

Solitary Waves and Property Management of Nonlinear Dispersive and Flattened Optical Fiber

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Abstract: In this work, we establish the Conditions that must satisfy the characteristic coefficients of the nonlinear and flattened dispersive optical fiber so that certain classes of solitary waves propagate there with fewer fluctuations. Once the conditions are established, we determine the exact solutions as well as the corresponding nonlinear partial differential equations that govern the propagation dynamics in this transmission medium. The propagation of the solutions obtained is also tested. The method used to obtain the analytical solutions is based on the control of the properties of the Bogning implicit functions whereas the numerical simulations are made through the split-step method which is very adapted to simulate the propagation of the signals.

Keywords: Flattened Optical Fiber, Solitary Wave, Characteristic Coefficient, Implicit Bogning Function, Propagation, Nonlinear, Dispersive, Partial Differential Equation

1. Introduction

Nonlinear physics in its branch of photonic optics has been at the center of many telecommunications technology applications in recent years. Among the transmission media and waveguides developed by these technologies, optical fiber is attracting even more interest, probably because of its high bandwidth and insensitivity to external electromagnetic disturbances. This great interest is also reflected by the large number of research works devoted to this ultimate transmission medium. The propagation dynamics of the waves in the fiber is generally modeled by partial differential equations of Schrödinger type with nonlinear terms, dispersion terms and dissipation terms characterized by their coefficients [1-5]. If the coefficient of nonlinearity is responsible for the unpredictable effects that the propagating wave may undergo, the dispersion coefficient is responsible for the spread of the signal and the dissipation coefficient responsible for the absorption or losses of the energy. These observations and remarks assume that these effects must be

taken into account during the fabrication of an optical fiber capable of better transmitting a signal. There are several types of optical fibers but the optical fiber that will be the focus of our study in this paper is the nonlinear dispersive and flattened optical fiber. The main characteristic of this one is that it is subject to strong dispersions and this can be understood because of its flattened shape. The nonlinear partial differential equation that models the propagation dynamics of waves in this type of fiber generally has higher order dispersion terms (greater than or equal to 4). Thus, the work that we have defined consists in establishing the constraint relations which make it possible to determine the type of wave (solitary wave) likely to propagate without difficulties in the waveguide. once these conditions are fixed, we make corrections to the initial differential equation to obtain an equation which admits for exact solution the analytical sequence attributed to the wave that must propagate in the optical fiber [6-8]. To achieve this, we will first assume that the fiber is immersed in a medium with

arbitrary characteristic coefficients n_i ($i=1,2,3,4$) and subsequently find a uniting relation such that the desired solution is effective. The method and technique used to obtain the results is the one we develop in recent years in our many works [9-33].

This article is organized as follows: Section 2 establishes coefficient range equations as well as the range of possibilities for obtaining solutions. In Section 3, we propose analytical solutions as well as constraint relations for this to be possible. In section 4, we numerically verify the propagation of solitary wave solutions obtained and finally we end the work with a conclusion.

2. Equation of Range of Coefficients and Possibilities of Solutions

We define the family of nonlinear partial differential equations that describes the propagation dynamics of the wave in a dispersive and flattened optical fiber as

$$in_1 \frac{\partial U}{\partial \xi} + n_2 \frac{\partial^2 U}{\partial \tau^2} + n_3 |U|^2 U + n_4 \frac{\partial^4 U}{\partial \tau^4} = 0 \quad (1)$$

where n_i ($i=1,2,3,4$) are the characteristic coefficients of the

$$\begin{aligned} & -n_1 k a J_{n,m} + n_2 \alpha^2 a [m(m-1)J_{n-2,m-2} - (2mn-m+n)J_{n,m} + n(n+1)J_{n+2,m+2}] \\ & + n_3 |a|^2 a J_{3n,3m} + n_4 \alpha^4 a m(m-1)(m-2)(m-3)J_{n-4,m-4} - n_4 \alpha^4 a m(m-1)(m-2)(n-3)J_{n-2,m-2} \\ & + n_4 \alpha^4 a [m(m-1)(n-2) + (2mn-m+n)m](m-1)J_{n-2,m-2} \\ & + n_4 \alpha^4 a [m(m-1)(n-2) + (2mn-m+n)m](n-1)J_{n,m} - n_4 \alpha^4 a [(2mn-m+n)n + n(n+1)(m+2)](m+1)J_{n,m} \\ & + n_4 \alpha^4 a [(2mn-m+n)n + n(n+1)(m+2)](m+1)J_{n,m} + n_4 \alpha^4 a [(2mn-m+n)n + n(n+1)(m+2)](n+1)J_{n+2,m+2} \\ & - n_4 \alpha^4 a n(n+1)(n+2)(m+3)J_{n+2,m+2} + n_4 \alpha^4 a n(n+1)(n+2)(n+3)J_{n+4,m+4} = 0 \end{aligned} \quad (5)$$

This equation is generally called the range equation of the coefficients, since it represents the equation whose analysis makes it possible to determine the coefficients. In the case of equation (5), the only coefficient to be determined is a . The different equations to be solved depend on the values assigned to n and m .

But the choice of the values of n and m is not hazardous; we have established in our previous work that the values of n and m for which are listed among the values of n and m for which some terms of equation (5) are grouped together. Thus, the

fiber, ξ is the spatial variable and τ is the temporal variable. The flattening of the fiber is materialized hereby the partial derivative of order 4 ($\frac{\partial^4 U}{\partial \tau^4}$). We propose to construct the solution of equation (1) in the form

$$U(\xi, \tau) = A(\tau) \exp i k \xi \quad (2)$$

where $A(\tau)$ is the temporal envelope of the wave solution and k the wave number. The introduction of the ansatz (2) in equation (1) gives

$$-n_1 k A + n_2 \frac{\partial^2 A}{\partial \tau^2} + n_3 |A|^2 A + n_4 \frac{\partial^4 A}{\partial \tau^4} = 0 \quad (3)$$

We propose to construct the solution of equation (3) in the form

$$A(\tau) = a J_{n,m}(\alpha \tau) \quad (4)$$

where a is a constant to be determined, $J_{n,m}(\alpha \tau)$ the implicit function whose numbers n and m are real to be determined and α a parameter to be determined as well. Thus, taking into account equation (4) in (3) gives

values of n and m for which some terms of equation (5) are related are given by

$$n, m \in \left\{ -2, -\frac{3}{2}, -1, -\frac{1}{2}, 0, \frac{1}{2}, 1, \frac{3}{2}, 2 \right\} \quad (6)$$

The various combinations of the pair (n, m) likely to lead to solutions are recorded in the table below called fields of possibilities for finding solutions.

Table 1. Fields of possibilities for finding solutions.

(n, m)	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
-2	$(-2, -2)$	$(-2, -\frac{3}{2})$	$(-2, -1)$	$(-2, -\frac{1}{2})$	$(-2, 0)$	$(-2, \frac{1}{2})$	$(-2, 1)$	$(-2, \frac{3}{2})$	$(-2, 2)$
$-\frac{3}{2}$	$(-\frac{3}{2}, -2)$	$(-\frac{3}{2}, -\frac{3}{2})$	$(-\frac{3}{2}, -1)$	$(-\frac{3}{2}, -\frac{1}{2})$	$(-\frac{3}{2}, 0)$	$(-\frac{3}{2}, \frac{1}{2})$	$(-\frac{3}{2}, 1)$	$(-\frac{3}{2}, \frac{3}{2})$	$(-\frac{3}{2}, 2)$
-1	$(-1, -2)$	$(-1, -\frac{3}{2})$	$(-1, -1)$	$(-1, -\frac{1}{2})$	$(-1, 0)$	$(-1, \frac{1}{2})$	$(-1, 1)$	$(-1, \frac{3}{2})$	$(-1, 2)$

(n, m)	-2	$-\frac{3}{2}$	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$\frac{3}{2}$	2
$-\frac{1}{2}$	$\left(-\frac{1}{2}, -2\right)$	$\left(-\frac{1}{2}, -\frac{3}{2}\right)$	$\left(-\frac{1}{2}, -1\right)$	$\left(-\frac{1}{2}, -\frac{1}{2}\right)$	$\left(-\frac{1}{2}, 0\right)$	$\left(-\frac{1}{2}, \frac{1}{2}\right)$	$\left(-\frac{1}{2}, 1\right)$	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	$\left(-\frac{1}{2}, 2\right)$
0	$(0, -2)$	$\left(0, -\frac{3}{2}\right)$	$(0, -1)$	$\left(0, -\frac{1}{2}\right)$	$(0, 0)$	$\left(0, \frac{1}{2}\right)$	$(0, 1)$	$\left(0, \frac{3}{2}\right)$	$(0, 2)$
$\frac{1}{2}$	$\left(\frac{1}{2}, -2\right)$	$\left(\frac{1}{2}, -\frac{3}{2}\right)$	$\left(\frac{1}{2}, -1\right)$	$\left(\frac{1}{2}, -\frac{1}{2}\right)$	$\left(\frac{1}{2}, 0\right)$	$\left(\frac{1}{2}, \frac{1}{2}\right)$	$\left(\frac{1}{2}, 1\right)$	$\left(\frac{1}{2}, \frac{3}{2}\right)$	$\left(\frac{1}{2}, 2\right)$
1	$(1, -2)$	$\left(1, -\frac{3}{2}\right)$	$(1, -1)$	$\left(1, -\frac{1}{2}\right)$	$(1, 0)$	$\left(1, \frac{1}{2}\right)$	$(1, 1)$	$\left(1, \frac{3}{2}\right)$	$(1, 2)$
$\frac{3}{2}$	$\left(\frac{3}{2}, -2\right)$	$\left(\frac{3}{2}, -\frac{3}{2}\right)$	$\left(\frac{3}{2}, -1\right)$	$\left(\frac{3}{2}, -\frac{1}{2}\right)$	$\left(\frac{3}{2}, 0\right)$	$\left(\frac{3}{2}, \frac{1}{2}\right)$	$\left(\frac{3}{2}, 1\right)$	$\left(\frac{3}{2}, \frac{3}{2}\right)$	$\left(\frac{3}{2}, 2\right)$
2	$(2, -2)$	$\left(2, -\frac{3}{2}\right)$	$(2, -1)$	$\left(2, -\frac{1}{2}\right)$	$(2, 0)$	$\left(2, \frac{1}{2}\right)$	$(2, 1)$	$\left(2, \frac{3}{2}\right)$	$(2, 2)$

3. Analytical Solutions and Constraint Relationships

In this section, we look for solutions corresponding to the different values of the pair (n, m) of the table 1.

1. case $(n, m) = (2, 0)$: We obtain from equation (5), the following equation

$$-2n_2\alpha^2 aJ_{2,0} + 6n_2\alpha^2 aJ_{4,2} - n_1kaJ_{2,0} + n_3|a|^2 aJ_{6,0} - 42n_4\alpha^4 aJ_{4,2} + 120n_4\alpha^4 aJ_{6,4} = 0 \quad (7)$$

Knowing that $J_{4,2} = J_{2,0} - J_{4,0}$ and $J_{6,4} = J_{2,0} - 2J_{4,0} + J_{6,0}$, equation (7) becomes

$$(4n_2\alpha^2 a - n_1ka + 78n_4\alpha^4 a)J_{2,0} + (-6n_2\alpha^2 a - 198n_4\alpha^4 a)J_{4,0} + (n_3|a|^2 a + 120n_4\alpha^4 a)J_{6,0} = 0 \quad (8)$$

Equation (8) is verified if for $a \neq 0$, we have

$$n_2 = -33n_4\alpha^2 \quad (13)$$

$$4n_2\alpha^2 - n_1k + 78n_4\alpha^4 = 0 \quad (9)$$

$$|a| = \alpha \sqrt{\frac{-120n_4}{n_3}}, n_4n_3 < 0, \alpha > 0 \quad (14)$$

$$n_2 + 33n_4\alpha^2 = 0 \quad (10)$$

We deduce from equation (14) that

$$n_3|a|^2 + 120n_4\alpha^2 = 0 \quad (11)$$

$$a = \alpha \sqrt{\frac{-120n_4}{n_3}} \exp i\theta, \theta \in R \quad (15)$$

The resolution of equations (9), (10) and (11) allows having

$$n_1 = -\frac{54n_4\alpha^4}{k} \quad (12)$$

The sought solution is given by

$$U(\xi, \tau) = \alpha \sqrt{\frac{-120n_4}{n_3}} J_{2,0}(\alpha\tau) \exp i(k\xi + \theta), n_4n_3 < 0, \theta \in R \quad (16)$$

Solution (15) is the exact solution of the equation below obtained by modifying the coefficients of equation (1)

$$54in_1\alpha^4 \frac{\partial U}{\partial \xi} + 33kn_4\alpha^2 \frac{\partial^2 U}{\partial \tau^2} - kn_3|U|^2 U - kn_4 \frac{\partial^4 U}{\partial \tau^4} = 0 \quad (17)$$

2. case $(n, m) = (2, 1)$: We obtain from equation (5), the following equation

$$(5n_2\alpha^2 a + n_1ka + 23\alpha^4 n_4 a)J_{2,1} - (6n_2\alpha^2 a - 12n_4\alpha^4 a)J_{4,3} - n_3|a|^2 aJ_{6,3} - 96n_4\alpha^4 aJ_{6,5} = 0 \quad (18)$$

The observation of equation (17) imposes for the purpose of simplifying the form of the equation to write the following

transformations

$$J_{4,3} = J_{2,1} - J_{4,1} \quad (19)$$

$$J_{6,3} = J_{4,1} - J_{6,1} \quad (20)$$

$$J_{6,5} = J_{2,1} - 2J_{4,1} + J_{6,1} \quad (21)$$

Taking into account relations (18), (19) and (20) in equation (17) leads to

$$\left(-n_2\alpha^2 a - 61n_4\alpha^4 a + n_1 k a\right)J_{2,1} + \left(6n_2\alpha^2 a + 180n_4\alpha^4 a - n_3|a|^2 a\right)J_{4,1} + \left(n_3|a|^2 a - 96n_4\alpha^4 a\right)J_{6,1} = 0 \quad (22)$$

Equation (21) is verified if we have

$$|a| = 4\alpha^2 \sqrt{\frac{6n_4}{n_3}}, n_4 n_3 > 0 \quad (23)$$

$$n_2 = -14n_4\alpha^2 \quad (24)$$

$$k = \frac{47n_4\alpha^2}{n_1}, n_1 \neq 0 \quad (25)$$

We obtain from equation (23)

$$a = 4\alpha^2 \sqrt{\frac{6n_4}{n_3}} \exp i\theta, \theta \in R \quad (26)$$

Taking into account the relationships and constraints above allows to write the solution

$$U(\xi, \tau) = 4\alpha^2 \sqrt{\frac{6n_4}{n_3}} J_{2,1}(\alpha\tau) \exp i\left(\frac{47n_4\alpha^4}{n_1}\xi + \theta\right) \quad (27)$$

Equation (26) is the exact solution of the corrected equation

$$in_1 \frac{\partial U}{\partial \xi} - 14n_4\alpha^2 \frac{\partial^2 U}{\partial \tau^2} + n_3|U|^2 U + n_4 \frac{\partial^4 U}{\partial \tau^4} = 0 \quad (28)$$

3. case $(n, m) = (-2, 0)$: We obtain from equation (5), the following equation

$$\left(2n_2\alpha^2 - n_1 k\right)J_{-2,0} + \left(2n_2\alpha^2 - 4n_4\alpha^4\right)J_{0,2} + n_3|a|^2 a J_{-6,0} = 0 \quad (29)$$

Equation (29) is verified if and only if $n_2 = 2n_4\alpha^2$, $n_3 = 0$ and $k = \frac{4n_4\alpha^4}{n_1}$ with $n_1 \neq 0$. In these conditions, the seeking solution is given by

$$U(\xi, \tau) = aJ_{-2,0}(\alpha\tau) \exp \frac{4in_4\alpha^4}{n_1}\xi, n_1 \neq 0 \quad (30)$$

Equation (30) is the exact solution of the following partial differential equation

$$in_1 \frac{\partial U}{\partial \xi} + 2n_4\alpha^2 \frac{\partial^2 U}{\partial \tau^2} + n_4 \frac{\partial^4 U}{\partial \tau^4} = 0 \quad (31)$$

We can see that equation (30) describes the propagation dynamics in the very weakly nonlinear flattened optical fiber ($n_3 \rightarrow 0$).

4. Numerical Study

In this section, we use the split-step method [34] to discretize the nonlinear partial differential equations (17) and (28) and to propagate their corresponding solutions. Thus, the constraint relations between the coefficients of the terms of the nonlinear partial differential equation allowed choosing the values of the parameters. We organized this numerical study in two cases.

1. First case

The nonlinear partial differential equation (17) is discretized so that the envelope $U(\xi, \tau)$ is given by the relation (16). The profiles obtained are as follows

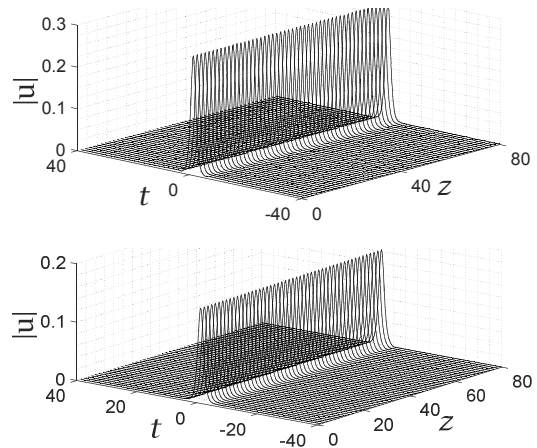


Figure 1. Propagation of the solitary wave (16) in equation (17): the left profile is obtained for: $n_4 = 0.01$, $n_1 = 0.01$, $n_3 = -10$, $\alpha = 0.8$, $\theta = \pi$ the right profile is obtained for $n_4 = 0.5$, $n_1 = 20$, $n_3 = -2$, $\alpha = 0.2$, $\theta = \pi/6$.

2. Second case

The nonlinear partial differential equation (17) is discretized so that the envelope $U(\xi, \tau)$ is given by the relation (16). The profiles obtained are as follows

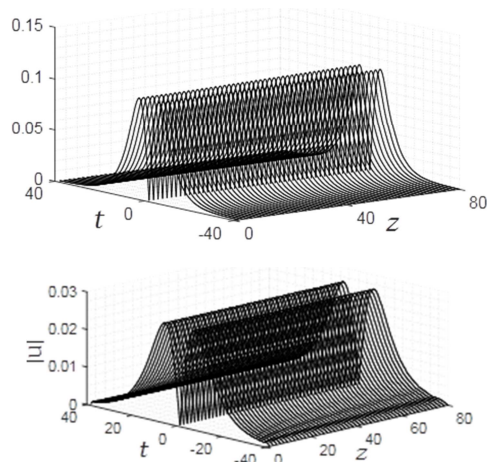


Figure 2. Propagation of the solitary wave (16) in equation (17): the left profile is obtained for: $n_4 = 0.5$, $n_1 = 20$, $n_3 = -2$, $\alpha = 0.2$, $\theta = \pi/6$ the right profile is obtained for $n_4 = 0.1$, $n_1 = 5$, $n_3 = -1$, $\alpha = 0.13$, $\theta = \pi$.

5. Conclusion

We have in the framework of this article studied how to choose the characteristic parameters of the single-mode fiber flattened so that the differential equations that govern the propagation dynamics in this transmission medium admit desired solutions. To achieve this, we divide the work into two major parts. A first part, where we have analytically established the relationships linking the parameters of the fiber or medium in which the fiber is immersed, so that the solutions we need have been obtained. To this end, we assigned the coefficients n_i ($i = 1, 2, 3, 4$) to the different terms of the nonlinear partial differential equation to solve and subsequently obtain the constraints that bind the n_i coefficients to n_i and other parameters of the studied system. We have found a field of possibilities of obtaining solutions through the different values that can take n and m . We note that in the case of the flattened optical fiber that is to say highly dispersive, the solitary wave solutions obtained are pulse of second order and kink of second order. All the values of the pairs (n, m) do not lead to the important solutions in physics domain. We get a second-order pulse for the pair (2,0) and a second-order kink solution for the pair (2,1). The split-step method is used to study the propagation of the solutions obtained. The study in this paper is very fascinating and interesting analytically because beyond its physical reach, it has a considerable mathematical significance. Numerically, we have checked the reliability of the solutions obtained. This study can potentially have very positive impacts in propagation phenomena in the optical fiber and naturally in mathematics for nonlinear physics.

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