

# Deriving $E = mc^2/22$ of Einstein's ordinary quantum relativity energy density from the Lie symmetry group $SO(10)$ of grand unification of all fundamental forces and without quantum mechanics

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**Abstract:** In the present short letter we aim at deriving the cosmic ordinary effective quantum gravity energy density as well as that of dark energy from the  $SO(10)$  Lie symmetry group of grand unification. Remarkably the derivation makes no use of quantum mechanics and remains largely classical except for nonclassical geometry and topology. Finally our main conclusions and results are reinforced using a nonlocal classical elastic field theory.

**Keywords:** Einstein's Quantum Gravity Energy, Dark Energy, Lie Symmetry Groups, Unification of Fundamental Forces

## 1. Introduction

To understand the mystery of dark energy is one of the most pressing issues in current cutting edge research efforts in physics and cosmology [1-3]. However the very meaning of fundamental forces is itself the subject of needed scientific understanding [1-27]. The present work is motivated by the belief that science represents a wholeness in which dark energy understanding depends on the understanding of the unification of fundamental forces which we use here [18-26]. In addition we show how all our results and conclusions can be obtained from a nonlocal elasticity theory due to A.C. Eringen which does not suffer from the technical and philosophical limitations of the continuum and Newton's differential calculus [4-7].

## 2. Analysis

We start not from quantum mechanics, nor in fact relativity but from Newtonian kinetic energy of particles which never "stop moving" and flight with maximal average fractal speed  $\langle v \rangle = c$  [2]. Thus we have [1,2]

$$E_N = \frac{1}{2} m \langle v \rangle^2. \quad (1)$$

Now we have to consider that there are two different but related things, namely spacetime degrees of freedom. In Einstein relativity we have first the four dimensions of spacetime made by fusing 3 space dimensions with the time dimension. Second we have the degrees of freedom of the Lagrangian of relativity. For Newton as well as for special relativity we have a single degree of freedom, namely a "single" Newtonian particle species because Newtonian material points are all the same like sand on the beach and a single messenger particle, the photon as far as special relativity is concerned. To deal with quantum gravity on the other hand we need unification of at least the non-gravitational interaction and hope that Newton's formula which formally at least seems to stand in an obvious self affinity if not similarity relation with Einstein's formula will come out intact. For such GUT unification we may chose the Lie symmetry group  $SO(10)$ . This group has  $|SO(10)| = 45$  generators representing "photon" like massless gauge bosons extending the standard model  $|SU(3)SU(2)U(1)| = 12$  by adding 23 more exotic messenger particles. Compared to  $E_N$  we have therefore  $45 - \gamma = 45 - 1 = 44$  more degrees of freedom of our nonconstructive Lagrangian. We may also mention on passing that the 44 degrees of freedom could also be seen as coming from the Vierbein representation  $[(D-1)(D-2)/2] - 1 = 44$  for  $D = 11$  of Witten's theory [12]. From these few simple thoughts

we see that we can scale  $E_N$  to  $E_{QR}$  of quantum gravity by involving a Lorentzian-like factor found as follows [2,3]

$$\begin{aligned}\gamma &= \frac{D^{(4)}}{|\text{SO}(10)| - \gamma} \\ &= \frac{4}{45 - 1} \\ &= \frac{4}{44} \\ &= \frac{1}{11}.\end{aligned}\quad (2)$$

Inserting in  $E_N = \gamma \frac{1}{2} m(v \rightarrow c)^2$  we find our quantum relativity energy density [16-27]

$$E_{QR} = \left(\frac{1}{2}\right) \left(\frac{1}{11}\right) mc^2 = mc^2 / 22. \quad (3)$$

Now we ask the question of what happens if SO(10) unification is replaced by the other Lie symmetry group SU(5) GUT? The answer is that in this case we have 24 gauge bosons which resemble a super symmetric minimal extension of the bosonic sector of the standard model. To keep things equal we imagine that there is also a super symmetric partner involved in Einstein's single photon Lagrangian so that effectively we have one photon and one photino making 2 gauge bosons altogether. Unlike in the previous SO(10) derivation we have to start from Einstein's formula and scale it to a quantum formula using the Lorentz-like factor [2,3]

$$\begin{aligned}\gamma &= \frac{1}{|\text{SU}(5)| - [\gamma + \gamma]} \\ &= \frac{1}{24 - 2} \\ &= \frac{1}{22}\end{aligned}\quad (4)$$

so that we find again

$$E = mc^2 / 22. \quad (5)$$

The analysis leading the  $E(D)$  of dark energy is now almost trivial since  $E(D) = E(\text{Einstein}) - (mc^2 / 22)$  which gives  $E(D) = mc^2 (21/22)$  [16-27]. We stress the importance of the present main feature of the present result, namely that we obtained a basically effective quantum gravity formula without using quantum mechanics. This could be interpreted as meaning that nonclassical fractal geometry and topology substitutes for the absence of quantum mechanics formalism [2,3].

### 3. Analysis Using the Theory of Nonlocal Elasticity [13-16]

In what follows we basically reargue the preceding conclusions and re-derive the basic formulas from the view point of a classical but nonlocal field theory due to A.C. Eringen [13,15,16].

Following the methodology explained by Challamel, Wang and Elishakoff in [13], the following nonlocal buckling instability Eigenvalue was found (see equations 79 to 84 of Ref. 13):

$$P(\text{nonlocal}) = \frac{P(\text{Euler})}{1 + \frac{P(\text{Euler})}{(EI)/L_c^2}} \quad (6)$$

where  $P(\text{Euler})$  is the buckling load familiar from strength of material of classical elasticity,  $EI$  is the bending stiffness and  $L_c$  is the critical buckling length [14]. Interpreting  $EI/L_c^2$  as a second Eigenvalue critical load due to nonlocal effects we could write  $P(\text{nonlocal})$  using Dunkerely's well known theorem [14] as

$$\frac{1}{P(\text{nonlocal})} = \frac{1}{P(\text{Euler})} + \frac{1}{EI/L_c^2} \quad (7)$$

which is nothing but the previous equation for  $P(\text{nonlocal})$  written in a different form. Note also that the vital nonlocal parameter  $n \Rightarrow e_0^2$  given by equation 80 of Ref. [10] may be continued transfinitely to give

$$e_0^2 \rightarrow \phi^4 = 0.145898034 \quad (8)$$

where  $\phi = (\sqrt{5} - 1)/2$  is the basis of the golden mean binary number system found from a simple two degrees of freedom golden oscillator [3].

In the next section we will see how the preceding two Eigenvalues interpretation of nonlocality in conjunction with the transfinite continuation to  $\phi^4$  will lead to extremely important conclusions regarding an extension of Einstein's famous formula  $E = mc^2$  and the nature of dark energy [16-27].

Let us first recall that the Schrödinger equation was inspired by diffusion and wave equations [4] and that it is basically extending the classical quantization of vibration frequencies and buckling loads to quantum energy states [7,14]. Seen that way we could regard  $E = mc^2$  of Einstein's maximal energy as an Eigenvalue while the Planck maximal energy, beyond which measurement is meaningless [7,9,11], is a second Eigenvalue related more to quantum mechanics rather than relativity although Witten's T-duality could be interpreted as setting the Hubble length and the Planck length on similar footing [20,21]. Thus we could start surmising if it would not be possible to combine the two Eigenvalues to find out information about a nonlocal Eigenvalue corresponding to quantum relativity which is effectively a quantum gravity theory [19]. This line of

thinking is far from being outlandish in engineering and was in fact used in a similar context to develop the well known Rankin-Merchant formula for elasto-plastic buckling [14]. In this formula the elastic buckling load which could be compared to Einstein's  $E = mc^2$  and the ultimate plastic load which could be compared to Planck energy are combined to obtain a reasonable estimation of the critical behaviour of columns in the complex elasto-plastic range [14]. That way we could write the following quantum relativity energy equation.

$$E(\text{nonlocal}) = \frac{mc^2}{1 + \frac{mc^2}{E_p}} \quad (9)$$

where  $m$  is the mass,  $c$  is the speed of light and  $E_p$  is the esoterically large Planck energy thought to be in the region of  $10^{19}$  GeV [3,4,6] [15-24].

In [20] Magueijo and Smolin combined quantum field theory, relativity and the idea of varying speed of light in a thoroughly ingenious way to produce a quantum gravity formula corresponding to Einstein's "non-quantum" formula  $E = mc^2$  [20,21]. We do not need to write the Magueijo-Smolín formula since it is nothing else but our equation No. 4  $E(\text{nonlocal})$  which we just derived using extremely simple analysis. This agreement speaks volumes about the unity of physics, engineering and mathematics. This formula was further extended by the author using unit interval Cantorian physics to reproduce both the ordinary measurable energy of the cosmos  $E(O)$  and the dark cosmic energy density  $E(D)$  [20,21,24]. Readers interested in the subject are referred to the concerned literature [22,23]. We only note that in the classical domain  $E_p \gg mc^2$  and  $E(\text{nonlocal}) \cong E(\text{local}) = E(\text{Einstein}) = mc^2$  because we can set  $1 + mc^2/E_p \approx 1$ . In addition it was found that  $E(O) \approx mc^2/22$  while  $E(D) \approx mc^2(21/22)$  which means that  $E(O) + E(D) = E(\text{Einstein})$ . In other words, quantum mechanics was well hidden inside Einstein's "classical" formula all this time [20-24].

## 4. Conclusion

Unification of all fundamental interactions clearly has a role to play in understanding many outstanding questions in physics. Here we show how  $SO(10)$  Lie symmetry group grand unification can contribute to the understanding of the dark energy density of the cosmos. Quantum physics and relativity may seem sometimes to be remote and very far away in concepts and mathematical formulation from classical mechanics [3,7]. The present analysis contests this view and asserts the unity of not only science but also science and engineering. Following ideas related in particular to C. Eringen nonlocal elasticity [13,15], we were able to derive basic equations related to fundamental questions in quantum physics and cosmology. Thus a formula on multi-Eigenvalues due to Dunkerley was found to combine Einstein's maximal energy  $E = mc^2$  with Planck energy  $E_p \approx (10)^{19}$  GeV [24] to give information on the

ordinary energy density  $E(O) = mc^2/22$  and dark energy density  $E(D) = mc^2(21/22)$  of the cosmos respectively. It is a little surprising if not surreal that a quasi-classical field theory like that of Eringen can be used as successfully as a gauge theory for gravity [26,27]. Application of the above to nanotechnology [28,29] is currently a focus of a small group working with the author and we hope to report upon that in the near future.

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