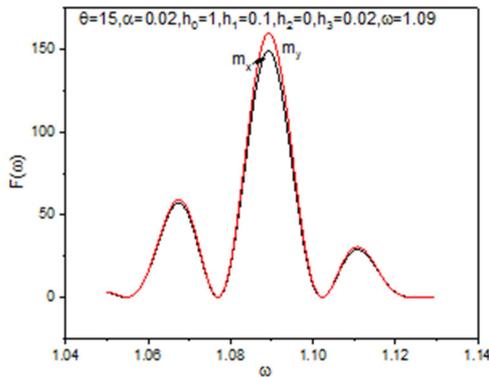


Figure 7. Frequency spectrum of  $m_x$  and  $m_y$  for  $\theta=15^\circ$ ,  $\alpha=0.02$ ,  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\omega=1.09$ .

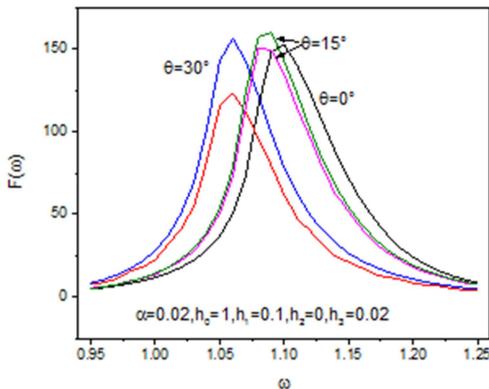


Figure 8. Ferromagnetic resonance spectra as functions of  $\omega$  for  $\alpha=0.02$ ,  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ , and  $\theta=0^\circ, 15^\circ, 30^\circ$ .

Figure 8 shows the ferromagnetic resonance spectra as functions of  $\omega$  for  $\alpha=0.02$ ,  $h_0=1$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ , and

$\theta=0^\circ, 15^\circ, 30^\circ$ , respectively. In the figure there are two curves for  $\theta=15^\circ$  and  $30^\circ$ , corresponding to  $m_y$  and  $m_x$ , respectively ( $m_y > m_x$ ). The resonance frequency  $\omega_0 \sim \cos \theta$  decreases as the  $\theta$  increases.

### 3.7. Effect of Spin-Transfer Torque (SST) Caused by Spin Current

We suppose that in the case of ferromagnetic resonance a spin current  $J$  is applied to the sample from the fixed ferromagnetic layer to the free ferromagnetic layer, which is represented by the  $a_J$  term in the Eq. (1). Now consider the effect of the SST caused by spin current. When the current value  $a_J$  is smaller than a critical value the magnetic moment  $m_z$  will not reverse, when the electric current  $a_J$  exceeds the critical value the  $m_z$  will reverse [5], this is the principle of the SST. We study the ferromagnetic resonance in this process.

Figure 9 shows the frequency spectra of  $m_x$  and  $m_y$  for  $\alpha=0.02$ ,  $h_0=0.907$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\omega=1$ , and  $J_H=0, 0.02, 0.04$ , respectively. Where  $J_H$  is the dimensionless current parameter, see [15]. When  $J_H=0.02$ , the  $m_z$  does not reverse, when  $J_H=0.04$ , the  $m_z$  reverses. From Fig.9 we see that along with the current increases there will be more resonance peaks, the peak frequency and height decrease. The SST effect will exceed the ferromagnetic resonance effect.

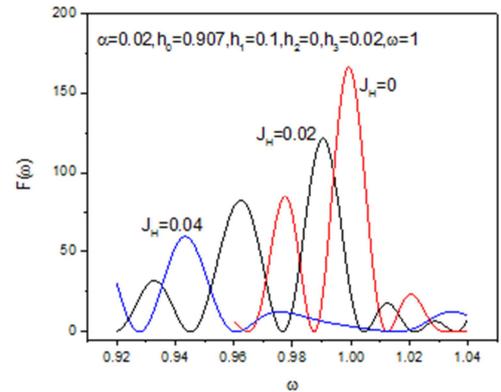


Figure 9. Frequency spectra of  $m_x$  and  $m_y$  for  $\alpha=0.02$ ,  $h_0=0.907$ ,  $h_1=0.1$ ,  $h_2=0$ ,  $h_3=0.02$ ,  $\omega=1$ , and  $J_H=0, 0.02, 0.04$ .

## 4. Summary

We use the LLG equation including the spin-transfer torque term, and without the constraint of magnetic moment conservation to study the dynamic process of ferromagnetic resonance. We studied the effects of damping factor  $\alpha$ , internal anisotropic field, magnetic field inclination, and spin-transfer torque caused by the spin current, and obtained the following results:

1. We obtained the ferromagnetic resonance spectra as functions of the frequency  $\omega$  for fixed magnetic field, and functions of magnetic field for fixed frequency from the solutions of LLG equation. It is found that the internal magnetic field also has contribution to the resonance field or

frequency. The resonant frequency (or field) is approximately determined by the sum of the two magnetic fields, though the properties of the two fields are different, the letter is  $m_z$  dependent. The resonant frequency  $\omega_0 \approx h_e + h_1$  (in unit of  $\gamma H_0$ ).

2. When the damping factor increases from 0.01 to 0.03, the resonance frequencies increases slightly, and the resonance strength decreases. The oscillatory waves of  $m_x$  and  $m_y$  reach their stable values more quickly.

3. The internal field perpendicular to the external field  $h_0$  as well as it parallel to  $h_0$  also has the effect to the resonant frequency. The resonant fields are  $h_0=0.907$  ( $h_1=0.1$ ,  $h_2=0$ ),  $0.95$  ( $h_1=0$ ,  $h_2=-0.1$ ),  $1.05$  ( $h_1=0$ ,  $h_2=0.1$ ),  $1.09$  ( $h_1=-0.1$ ,  $h_2=0$ ) for  $\omega=1$ ,  $h_3=0.02$ , respectively. The positive and negative internal field will have reversed effects to the resonance field or frequency.

4. When the spin current becomes larger the STT effect becomes stronger, even exceeds the ferromagnetic resonance effect, makes  $m_z$  reversed, and  $m_x$  and  $m_y$  decreased.

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