

# Annular Axisymmetric Stagnation Flow of a Casson Fluid Through Porous Media in a Moving Cylinder

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## To cite this article:

Muhammad Umar Farooq, Abdul Rehman, Naveed Sheikh. (2023). Annular Axisymmetric Stagnation Flow of a Casson Fluid Through Porous Media in a Moving Cylinder. *American Journal of Modern Physics*, 12(3), 47-53. <https://doi.org/10.11648/j.ajmp.20231203.12>

**Received:** November 14, 2023; **Accepted:** December 4, 2023; **Published:** December 14, 2023

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**Abstract:** The present investigation deals with the study of annular axisymmetric stagnation flow and heat transfer of a Casson fluid through porous medium in a moving cylinder. The inner cylinder is rotating with a fixed angular velocity and is also translating along axial axis with constant axial velocity while the outer cylinder is assumed fixed. Fluid is injected from the top surface of the fixed outer cylinder towards the translating and rotating inner cylinder. The motion of the Casson fluid is assumed under the influence of some porous media. The governing nonlinear partial differential equations of conservation of mass, conservation of linear momentum and heat transfer are obtained and then simplified with the help of a set of suitable similarity transformations which reduces the original set of partial differential equations into a new simplified set of ordinary differential equations. The resulting system of ordinary differential equations is then solved numerically with the aid of fifth order Runge-Kutta Fehlberg method. A comparison of special cases of the present numerical solution with the already available work is also included through tables. The behavior of important involved physical non-dimensional parameters like Prandtl numbers, porosity parameter and Reynolds numbers is also presented at the end.

**Keywords:** Axisymmetric Stagnation Flow, Casson Fluid, Moving Cylinder, Porous Medium

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## 1. Introduction

The study of stagnation point flow was initiated by Hiemenz [1]. Since then many researchers have contributed to the study of stagnation point flow for viscous and non-Newtonian fluids for different flow geometries. Weidman and Putkaradze [2] have classified stagnation flows as viscous or inviscid, steady or unsteady, two or three dimensional, symmetric or asymmetric, normal or oblique, homogenous or inhomogeneous and forward or reverse. In certain situations, fluid is been stagnated by the surface of the object, while in some cases a free stagnation point or line exists in the interior of the fluid domain. The problem of mixed convection stagnation point flow of a non-Newtonian micropolar nanofluid flowing over a vertical slender cylinder is studied by Rehman and Nadeem [3]. In another work, Robert et al [4] have analyzed the hydromagnetic stagnation point flow of a second grade fluid over a stretching sheet. Chiam [5] has

examined the variable conductivity heat transfer in a stagnation flow towards a stretching sheet. In a very recent work, Rehman et al [6] have presented the solution for the problem of boundary layer stagnation flow of a third grade fluid that is flowing over an exponentially stretching surface. The non-orthogonal stagnation point flow towards a stretching vertical plate was examined by Yian et al [7]. The results obtained in [7] indicate that when the stretching velocity of the surface is greater than the free stream stagnation velocity the boundary layer gains an inverted structure. Few other interesting works concerning the stagnation flows are referred in [8-22].

Recently, Nadeem et al [23] have discussed the axisymmetric stagnation flow of a micropolar fluid containing nanoparticles and is flowing through the annular region between the concentric cylinders. The present work provides an investigation for the flow of a non-Newtonian Casson fluid flowing through porous medium. The fluid is assumed to be flowing through the annular region between the two

concentric cylinders. The infinite inner cylinder is rotating as well as translating along the axial direction while the finite outer cylinder is kept fixed. The solution of such finite domain stagnation flow problem is very rare. For solution of the problem, first the governing equations are nondimensionalized using a similarity transformation; the obtained system of ordinary differential equations is solved using the numerical scheme the Fehlberg method. The behavior of velocity and temperature profiles is discussed in detail for the parameters involved at the end.

## 2. Formulation

Consider an incompressible flow of a Casson fluid between two cylinders flowing through porous medium. The flow is assumed to be axisymmetric about  $z$ -axis. The inner cylinder is of radius  $R$  rotating with an angular velocity  $\Omega$  and moving with velocity  $W$  in the axial  $z$ -direction. The inner cylinder is enclosed by an outer cylinder of radius  $b_0 R$ . The fluid is injected radially with velocity  $U$  from the outer cylinder towards the inner cylinder. The governing equations of motion and heat transfer are [24-30].

$$rw_z + (ru)_r = 0, \quad (1)$$

$$uu_r + wu_z - \frac{v^2}{r} = -\frac{1}{\rho} p_r + \nu \left(1 + \frac{1}{\beta}\right) \left(u_{rr} + \frac{1}{r} u_r + u_{zz} - \frac{u}{r^2}\right) - \frac{\nu \phi_p}{k_0} u, \quad (2)$$

$$uv_r + wv_z + \frac{uv}{r} = \nu \left(1 + \frac{1}{\beta}\right) \left(v_{rr} + \frac{1}{r} v_r + v_{zz} - \frac{v}{r^2}\right) - \frac{\nu \phi_p}{k_0} v, \quad (3)$$

$$uw_r + ww_z = -\frac{1}{\rho} p_z + \nu \left(1 + \frac{1}{\beta}\right) \left(w_{rr} + \frac{1}{r} w_r + w_{zz}\right) - \frac{\nu \phi_p}{k_0} w, \quad (4)$$

$$uT_r + wT_z = \alpha \left(T_{rr} + \frac{1}{r} T_r + T_{zz}\right), \quad (5)$$

where  $(u, v, w)$  are the velocity components along  $(r, \theta, z)$  axes,  $\mu$  is the viscosity,  $\rho$  is the density,  $\nu$  is the kinematic viscosity,  $p$  is pressure,  $\beta$  is the Casson fluid parameter,  $\phi_p$  is porosity of porous space,  $k_0$  is permeability of porous space,  $\alpha$  is the thermal diffusivity and  $T$  is temperature. The corresponding boundary conditions are

$$u(R, z) = 0, \quad v(R, z) = \Omega a, \quad w(R, z) = W, \quad T(R, z) = T_1, \quad (6)$$

$$u(b_0 R, z) = -U, \quad v(b_0 R, z) = 0, \quad w(b_0 R, z) = 0, \quad T(b_0 R, z) = T_2, \quad (7)$$

where  $T_1$  is the fluid temperature at the surface of the inner cylinder while  $T_2$  is the fluid temperature at the surface of the outer cylinder.

## 3. Solution of the Problem

The system of nonlinear partial differential equations (1-5) subject to the boundary conditions (6-7) are simplified using the similarity transformations [16]

$$u = -\frac{Uf(\eta)}{\sqrt{\eta}}, \quad v = \Omega a h(\eta), \quad w = 2Uf'(\eta)\xi + Wg(\eta), \quad (8)$$

$$\theta = \frac{T - T_1}{T_b - T_1}, \quad \eta = \frac{r^2}{R^2}, \quad \xi = \frac{z}{R}. \quad (9)$$

With the help of above transformation, Eq.(1) is identically satisfied and Eqs.(2) to (5) take the following form

$$\left(1 + \frac{1}{\beta}\right)(\eta f^{(IV)} + 2f''') + \text{Re}(ff''' - f'f'') - kpf'' = 0, \quad (10)$$

$$\left(1 + \frac{1}{\beta}\right)(\eta g'' + g') + \text{Re}(fg' - f'g) - kpg = 0, \quad (11)$$

$$\left(1 + \frac{1}{\beta}\right)(\eta h'' + h' - \frac{h}{4\eta}) + \text{Re}\left(2fh' + \frac{fh}{\eta}\right) - kph = 0, \quad (12)$$

$$\eta\theta'' + \theta' + \text{Pr Re } f\theta' = 0, \quad (13)$$

in which  $\text{Re} = UR/2\nu$  is the Reynolds number,  $kp = R^2\phi_p/4k_0$  is the porosity parameter and  $\text{Pr} = \nu/\alpha$  is the Prandtl number. The boundary conditions in nondimensional form are

$$f(1) = 0, \quad f'(1) = 0, \quad g(1) = 1, \quad h(1) = 1, \quad \theta(1) = 0, \quad (14)$$

$$f(b) = \sqrt{b}, \quad f'(b) = 0, \quad g(b) = 0, \quad h(b) = 0, \quad \theta(b) = 1. \quad (15)$$

Where  $\sqrt{b} = b_0$ . For numerical solution of the problem, we consider  $F_1 = f'$ ,  $F_2 = F_1'$ ,  $F_3 = F_2'$ ,  $G = g'$ ,  $H = h'$  and  $\phi = \theta'$  then the resulting system is

$$(1 + \frac{1}{\beta})(\eta F_3' + 2F_3) + \text{Re}(fF_3 - F_1F_2) - kpF_2 = 0, \quad (16)$$

$$(1 + \frac{1}{\beta})(\eta G' + G) + \text{Re}(fG - F_1g) - kpg = 0, \quad (17)$$

$$(1 + \frac{1}{\beta})(\eta H' + H - \frac{h}{4\eta}) + \text{Re}\left(2fH + \frac{fh}{\eta}\right) - kph = 0, \quad (18)$$

$$\eta\phi' + \phi + \text{Pr Re } f\phi = 0, \quad (19)$$

The above system of first order odes is solved numerically with the help of fifth order Fehlberg method and the obtained results are discussed in the next section.

## 4. Results and Discussion

The numerical solution of the problem of steady, incompressible, non-Newtonian Casson fluid flowing through the annular region between the concentric cylinders is obtained through the fifth order Fehlberg method. The outer cylinder is assumed to be fixed while the inner cylinder is translating with a constant axial velocity and is also rotating axisymmetrically about the axial direction through some porous medium. The behavior of non-dimensional velocity profiles for the involved parameters is plotted in figures 1 and 2 [31-35]. In these figures  $f$  is associated with the fluid flow in radial direction,  $g$  is concerned with the fluid flow in axial direction while  $h$  represents the rotation effects in the fluid flow caused due to the rotation of inner cylinder. Figures (1-3) show the behavior of radial velocity with respect to the Reynolds numbers  $\text{Re}$ , Casson fluid parameter  $\beta$  and the porosity parameter  $kp$  respectively. From figure 1 it is noted that with the increase in Reynolds numbers  $\text{Re}$  the non-dimensional radial velocity increases. This happen because higher radial velocity is concerned with a higher injunction velocity (and so a higher Reynolds number) from outer cylinder towards the inner cylinder. Figure 2 shows that with increase in the Casson fluid parameter  $\beta$  increases the radial velocity  $f$ . figure 3 indicates that by increasing the porosity parameter  $kp$  the radial velocity decreases because higher  $kp$  corresponds to porous medium with lesser permeability and thus fluid flow encounters more resistance. The influence of Reynolds numbers  $\text{Re}$  over the velocity gradient  $f'$  is sketched in figure 4 that indicates that near the surface of inner cylinder velocity gradient increases with the increase in  $\text{Re}$  while near the surface of outer cylinder, increase in  $\text{Re}$  decreases the velocity gradient. Figures 5 and 6 give the impact of Casson fluid parameter  $\beta$  and the porosity parameter  $kp$  over the velocity gradient  $f'$ . From these graphs it is observed that near the surface of inner cylinder

increase in  $\beta$  increases  $f'$  while increase in  $kp$  decreases  $f'$ , whereas near the surface of outer cylinder, increase in  $\beta$  decreases  $f'$  while increase in  $kp$  increases  $f'$ . figures 7-9 show the behavior of axial velocity profile  $g$  for Reynolds numbers  $\text{Re}$ , Casson fluid parameter  $\beta$  and the porosity parameter  $kp$ . From these plots it is noticed that the axial velocity  $g$  decreases with increase in all the parameters  $\text{Re}$ ,  $\beta$  and  $kp$ . figures 10-12 shows the pattern adopted by the non-dimensional rotation velocity  $h$  for the involved parameters  $\text{Re}$ ,  $\beta$  and  $kp$  respectively. From these figures it is evident that with increase in all the three parameters the angular velocity profile  $h$  decreases. The variation in temperature flow  $\theta$  for Prandtl numbers  $\text{Pr}$  and Reynolds numbers  $\text{Re}$  is presented in figures 13 and 14. From these sketches it is noted that with increase in both  $\text{Pr}$  and  $\text{Re}$  the non-dimensional temperature profile increases [36-42].

To validate the accuracy of the current numerical results the present Fehlberg solutions are compared as a special case with the existing work of Hong and Wang [16] in table 1. The boundary derivatives obtained for the velocity profiles are in excellent agreement. Table 2 predicts the behavior of boundary derivatives for the velocity profiles  $f$  and  $g$  computed at the surface of the inner cylinder that proportional to the skinfriction at the surface of the inner cylinder computed for different values of the Casson fluid parameter  $\beta$  and the porosity parameter  $kp$ . From table 2 it is evident that increasing  $\beta$  and  $kp$  also increases the skinfriction coefficient. Table 3 contains the values for the boundary derivative of the angular velocity profile that are computed for different Reynolds numbers and Prandtl numbers and that corresponds to the azimuthal shear stress at the surface of the inner cylinder. From table 3 it is observed that increase in both  $\text{Re}$  and  $\text{Pr}$  increases the azimuthal shear stress. Table 4 shows the numerical values of the boundary derivative of the temperature flow function  $\theta$  that predicts the behavior of Nusselt numbers  $Nu$  against Prandtl numbers  $\text{Pr}$  and Reynolds numbers  $\text{Re}$ . From table 4 it is observed that the Nusselt numbers increases with the increase in both  $\text{Pr}$  and  $\text{Re}$  [43-45].

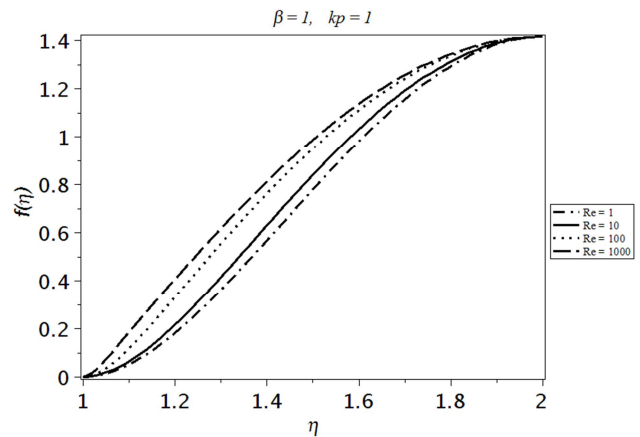
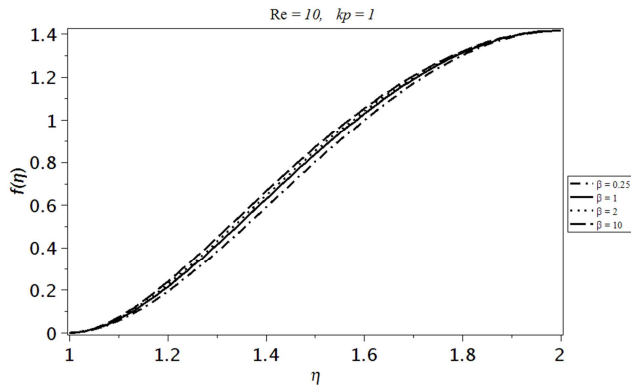
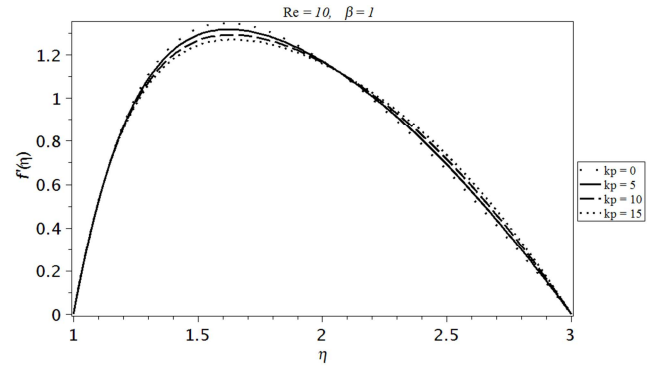
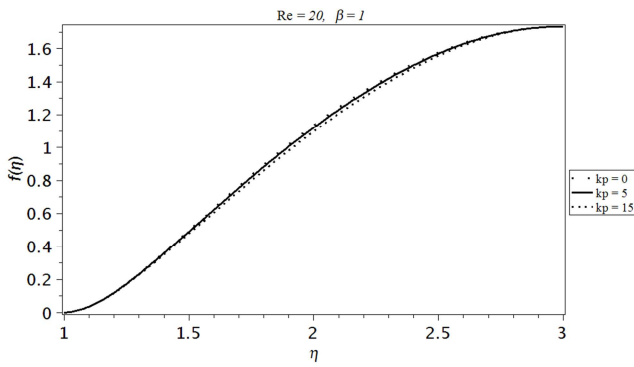
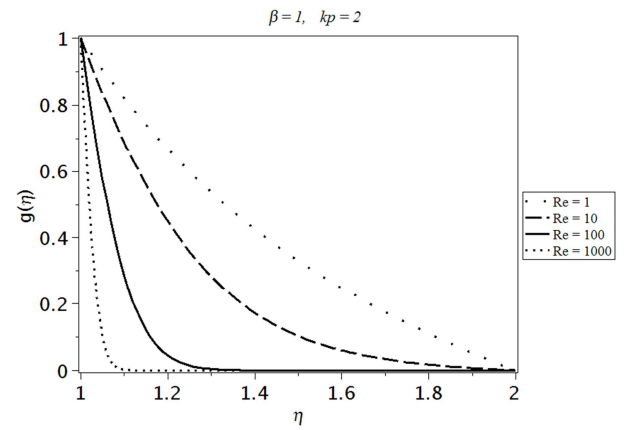
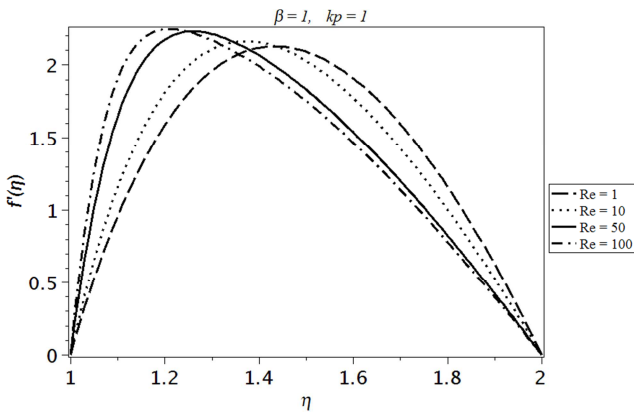
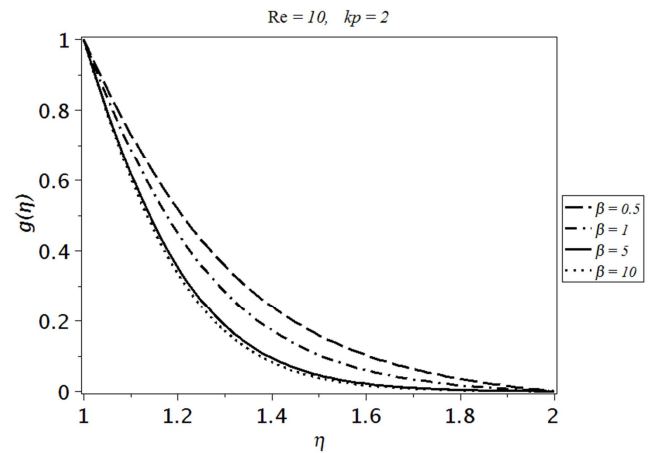
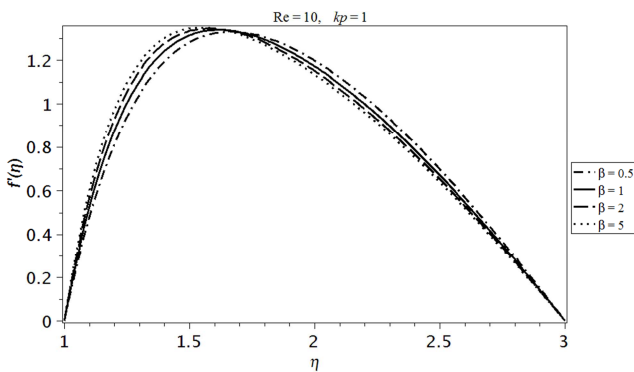
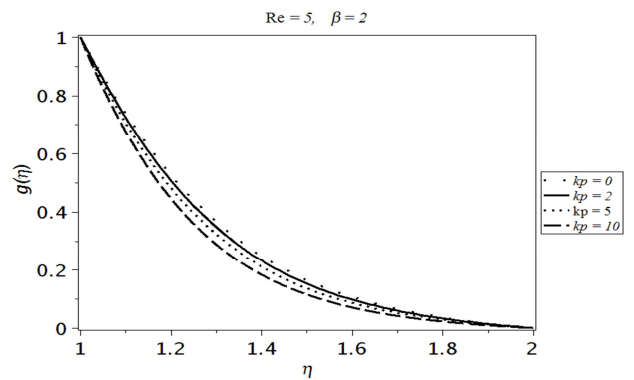
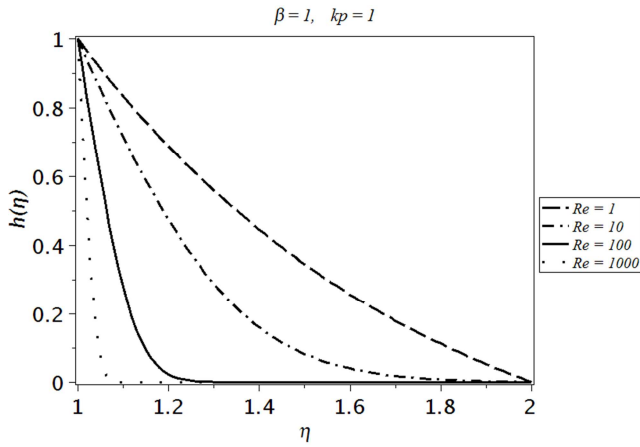
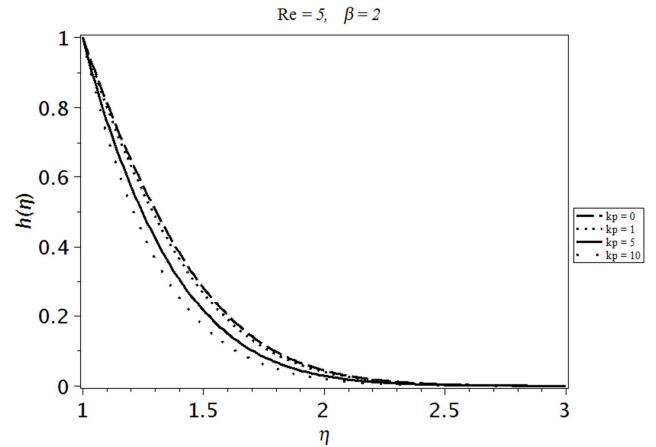
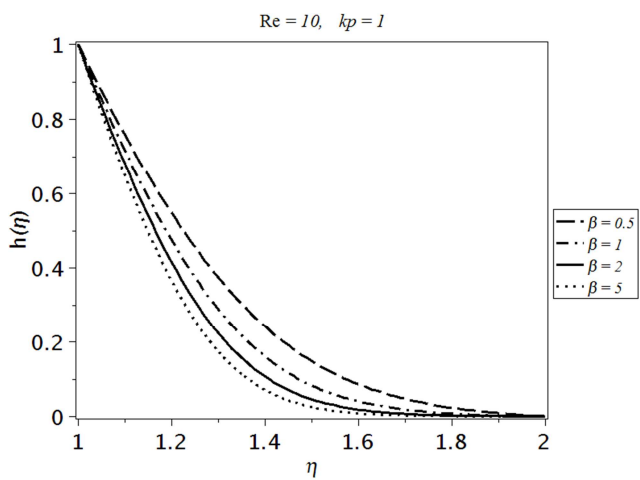
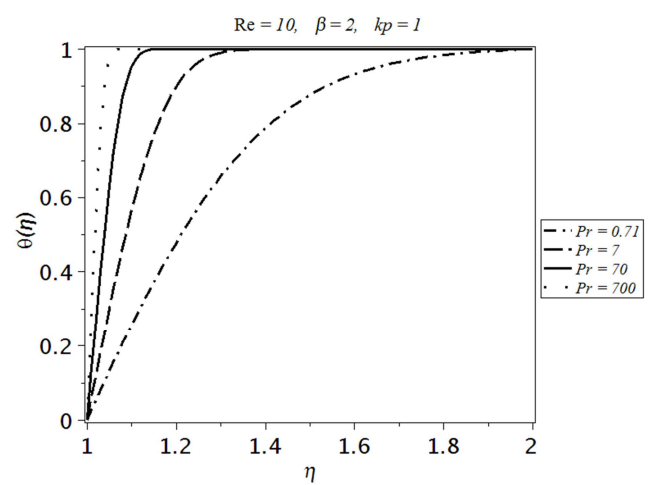


Figure 1. Influence of  $\text{Re}$  over  $f$ .

Figure 2. Influence of  $\beta$  over  $f$ .Figure 6. Influence of  $k_p$  over  $f$ .Figure 3. Influence of  $k_p$  over  $f$ .Figure 7. Influence of  $Re$  over  $g$ .Figure 4. Influence of  $Re$  over  $f$ .Figure 8. Influence of  $\beta$  over  $g$ .Figure 5. Influence of  $\beta$  over  $f$ .Figure 9. Influence of  $k_p$  over  $g$ .

Figure 10. Influence of  $Re$  over  $h$ .Figure 12. Influence of  $k_p$  over  $h$ .Figure 11. Influence of  $\beta$  over  $h$ .Figure 13. Influence of  $Pr$  over  $\theta$ .Table 1. Comparison of the boundary derivatives of the present results with the work in [16] when  $\beta=20000$  and  $k_p=0$ .

$\beta/k_p$	$f''(1)$					$-g'(1)$				
	0.2	0.5	1	2	5	0.2	0.5	1	2	5
0.5	13.366	13.374	13.386	13.409	13.481	2.805	2.821	2.847	2.899	3.051
1	14.499	14.509	14.523	14.552	14.639	3.288	3.308	3.341	3.407	3.598
5	16.583	16.593	16.608	16.640	16.739	4.071	4.097	4.141	4.226	4.476
50	17.431	17.434	17.455	17.487	17.586	4.367	4.395	4.442	4.535	4.807
500	17.530	17.539	17.554	17.585	17.685	4.401	4.429	4.477	4.571	4.845

Table 2. Behavior of boundary derivatives for linear velocity profiles computed for different  $\beta$  and  $k_p$  when  $Re=10$ .

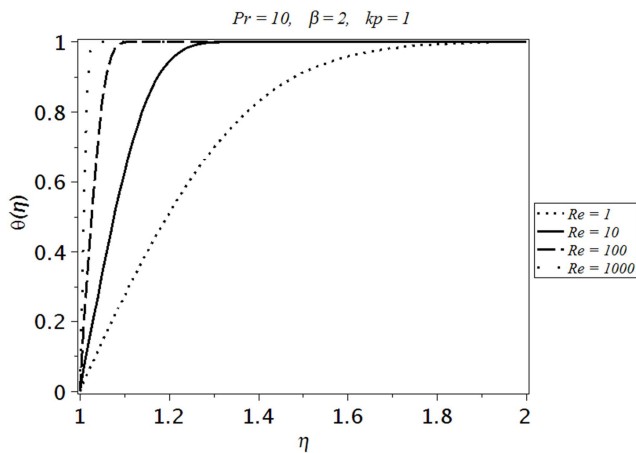
$Re$	$f''(1)$		$-f'''(1)$		$-g'(1)$		$-h'(1)$	
	Present	[16]	Present	[16]	Present	[16]	Present	[16]
0.1	11.001	11.0010	36.1443	36.1443	1.4963	1.4963	10.5151	10.5151
1	11.677	11.6772	41.0795	41.0797	1.9309	1.9309	10.6511	10.6511
10	17.5345	17.5348	93.565	93.5670	4.3856	4.3856	12.0407	12.0407
100	44.449	44.4492	790.731	590.738	12.6450	12.6450	24.8226	24.8226

Table 3. Behavior of boundary derivatives for angular velocity profiles computed for different  $Pr$  and  $Re$  when  $\beta=5$ .

$Re/k_p$	$-h'(1)$				
	0.2	0.5	1	2	5
0.5	1.6705	1.7335	1.8358	2.0309	2.5534
1	1.7987	1.8584	1.9555	2.1412	2.6422
5	2.7703	2.8095	2.8740	3.0003	3.3588
10	3.6861	3.7142	3.7608	3.8529	4.1216
100	10.5678	10.5766	10.5912	10.6203	10.7076

**Table 4.** Behavior of boundary derivatives for temperature profile computed for different  $pr$  and  $re$  when  $\beta=5$ ,  $kp=2$ .

Re/Pr	(1)				
	0.7	7	20	70	700
0.5	1.5063	2.0426	2.8658	4.4448	9.7080
1	1.5707	2.5416	3.6821	5.6738	12.3513
5	2.0910	4.6744	6.7300	10.3204	22.4300
10	2.6897	6.2203	8.9540	13.7367	29.8783
100	7.6624	17.8594	25.7870	39.6852	

**Figure 14.** Influence of  $Re$  over  $\theta$ .

## 5. Conclusion

In the present paper the authors have analyzed the problem of Casson fluid flow through the annular region formed between to concentric cylinders. Main findings of the study are as under

1. With increase in the porosity parameter the Reynolds number the angular velocity decreases
2. With increase in the Prandtl number the temperature profile the thermal layer decreases.

## Conflicts of Interest

The authors declare no conflicts of interest.

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