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On the Evaluation of the Derived Khartoum Geoid Models

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Abstract: This study was carried out to evaluate a newly developed local geoid model for the Khartoum State. Firstly, the geoid heights were obtained from the differences between observed WGS84 ellipsoidal heights and Khartoum State known orthometric heights. Secondly, the global well known EGM2008 geoid model was used to extract the geoid heights at 48 ground control stations, followed by generation of a geoid surface corrector. Thirdly, the geoid heights were also obtained by using an interpolation method. The three methods used for geoid determination were compared and evaluated. The geoid models' uncertainties were evaluated at 6 test ground control points. The average difference between the derived geoid heights obtained from the geometrical geoid model, and their corresponding EGM2008 geoid heights was determined with a bias of about -0.28m. EGM2008 geoid model values in the area were corrected by the local bias value of 28 cm. The same known ground control points were used for the implementation of the interpolation geoid model. The differences between the geometrical geoid heights obtained from the differences between WGS84 ellipsoidal heights and orthometric heights at the given ground control points were compared with the geoid height values obtained from the improved local EGM2008 geoid and the interpolation method, the differences were found to be in the range of 4 cm and 5 cm respectively. This study showed that, the geoid heights in Khartoum State can be determined with the above two methods, namely the improved EGM2008 and the interpolation method with typical accuracy of better than 5cm.

Keywords: GPS, GNSS, EGM2008, WGS84, UTM, ITRF2008, GDS, TBC

1. Introduction

The Department of Surveying Engineering of the University of Khartoum, continued to perform other techniques in addition to Neural Network Khartoum Geoid Model given in the research [1], for the determination of Khartoum WGS84 local geoid, with the intention to derive the local geoid model for entire Sudan territory. By obtaining the WGS84 coordinates of 48 ground control points by GNSS static observations and their corresponding first order levelling orthometric heights, the geometrical geoid heights were computed from the differences between the ellipsoidal heights and orthometric heights at each ground control point. These differences were compared with geoid heights obtained from the improved local EGM2008 geoid model and an adopted interpolation method. Many contributions have been made in the previous studies, which have been carried out using other techniques and data, for the determination of Sudan geoid, outlined in the articles [1, 4, 8].

The main objective of this study, is to determine and evaluate a local geoid model for Khartoum State derived from two approaches. The study started by the evaluation of the given ground control coordinates and heights in Khartoum State. Here considerations have been taken for all ground control points and bench marks, their sources, stability, accuracy and quality of their original surveys.

The study adopts the use of the well-known geometrical geoid determination method, illustrated in the books [10, 12] based on the difference between ellipsoidal heights and orthometric heights as geoid height reference for the evaluation process. The WGS84 GNSS/levelling geoid is computed as well as EGM2008 global geoid determination, then their results were compared and evaluated to determine the average local EGM2008 surface corrector (bias). This

EGM2008 surface corrector was used to derive the improved local EGM2008 geoid model for Khartoum area. The geoid heights were obtained from the implementation of improved local EGM2008 geoid model and from the interpolation method based on the computation of coefficient parameters representing the bias and tilts of the geoid [5].

The study final results illustrated that, the orthometric heights were derived with accuracies of about ±5 cm from the two geoid determination methods i.e. the improved local EGM2008 geoid and the interpolation method.

Figure 1. *The study area- Khartoum State – Sudan.*

2. Study Area and Data Collection

The study area and the ground control stations used are the same as those used before on the evaluation of the Neural Network Khartoum Geoid Model [1]. Khartoum area is relatively flat surface with slight slope towards to the Nile, at elevation of about 385 m above mean sea level. The state lies between longitudes 31° 37′ 11" to 34° 23′ 08" E and latitudes 15° 10′ 54" to 16° 38′ 54" N (Figure 1) and the WGS84 global geoid is fitting well in Sudan territory.

The Department of Surveying Engineering of the University of Khartoum, acknowledged the cooperation of the General Directorate of Survey (GDS) of Khartoum state for their provision of data and collaboration in all geomatics aspects. The data was in a form of geodetic UTM coordinates, Easting, Northing, and geodetic latitude, longitude, ellipsoidal and orthometric heights.

The main hardware and software used for the data collection and processing are mainly consisted of the Global Navigation Satellite System (GNSS) Trimble receivers and Trimble Business Center (TBC) software. The primary software packages used for extracting the EGM2008 geoid heights and geoid modelling are based on Arc GIS.

The investigations carried out in this study consist of four main steps (figure 2). In step one, the data was acquired and evaluated. In step two, geoid heights were determined by three methods. In step three, the results obtained were analyzed and assessed, then followed by a comparison and quality control processes of the obtained geoid heights.

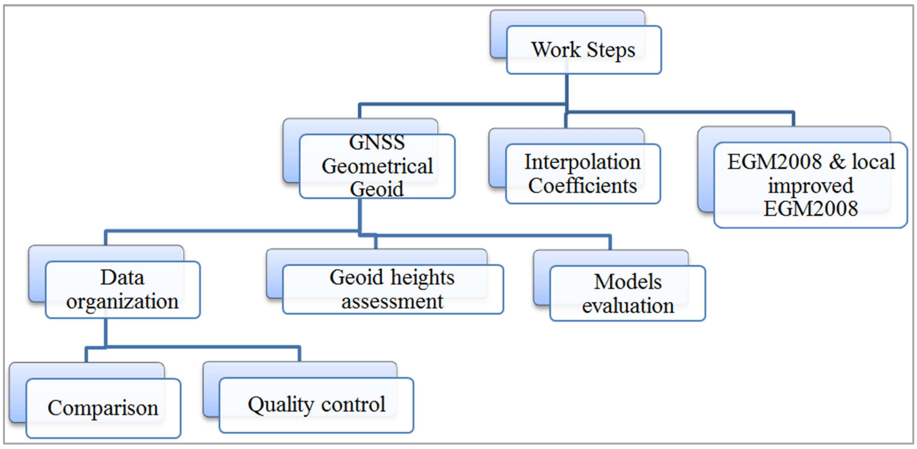

Figure 2. *Frame of Study work.*

The current height network in Khartoum State is based mainly upon a series of leveling runs carried out by the General Directorate of Survey (GDS) of Khartoum State in 2008. The accuracy was specified as 3 mm per square root of levelling run in kilometers. The aim was to provide one point for every 4 km². The datum used for this levelling [14] was the Alexandria vertical datum, established in 1890s.

From the analysis of vertical control, it can be strategically stated that, in the future, it is essential to transform Alexandria heights to a local Sudan vertical datum in the Red Sea at Port Sudan. This is also vital, since the Country intended to move to use ITRF2008-WGS84 as a unified geocentric datum or reference system for national geomatics and geospatial activities [17], as well as to comply with the United Nation Global Geodetic Reference Frame (UN-GGRF) and the regional intention for the unification of African reference frame (AFREF) and geoid. As well known, that, the ellipsoidal heights are not usually used directly for practical surveying and engineering, as they have no physical meaning [5, 13, 18]. Due to the current effective use of GNSS technology, it will be a cost-effective means to derive the orthometric heights from GNSS ellipsoidal heights through the implementation of geoid models.

As mentioned above, in Sudan, orthometric heights are assumed to be referred to Alexandria vertical datum since the year 1890, but the coincidence of this vertical datum with the geoid surface needs further investigation. There are many approaches to transform ellipsoidal heights to orthometric heights, such as by using gravimetric geoid methods, global and geometrical methods [14]. In this paper, only the geometric method of geoid determination, EGM2008 global geoid model, and the interpolation method were considered. Ellipsoidal and orthometric heights are given at 48 benchmarks to provide the geoid heights at these common points.

3. Geometric Geoid Determination

In Sudan, most commonly known and used vertical datums are Alexandria and Sudan irrigation datums. Each system has separate observation and computation procedures, data availability, accuracy requirements, compatibility with GNSS, and the topographic settings in which the heights are used. Unfortunately, even the relationship between these two datums is not well investigated, but in general the difference between the two datums was about 3 meters.

Orthometric heights are usually referenced to the geoid which is an equipotential reference surface, that approximate the mean sea level. The orthometric height, H, of a point on the earth's surface is the distance from a point measured along the plumb line and vertical to geoid surface. While the ellipsoidal heights, h, are referenced to WGS84 reference ellipsoid; and usually defined as the distance from the point at the earth surface to the WGS84 reference ellipsoid. At the same point on the surface of the earth the difference between an ellipsoid height and orthometric height is defined as the geoid height, N, and can be derived by equation (1).

$$N = h - H \quad (1)$$

With regards to the data used in this study, all leveling benchmarks and previous horizontal ground control points were occupied by GNSS and their 3-D geodetic coordinates were determined. Equation (1) was used to compute the values of geometrical geoid heights. The total number of 48 GNSS/leveling benchmarks were measured. The measurements were performed using Trimble dual frequency receivers R8 and DNK3 precise levels. All 48 GNSS/Levelling benchmarks coordinates were computed on the known geodetic reference stations, using at least one-hour GNSS observations in order to define them in the same system at the specific epoch. The combined use of global Navigation Satellite System (GNSS) and leveling is considered to be a key procedure in deriving and evaluating the two adopted Khartoum local geoid models. Although these three types of height information are considerably different in term of accuracy, physical meaning, surface realization, approximations, observation method; but since the geoid heights in the study area are in the range of ±3 m, which means the geoid and ellipsoid surfaces are very close i.e., the deviation of the vertical assumed to be negligible. It can be stated that the relationship between the ellipsoidal heights and their corresponding orthometric heights, should fulfill the following geometrical relationship [16]:

$$h - H - Ng = 0 \quad (2)$$

Where h are ellipsoidal heights obtained from GPS observations, H are orthometric heights derived from leveling methods and Ng are the geoid heights. In practice, due to the random noise in values of H, h and Ng, equation (2) is never satisfied [15, 17]; as affected by many sources of errors such as inconsistencies of datum, systematic distortions; geodynamic effects, and other appropriations in the computation of ellipsoidal and geoid heights.

Featherstone [5] stated that for areas where the bench marks are well geometrically distributed, a geoid surface can be created using equation (3):

$$h - H = Ng + \alpha 0 + \alpha 1\, \mathrm{E} + \alpha 2\, \mathrm{N} \quad (3)$$

where α_0 is the coefficient representing the bias, α_1 and α_2 are the coefficients represent tilts of the geoid surface will respect to WGS84 ellipsoid, and E, N are the UTM easting and northing coordinates. To determine the three coefficients α_0, α_1 and α_2, WGS84 coordinates at 42 ground control points were used, for the determination of the geoid heights. This followed by the application of least squares model to compute the three coefficients $\propto_0$, $\propto_1$ and $\propto_2$. Then in any GNSS survey in the area, orthometric heights can be determined either considering absolute or relative height determination approaches.

There are some important precautions that must be taken when using geometrical interpolation to determine orthometric heights from GNSS, these may include the best accuracy of ellipsoidal and orthometric heights and independent checks should always be used at points with known coordinates and orthometric height to ensure that the geoid is accurately modeled.

4. Improved EGM2008 Geoid Model

To improve the global EGM 2008 geoid model, in the first step, equation (1) was used to determine the geometrical geoid heights, then to be subtracted from the values obtained from the original EGM 2008. The discrepancies of geoidal heights are computed by using equation (4):

$$\Delta N = N_{(known)} - N_{(EGM\ 2008)} \quad (4)$$

Where:

ΔN = discrepancy in the geoidal heights.

$N_{(known)}$ = geoidal heights using geometrical geoid at known control points.

$N_{(EGM\ 2008)}$ = geoidal heights obtained through EGM 2008.

The second step is computation of the average value of the discrepancies of geoidal heights by using equation (5):

$$\Delta \mathrm{N}_{Average} = \sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\Delta \mathrm{N}_{\mathrm{i}}}{\mathrm{n}} \quad (5)$$

In the third step, the corrected values of EGM 2008 were obtained by subtracting the average discrepancy $\Delta N_{Average}$ (considered to be the surface corrector) from original EGM2008 model (Figure 3) by using equation (6):

$$N_{\text{crrected EGM 2008}} = N_{(EGM\ 2008)} - \Delta N_{average} \quad (6)$$

In the fourth step the corrected geoid height values of EGM 2008 was used to construct a grid in Arc GIS, then using the values of gridded nodes to build a model of improved EGM2008 geoid model (or local enhanced EGM2008).

5. Implementation of the Adopted Geoid Models

5.1. Geoid Model Using Interpolation Method

The data was used to build the model of the geoid height by using the adopted interpolation method based on equation (3), in which the coefficients $\propto_0$ is representing the bias, $\propto_1$ and $\propto_2$ are the coefficients represent tilts of the geoid plane with respect to WGS84 ellipsoid, and E, N are the UTM Easting and Northing coordinates.

To determine the three coefficients $\propto_0$, $\propto_1$ and $\propto_2$, the WGS84 coordinates at 42 ground control points/bench marks were used, applying the following least squares observation equation.

$$\begin{bmatrix} \propto_0 \\ \propto_1 \\ \propto_2 \end{bmatrix} = \begin{bmatrix} 1 & E_i & N_i \\ 1 & E_{i+1} & N_{i+1} \\ 1 & E_{i+n} & N_{i+n} \end{bmatrix} \begin{bmatrix} (h-H)_i \\ (h-H)_{i+1} \\ (h-H)_{i+n} \end{bmatrix} \quad (7)$$

$$= A\ X\ B$$

In Least squares, this can be given in a form of observation equation in which i, ranges from 1 to 42:

$$\begin{bmatrix} \propto_0 \\ \propto_1 \\ \propto_2 \end{bmatrix} = (A^T A)^{-1} A^T B \quad (8)$$

Table 1 shows the computed values of the coefficients representing the bias $\propto_0$, and $\propto_1, \propto_2$ tilts of the geoid plane will respect to WGS84 ellipsoid and later used to model the Geoid surface for Khartoum state. The interpolation coefficients were found to be as given in table 1:

Table 1. *Computed coefficients $\propto_0$, $\propto_1$ and $\propto_2$.*

Coefficients	Value
$\propto_0$	-27.74077941
$\propto_1$	0.000003428196402
$\propto_2$	0.000018531773215

Then, equation (3) was used to determine the values of geoid heights at any point with known UTM coordinates in Khartoum State. As well the geoid heights (in Meters), were computed from equation (3) at the six test points and the results were illustrated in table 2.

Table 2. *Geoid heights obtained from the interpolation coefficients, $\propto_0$, $\propto_1$ and $\propto_2$.*

Points	$N_{(known)}$	$N_{(Interpolation)}$	$\Delta N_{(Interpolation)}$
1	2.578	2.538	0.040
2	2.416	2.444	-0.028
3	2.482	2.434	0.048
4	2.632	2.687	-0.052
5	2.517	2.547	-0.030
6	2.451	2.455	-0.004
RMSE			±3.7 cm

Table 2 shows the results of the processed geoid at the 6 ground control test points. The first column shows the points IDs; the second the geometrical geoid heights and column three showed, the geoidal heights obtained by interpolation model. The fourth column highlighted the geoid height differences between geometrical geoid heights and the geoid height values obtained from the interpolation method. The results indicated that geoid heights can be determined from the interpolation method within the accuracy of about ± 5 cm in Khartoum State.

5.2. The Earth Geo-Potential Model EGM2008

The EGM2008 model used to model the geoid within an accuracy of about 30 cm (global average). For a further refinement of the EGM2008 in Khartoum area, surface point information has been introduced as given in the articles [6], and [7]. In the studied area, a steep increase of the geoidal height of about 3m (ranges from -1.24m to 3.6m) occurs over distance of about 100km. The initial analysis of the first processing step revealed an average difference between orthometric heights and the corresponding EGM2008 heights with the magnitude of about 0.28m (table 3).

EGM2008 values were then uniformly corrected by this regional default value. However, the mismatch between bias corrected EGM2008 geoid values and geometrical geoid heights (ellipsoidal heights minus orthometric heights) was found to be with the maximum of ± 8 cm. The discrepancy between the EGM2008 model (bias corrected) and the observations was agreed within ±8 cm, which is better than the overall quality of the EGM2008 model of 28 cm in Khartoum State area.

Table 3. *Geoid heights obtained from the EGM2008.*

Points	$N_{(known)}$	$N_{(EGM2008)}$	$\Delta N_{(EGM2008)}$
1	2.578	2.259	0.319
2	2.416	2.127	0.290
3	2.482	2.182	0.300
4	2.632	2.435	0.197
5	2.517	2.255	0.262
6	2.451	2.136	0.315
RMSE			±28 cm

The EGM2008 average bias (-28 cm) is very close to the bias obtained from the interpolation method (-27.7 cm). This bias closeness indicated that EGM2008 values can be directly used for the determination of the three coefficients of the interpolation method.

Table 4. *Geoid heights obtained from the improved EGM2008.*

Points	$N_{(Known)}$	$N_{(improved\ EGM)}$	ΔN
1	2.578	2.542	0.036
2	2.417	2.410	0.007
3	2.482	2.465	0.017
4	2.632	2.712	-0.080
5	2.451	2.419	0.032
6	2.517	2.538	-0.021
RMSE			±4.2 cm

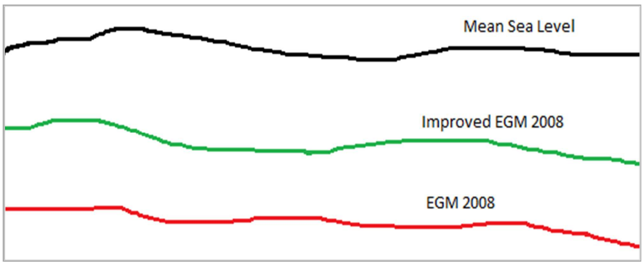

Figure 3. *Shows the surfaces of Global EGM2008 and the improved EGM 2008.*

This approach can be considered as a fundamental component of determining a local geoid using an accurate homogeneous control network extending throughout the area under consideration. The exploitation of this valuable resource to derive the relationship between the heights from GNSS surveys and the orthometric heights and those obtained from the adopted geoid models [9, 11] will certainly allow the various user groups of geospatial data to maximize their use of the adopted models and their investment in GNSS technology and other related information.

6. Conclusions

The determination of Orthometric heights is important for many tasks in engineering studies, design and construction, mapping and geospatial activities. Geometrically, this can be determined by computing the difference between ellipsoidal height and geoid height. This relationship is different and unstable from one area to another. This study can be considered as an attempt to drive a cost-effective orthometric heights with accuracies ranges between ±5 and ±8cm from the interpolation method and the improved EGM2008 respectively. Investigation of the height system of Khartoum area, precise evaluation and analysis on the values of vertical control networks and benchmarks revealed that the interpolated and EGM2008 improved geoid heights can be used in mapping and many geospatial applications.

These Geoid surfaces at Khartoum state, are compared, based on the results and the statistical analysis carried out in this study. Considering the interpolation method with three coefficients representing the bias and tilts of the geoid plane, the discrepancies at the test points was calculated to be in the range of ± 5 cm. While, the discrepancy between the EGM2008 model (bias corrected) and the observations was agreed within ±8cm, with the consideration of the overall quality of the EGM2008 model of about 28cm in Khartoum area. The study also highlighted that the EGM2008 values can be directly used for the determination of the three coefficients $\propto_0$, $\propto_1$ and $\propto_2$ of the interpolation geoid model.

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