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# Determining the Permittivity of a High-Loss Liquid by Resonant Method in Ka-Waveband

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**Abstract:** Dielectric resonators excited by higher types of azimuthal electromagnetic modes, that are whispering gallery modes, are promising to solve physical problems including studying the electrophysical parameters of substances. Disk resonators with a capillary filled with liquid and located in the whispering gallery field region are suitable to determine the permittivity of liquid. Semi-cylinder resonator located on a flat conducting mirror is of particular interest. Ensuring high accuracy and resolution in determining the permittivity of aqueous solutions under study requires a detailed study of the location of the capillary and improvement of the excitation features of the resonator. The work's object is to develop a technique for determining the permittivity of liquid filling the capillary of semi-cylindrical dielectric resonator, and to ensure an effective interaction of the electromagnetic field of resonator whispering gallery eigenmodes with the liquid and to define the conditions for effective excitation of the resonator. The technique for determining the permittivity of liquid is based on solving the inverse problem of electrodynamics. Experimental studies are based on methods for registration the coefficients of the resonator scattering matrix. As a result, the broadband resonant method is presented for determining and monitoring the dielectric properties of liquids that occupy small volumes. The method for determining the complex value of the permittivity of liquid that fills a capillary located in the maximum electric field variation of semi-cylindrical dielectric resonator eigenmode is represented. The inherent parameters of resonator with whispering gallery $EH_{n1\delta}$ modes, in which $30 \le n \le 36$ and $0 < \delta < 3$, were experimentally studied in the frequency range of 31–37.5 GHz. Eigenmodes were excited by the coupling slot in the flat mirror of semi-cylindrical dielectric resonator or by the dielectric waveguide with the distributed weak coupling to the resonator. It has been shown that the most effective excitation of the $EH_{36\,1\delta}$ modes of resonator, the capillary of which is filled with water, is achieved when the coupling slot is located in the region of maximum field intensity along the radial coordinate. It has been established that the quality factor of resonator, whose capillary is filled with distilled water, is higher than that of resonator with an air capillary. It is shown that this effect is made conditional upon the redistribution of the field energy of resonant modes due to dielectric losses in the capillary's aqueous medium and diffraction losses at the slot.

**Keywords:** Dielectric Resonator, Whispering Gallery Modes, Determining the Permittivity of Liquid

# 1. Introduction

In recent years, there has been a significant increase in interest in the study of the electrical properties of water and aqueous solutions at extremely high frequencies (EHF). The main known methods for analyzing liquids by using electromagnetic waves are the followings: the interferometry method [1–5], waveguide methods with samples located both inside the waveguide [6–10] and at its free end [11], resonant methods [12–18]. Naturally, resonator methods are the most precision and most sensitive. Unfortunately, some of their modifications are quietly difficult to implement at studying the electrical properties of water in the EHF range of the electromagnetic spectrum. In particular, in the mm-waveband, cavity resonators are so reduced in volume that even a drop of water leads to almost complete absorption of the energy of the basic operating mode. Quasi-optical resonators with metal mirrors (like Fabry-Perrot one) are able to determine the permittivity of thin film samples with low dielectric losses [8, 11].

All noted methods are based on solving the inverse task of electrodynamics and allow determining the dielectric constants of substances that are contained in integral parts of measuring systems. Unfortunately, they do not make possible determining the chemical composition of water and aqueous solutions, identifying components of organic and inorganic origin. However they are able to quickly assess their impurity based on the complex values of permittivity. This is relevant both for solving scientific tasks related to the propagation of electromagnetic waves in the aquatic environment, and for applied applications. In the latter case, this is the protection of water resources and control of their purity. Such a problem is one of the most important between global ones.

In this regard, it seems promising to use dielectric resonators (DRs) excited on azimuthal modes of high orders, such as, whispering gallery modes (WGMs) [4, 5, 13–19]. Among the modifications of measuring cells based on DRs, using a radially layered disk resonator [12, 13–18] or a disk resonator with a capillary filled with water and located in the whispering gallery field caustic [20] is promising. The last version of the measuring cell for determining the electrical properties of water is very attractive due to the small amount of the studied liquid. Its modification based on using a dielectric half-cylinder located on a flat conducting mirror [19, 21] makes it possible to expand the possibilities for improving the accuracy and resolution of determining the permittivity of the studied aqueous solutions. However, this task requires a detailed study of the location of the capillary and clarifying the features of the operating WGMs excitation in the used DR. The presented studies are aimed at solving these problems and developing a method for determining the permittivity of a liquid filling a capillary.

# 2. Semi-Cylindrical Dielectric Resonator with Capillary Inhomogeneity (Theory of Eigen Axial-Homogeneous Modes)

### *2.1. The Resonant Structure*

Let us consider a semi-cylindrical DR with a cylindrical-shaped capillary inhomogeneity (Figure 1). The resonator with the flat side of the half-cylinder is located on a perfectly conducting plane. The sides of the half-cylinder and the inhomogeneity are parallel, and their longitudinal dimensions $L$ are the same. The distance between longitudinal axis $Z$ of the resonator and $Z'$ of the cylindrical inhomogeneity is equal $\rho_c$. Radiuses of the DR's cylindrical surface and the inhomogeneity can be represent as $\rho_0$ and $\rho_{cl}$, respectively. The distance between the side surface of the half-cylinder and the center of the inhomogeneity exceeds the radius of the latter one, i.e. $\rho_0 - \rho_c > \rho_{cl}$. A diameter 2 $\rho_{cl}$ of the capillary inhomogeneity does not exceed a half-wavelength of the eigenmode of the semi-cylinder. This allows placing the inhomogeneity to the region of one field variation. An azimuthal angle between the axis $X$ and the direction to the inhomogeneity center will be denoted as $\phi_c$. Semi-cylinder is made from isotropic material with complex permittivity $\varepsilon_1 = \varepsilon_1' + i\text{tg}\delta_1$ and permeability $\mu_1$. The walls of capillary inhomogeneity have the same permittivity and permeability. The considered resonator is placed inside the medium with permittivity $\varepsilon_2 = \varepsilon_2' + i\text{tg}\delta_2$ and permeability $\mu_2$. Note, that the coordinate systems (figure 1) — with origin on the longitudinal axis of the cylinder (semi-cylinder in the considered task) ( $\rho$ , $\phi$, $z$ ) and with origin on the longitudinal axis of the capillary inhomogeneity ( $\rho'$, $\phi'$, $z'$ ) are related by following equations [18, 19]:

$$\rho'^2 + \rho_c\rho'\cos(\phi_c - \phi') = \rho^2 - \rho_c\rho\cos(\phi_c - \phi),$$
$$\rho'\sin(\phi_c - \phi') = \rho\sin(\phi_c - \phi).$$

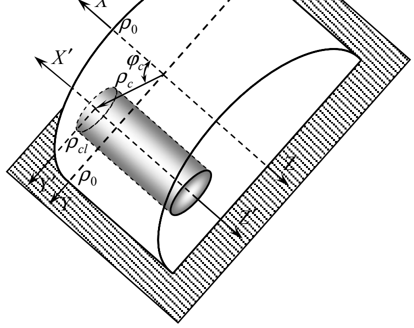


***Figure 1.*** *Semi-cylindrical DR with the capillary inhomogeneity.*

### *2.2. Dispersion Equation of the Resonator with Axial-Uniform Modes*

The field components of eigen electromagnetic modes of the composed elements of the resonator (semi-cylinder and capillary inhomogeneity) are determined by solutions of the Maxwell equations. Satisfying the boundary conditions, related with Maxwell equations, exactly, on the boundaries of the composed elements of the resonator, flat conducting surface and surrounding space leads to the quadratic system of homogeneous linearly independent equations. The condition for the existence of its nontrivial solutions leads to the dispersion equation of the resonator. In the case of axial-uniform TM$_{ns0}$ modes (where $n$ and $s$ — an azimuthal and radial modal indexes, respectively, and an axial (longitudinal) index $\delta$ = 0) a dispersion equation has the following form [18, 19].

$$\sqrt{\frac{\varepsilon_1}{\mu_1}}\frac{J_n'(q_1\rho_0)-W_{nm}}{J_n(q_1\rho_0)-Q_{nm}}=\sqrt{\frac{\varepsilon_2}{\mu_2}}\frac{H_n^{(1)\prime}(q_2\rho_0)}{H_n^{(1)}(q_2\rho_0)}, \quad (1)$$

where $q_\nu^2=\varepsilon_\nu\mu_\nu k_p^2$ ; $k_p=\omega_p/c$ , $\omega_p=\omega_p'+i\omega_p''$ — complex frequency of the $p\equiv ns\delta$ -order mode, $c$ — light velocity, $\nu$ — medium indicator (for semi-cylinder $\nu$ = 1 and for surrounding space $\nu$ = 2), $J_n(u)$ and $H_n^{(1)}(u)$ — cylindrical Bessel and Hankel functions of the first kind and $n$ - order, respectively. The prime at cylindrical functions means their differentiation with respect to the argument. The functions of the influence of a capillary inhomogeneity on the spectral and energy characteristics of a semi-cylindrical DR are the following:

$$W_{nm}=\frac{2i}{\pi}\sum_{m=-\infty}^{+\infty}\frac{\Delta_D(\omega_p)}{\Delta_m(\omega_p)}\int_0^{\pi}\left[\frac{im}{q_1}\frac{\rho_c\sin(\phi_c-\phi)}{\rho_0'^2}J_m(q_1\rho_0')+\right.$$
$$\left.+\frac{\rho_0-\rho_c\cos(\phi_c-\phi)}{\rho_0'}J_m'(q_1\rho_0')\right]\sin(n\phi)\exp(im\phi_0')d\phi,$$

$$Q_{nm}=\frac{2i}{\pi}\sum_{m=-\infty}^{+\infty}\frac{\Delta_D(\omega_p)}{\Delta_m(\omega_p)}\int_0^{\pi}J_m(q_1\rho_0')\sin(n\phi)\exp(im\phi_0')d\phi,$$

where

$$\rho_0'^2=\rho_c^2+\rho_0^2-2\rho_c\rho_0\cos(\phi_c-\phi),$$

$$\phi_0'=\mathrm{arctg}\frac{\rho_0\sin\phi-\rho_c\sin\phi_c}{\rho_0\cos\phi-\rho_c\cos\phi_c},$$

$$\Delta_D(\omega_p)=\frac{1}{J_m(q_1\rho_{cl})}\left[G_{nm}-F_{nm}\sqrt{\frac{\varepsilon_{1'}}{\mu_{1'}}}\frac{J_m'(q_{1'}\rho_{cl})}{J_m(q_{1'}\rho_{cl})}\right],$$

$$\Delta_m(\omega_p)=\sqrt{\frac{\varepsilon_{1'}}{\mu_{1'}}}\frac{J_m'(q_{1'}\rho_{cl})}{J_m(q_{1'}\rho_{cl})}-\sqrt{\frac{\varepsilon_1}{\mu_1}}\frac{J_m'(q_1\rho_{cl})}{J_m(q_1\rho_{cl})},$$

$$G_{nm}=\frac{i}{2\pi\mu_1}\times$$
$$\times\int_0^{2\pi}\frac{1}{k_p\rho_{cl}'}[J_n'(q_1\rho_{cl}')q_1\left(\rho_{cl}+\rho_c\cos(\phi_c-\phi')\right)\sin(n\phi_{cl}')-$$
$$-\frac{n\rho_c}{\rho_{cl}'}J_n(q_1\rho_{cl}')\sin(\phi_c-\phi')\cos(n\phi_{cl}')]\exp(-im\phi')d\phi',$$

$$F_{nm}=\frac{i}{2\pi}\int_0^{2\pi}J_n(q_1\rho_{cl}')\sin(n\phi_{cl}')\exp(-im\phi')d\phi',$$

in which

$$\rho_{cl}'^2=\rho_c^2+\rho_{cl}^2+2\rho_c\rho_{cl}\cos(\phi_c-\phi'),$$

$$\phi_{cl}'=\mathrm{arctg}\frac{\rho_{cl}\sin\phi'+\rho_c\sin\phi_c}{\rho_{cl}\cos\phi'+\rho_c\cos\phi_c}.$$

The sum over the index $m$ shows that the field inside the inhomogeneity is a superposition of its eigen axial-uniform TM$_{ms'0}$-modes, excited by the TM$_{ns0}$-mode of the semi-cylindrical DR. Exactly, field of TM$_{ns0}$-mode of the dielectric semi-cylinder is mostly perturbed by the eigenmodes with azimuthal indexes $m << n$ of the capillary inhomogeneity. This is determined by the small circumference in cross-section of the capillary inhomogeneity, since $m$ is equal to the number of wavelengths along the capillary circle.

### *2.3. Energy Absorption*

Function $\Delta_m(\omega_p)$ , included in the expressions for $Q_{nm}$ and $W_{nm}$ , looks like the equation $\Delta_m(\omega_{p'})=0$ . Its solutions determine the complex frequencies $\omega_{p'}$ of eigen $p'\equiv ms'0$ -order modes of the capillary inhomogeneity. At certain parameters of the resonant structure with operating TM$_{ns0}$-mode function $\Delta_m(\omega_p')\to\Delta_m(\omega_{p'}')$ . At approaching the frequency $\omega_p'$ of TM$_{ns0}$-mode to the frequency $\omega_{p'}'$, the resonant excitation of eigen axial-uniform TM$_{ms'0}$-mode of capillary inhomogeneity is carried out. This leads to the resonant absorption of the TM$_{ns0}$-mode energy. As a result, the inhomogeneity shifts the eigen frequency of the semi-cylindrical DR and leads to additional energy loss of TM$_{ns0}$-mode.

### *2.4. Solution of the Dispersion Equation*

A semi-cylindrical Teflon ($\varepsilon_1'$ = 2.07, tg$\delta_1$ = 1.7×10$^{-4}$ and $\mu_1$ = 1) resonator with operating TM$_{36\ 1\ 0}$-mode has been studied numerically. Resonator was placed inside free space ($\varepsilon_2$ = 1 и $\mu_2$ = 1). Radius $\rho_0$ of the resonator and radius $\rho_{cl}$ of the capillary inhomogeneity is equal 39 mm and 0.55 mm, respectively. A center of the inhomogeneity was placed in the point with coordinates $\rho_c$ = 37 mm and $\phi_c$ = 32°30'. Such locating the capillary corresponds to the E-field antinode of the $TM_{36\,1\,0}$-mode of the considered resonator. In this case, the eigen frequency of a homogeneous semi-cylindrical Teflon resonator (without inhomogeneity, when $\rho_{cl}$ = 0) with the same operating mode $\omega_p'/2\pi$ = 34.75 GHz. The capillary could be filled by air or water. Eigen frequency of the resonator, according to equation (1) solutions, are equal 34.767 GHz at air-filled capillary ($\varepsilon_{1'}$ = 1 and $\mu_{1'}$ = 1) and 34.842 GHz at water-filled capillary ($\varepsilon_{1'}$ = 17.92+1.59$i$ and $\mu_{1'}$ = 1). Any from these inhomogeneities leads to increasing the eigen frequency of non-uniform resonator compared to the frequency of uniform DR (without capillary). Shifting the frequency is determined by the excitation $TM_{0\,1\,0}$-mode in the capillary. Taking into account the term with index $m$ = 0 in the functions $W_{nm}$ and $Q_{nm}$ indicates that. Taking into account the terms with higher-order indexes $|m| \geq 3$ leads to a refinement of the eigen frequency of the non-uniform resonator in units of kHz.

## 3. Method for Determining the Permittivity of a Liquid

Equation (1) makes possible to determine the permittivity of the substance used like cylindrical inhomogeneity under accounting experimentally measured resonant frequency $\omega_p'/2\pi$ and unloaded Q-factor $Q$ of the resonator. In this case, both the electrical parameters ($\varepsilon_1$ и $\mu_1$) of the semi-cylinder and surrounding space ($\varepsilon_2$ и $\mu_2$), geometric dimensions of the resonant structures ($\rho_0$, $\rho_{cl}$, $\rho_c$ и $\phi_c$) should be known.

For this it is necessary to provide:

1) excitation of operating $TM_{ns0}$-mode in the resonator,
2) definition of the modal indexes $n$ and $s$,
3) measurement the resonant frequency $\omega_p'/2\pi$,
4) measurement of the unloaded Q-factor $Q$ of the resonator taking into account the dielectric and radiative losses,
5) determination of the imagine party of resonant frequency $\omega_p'' = \omega_p'/2Q$,
6) determination of the eigen complex frequency $\omega_p = \omega_p' - i\omega_p''$ of the resonator.

Permittivity of a substance (liquid) $\varepsilon_{1'}$ is determined by solution of the dispersion equation (1) of the resonator with identified operating mode at known (or supposed) value of permeability $\mu_{1'}$. Note, that the root of the transcendental equation (1) should be corresponded to the radial index $s$ of the operating $TM_{ns0}$-mode.

In real conditions an axial index $\delta$, indicating the fraction of field variations along the longitudinal dimension of the resonator, is different from zero [22]. In this regard, hybrid $HE_{ns\delta}$- and $EH_{ns\delta}$-modes are excited in the resonator. Therefore, the longitudinal size of the resonator should be chosen in such manner that the distribution of the E-field intensity along an axial axis $Z$ of the resonator is the most uniform $\delta \to 0$. In this case, the distribution of resonant fields of $HE_{ns\delta}$-modes in real conditions (at experiment) and $TM_{ns0}$-modes in ideal conditions (at theory) are almost the same. Covering the side bases of a dielectric semi-disk by ideally conducting planes always allows exciting $TM_{ns0}$-modes in it due to fulfillment of the boundary conditions on these planes [18, 19].

## 4. Experimental Setup and Measurement Instruments

A semi-cylindrical Teflon resonator (Figure 2) with radius of $\rho_0$ = 39 mm and longitudinal size of 7.2 mm was used for measuring the electric properties of water. Semi-cylinder was placed on the flat brass mirror. The capillary with thin Teflon walls had an inner diameter of 2 $\rho_{cl}$ = 1 mm. It was alternately filled with the following water samples: distilled water; sea water of the Black Sea; drinking water, mineralized by salt of potassium, sodium, calcium and magnesium. A capillary hole inside the resonator was placed at a distance of 3 mm from curved surface of the semi-cylinder and oriented parallel to its generator. The angle between the center of the capillary and the brass mirror was 57°30'. In this case, the capillary is located in the region of a "strong" resonant field with a maximum of E-field intensity of the operating WGM with an azimuthal index $n$ = 36 and radial index $s$ = 1.

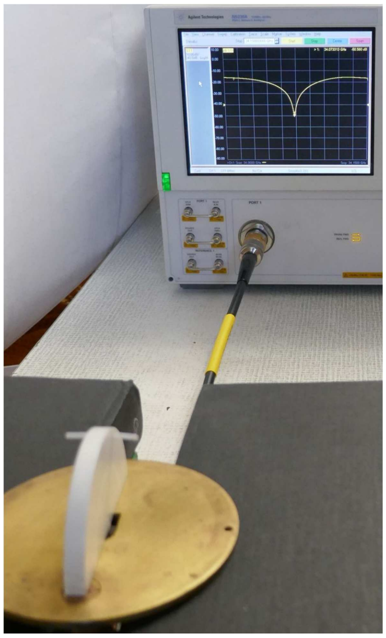

***Figure 2.*** *The experimental measuring setup.*

The excitation of the operating WGM in the semi cylindrical DR was achieved by the rectangular coupling slot 7.2x0.1 $mm^2$ in the. Using a transformer connection, it was connected in a consistent manner with a rectangular metal waveguide with a cross section of 7.2x3.4 $mm^2$. The center of the coupling slot was located on the surface of a flat mirror at a point with coordinates ($\rho_s$, $\phi_s = \pi$, $z_s = L/2$). The experimental setup had an opportunity to change the arrangement of the coupling slot center (coordinate $\rho_s$) by moving the semi-cylinder on the flat mirror.

Choosing the orientation of the coupling slot with respect to the normal to the cylindrical surface of the DR (parallel or perpendicular) makes it possible to select the excited eigenmodes of the resonator ($HE_{ns\delta}$ or $EH_{ns\delta}$, respectively). The results of the measurements, represented below, are mainly obtained at excitation of $EH_{ns\delta}$ WGMs of the semi-cylindrical DR.

All measurements were carried out in the frequency range of 31–37.5 GHz by Pna-L Network Analyzer N5230a 10 MHz÷40 GHz Agilent Technologies. To determine the distinguishing features of the studied water samples, the resonant frequencies and Q-factors of the DR were experimentally measured. In addition, the operating WGM was identified according to the measured distribution of the intensity of E-field longitudinal component in the plane of the semi-cylinder cross section). As noted above, these parameters are informative for determining the complex permittivity of a liquid. For comparison, the spectral and energy characteristics of a uniform resonator (without a capillary) and a DR with air-filled capillary were measured. In addition, the amplitude distributions of the longitudinal components intensity of the electrical ($E_z$) and magnet ($H_z$) fields of resonant WGMs in the в плоскости его поперечного сечения. This made it possible to identify both the type of modes (by comparing the maximum values of $|E_z|$ and $|H_z|$) and their azimuthal $n$ and radial $s$ modal indexes [18]. The type of modes was also identified by the displacement of the resonant frequencies of the DR at using the method of passive probe [22]. Determination of Q-factor was carried out by resonant method according to the width of the resonant response at the level of –3 dB [22]. The error of its determination at the experiment did not exceed 0.1%. At the same time, the value of the measured Q-factor of the DR is an analogue of its unloaded Q-factor. However, at excitation of the WGMs in the DR by a local source, its Q-factor depends on the choice of the location of the source $\rho_s$ in relation to the region of "strong" or "weak" field. Briefly, this can be explained as follows. A local source, such as, a coupling slot in the mirror is an inhomogeneity in relation to the resonator field. It causes additional energy loss - diffractive loss at the boundaries of the slot. Their value is largely determined by the field intensity in the location of the coupling slot. It is obvious that such losses will reach maximum values when the coupling slot is located in the region of maximum resonant field intensity. Such an arrangement of the coupling slot provides the maximum efficiency of excitation of the operating WGM in the resonator. The presented resonant system, on the one hand, is quite complex, since it includes two sources of perturbation of the fields of resonant oscillations (a coupling slot and a capillary with liquid). But, on the other hand, it is attractive in that it is possible to control the degree of perturbation of the fields by changing the position of the coupling slot. This makes it possible to control and change the degree of field perturbation by the studied water samples. In addition, as shown in [22], shifting the coupling slot along the radial coordinate leads to a significant changing the distribution of the fields of excited WGMs and the appearance of new effects.

The research of the field distributions of the resonant WGMs in the DR was carried out by the method of a passive probe. The probe dimensions are much smaller than the operating wavelength in the resonator [23]. To study the distribution of fields, a passive probe made of an absorbing material was used. It moved smoothly in the coordinate plane (X, Y) parallel to the flat side surface of the semi-cylindrical DR at the minimum possible distance from it (less than 1mm). Additionally, the radial distributions of fields on the flat surface of the semi-cylinder were measured using a passive slot 7.2x0.1 mm$^2$ in a movable mirror. This made it possible to move it along the radial coordinate at $\phi = 0$ or $\phi = \pi$ and $z = L/2$ (i.e. along the Y axis).

# 5. Experimental Results and Discussion

## *5.1. Perturbation of the Resonant Frequencies Grid*

At excitation of operating WGMs in uniform semi-cylindrical DR by a coupling slit in a mirror oriented for selection $EH_{n1\delta}$-mode (at index values $0 < \delta < 3$), its spectrum contains a sequence of resonant responses with a period of about 0.9 GHz. The modes corresponding to neighboring responses differ in the value of the azimuth index $n$ by one. Behavior of the frequency period between adjacent resonant responses in the spectrum (grid) of resonant frequencies of the DR $\delta f = (\omega'_{n1\delta} - \omega'_{(n-1)1\delta})/2\pi$ depending on intermodal azimuthal indexes ($n$–1, $n$) as a histogram is shown in Figure 3 by red columns. Monotonic decrease in intermodal frequency period $\delta f$ at increasing azimuthal indexes indicates a shift of the resonant fields deep into the dielectric and decreasing the radiative loss, as a consequence.

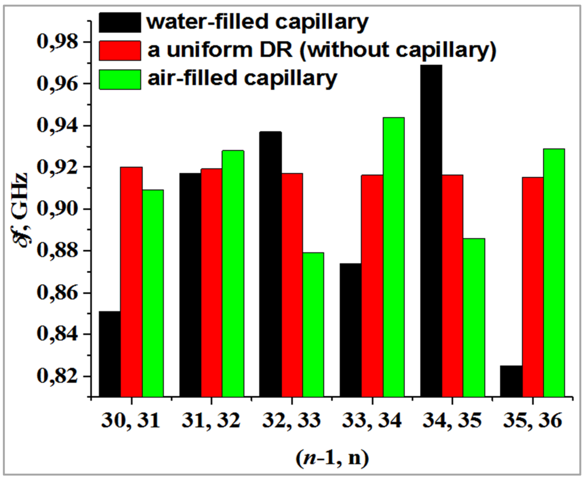


***Figure 3.*** *Changing the intermodal frequency period in the resonant frequency grid of the DR.*

In the DR with air-filled capillary, the dependence $\delta f = \psi(n-1, n)$ becomes oscillatory (green columns 2 in Figure 3). An intermodal frequency period $\delta f$ can either exceed the $\delta f$ value for a uniform DR in certain sections of the graph, or be less than it. It has been found, that the smallest values of $\delta f$ correspond to the position of the capillary in the regions of the "weak" field, and the largest ones, near the maximum field intensity.

In the DR with distilled water-filled capillary, there is also an oscillating dependence $\delta f = \psi(n-1,n)$ (black columns in Figure 3). However, the maxima and minima of this dependence change places with respect to the air filling of the capillary. In this case, the intensity distribution of the resonant field is changed significantly along the azimuthal coordinate. For example, at excitation of the $EH_{36\,1\delta}$-mode in the DR, моды ДР в области воздушного капилляра a high-intensity field is observed, and in the region of the water capillary a "weak" field is observed (the intensity is close to zero). Measurement of the E-field intensity distribution of this mode showed, that at filling capillary with water, the maximum of E- field intensity is shifted along an arc parallel to the cylindrical surface of the DR by 1.6 mm, which corresponds to more than a quarter of the wavelength.

A histogram in Figure 4 shows changing the resonant frequencies $\Delta f = \psi(n)$ (tallow columns) and Q-factors $\Delta Q = \psi(n)$ (blue columns) of DR with distilled water-filled capillary relative to a DR with an air-filled capillary at excitation of $EH_{n1\delta}$-modes with azimuthal indexes $30 \le n \le 36$. The dependences $\Delta f = \psi(n)$ and $\Delta Q = \psi(n)$ are the oscillatory.

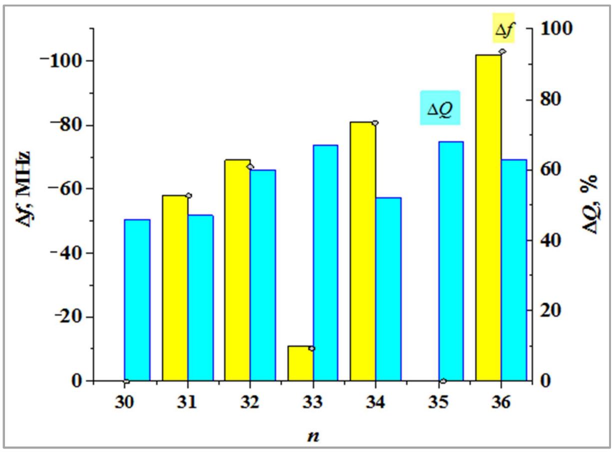


*Figure 4. Differences of resonant frequencies and Q-factors of DRs with an air- and water-filled capillaries at excitation of $EH_{n1\delta}$ modes.*

Note that filling the capillary with water leads to decreasing the resonant frequencies of the resonator. The largest shift of the DR resonant frequency, equal to 102 MHz, corresponds to the excitation of $EH_{36\,1\delta}$-mode. Its field least penetrates into the capillary with water filling. This corresponds to the location of the capillary in the range of a "strong" field with the highest E-field intensity.

Q-factor of DR with water-filled capillary is significantly (more than 60%) differed from Q-factor of DR with air-filled capillary at excitation of $EH_{36\,1\delta}$-mode. It is need to note, that Отметим, что the maximum differences in the Q-factors of the resonators with air- and water-filled capillaries correspond to the minimum differences in their resonant frequencies at excitation of modes with the same field intensity distributions in them. This is explained by the smallest differences in the Q-factors of the DRs are observed at location of the capillary in the region of a "weak" field, when the loss of the resonant field in water is insignificant.

The dependences, represented in Figures 3 and 4 is obtained at an arrangement of the coupling slot near the border of the resonator (when $\rho_s = \rho_0$) — in the region of decreasing E-field intensity of WGMs. In this case, the perturbation of the resonant fields by the coupling slot is weak. Thus, the presented results suggest the application of the DR with capillary inhomogeneity as a measuring cell of a dielectrometer (a device for measuring dielectric characteristics) at excitation $EH_{36\,1\delta}$-mode.

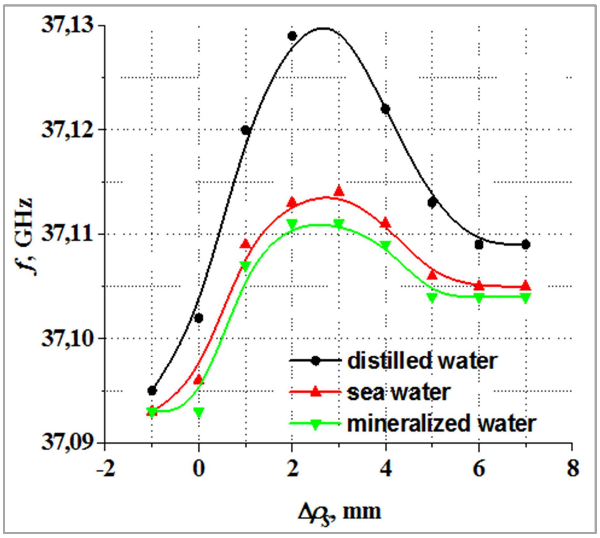


*Figure 5. DR resonant frequencies at filling the capillary with air (1) or distilled (2), sea (3), or mineralized (4) water.*

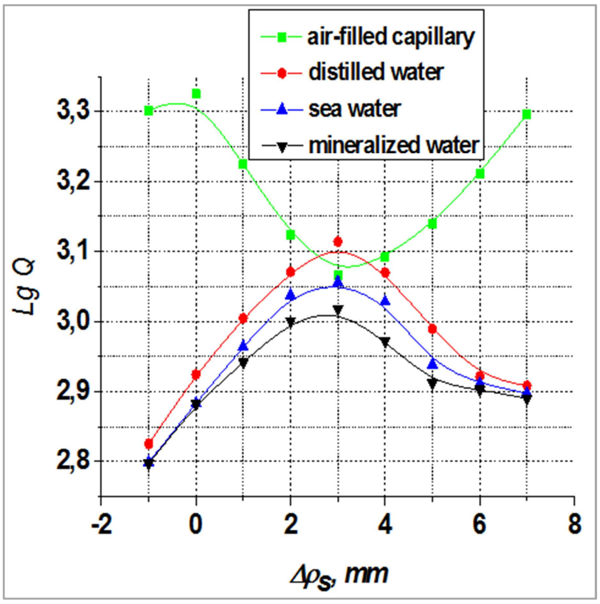


*Figure 6. Q-factor of DR at filling the capillary with air (1) or distilled (2), sea (3), or mineralized (4) water.*

### *5.2. Influence of the Location of the Excitation Source on the Spectral and Energy Characteristics of the Non-Uniform DR*

Filling the capillary by different studied water samples shown, that the difference in spectral and energy characteristics of DR with operating $EH_{36\,1\delta}$-mode at placing the coupling slot at border of the resonator ( $\rho_s = \rho_0$ ) is insignificant. They do not exceed the measurement errors. At shifting the coupling slot into DR it is possible to detect the distinctive features of the studied water samples by changes in the resonant frequency $f = \omega'_{361\delta} / 2\pi$ and Q-factor $Q$ of the resonator. The dependences of $f$ and $Q$ (on a logarithmic scale) on the distance between the flat side surface of the DR and the center of the coupling slot $\Delta\rho_s = \rho_0 - \rho_s$ , represented by Figure 5 and Figure 6, respectively, clearly demonstrate this. The positive values of $\Delta\rho_s$ correspond to the coupling slot location inside the DR, and its negative values – to outside the resonator. The greatest difference in the characteristics of the non-uniform DR is observed when the source of oscillation excitation is located at a distance $\Delta\rho_s$ = 3 mm — at the point corresponding to the radial coordinate of the maximum of E-field intensity in the resonator. In this case, the displacement of the maximum of E-field intensity along the azimuthal coordinate does not exceed 0.5 mm (this corresponds to the angular measure $44'4''$ ) regarding its position at $\Delta\rho_s$ = 0. At filling the capillary with distilled or sea water, the difference between the resonant frequencies of non-uniform DR is 17 MHz (Figure 5), and Q-factors — 12% at normalizing to the average value $\langle Q \rangle$ = 1200 (Figure 6).

Additionally, studies of electrical properties of water at using the distributed weak electromagnetic coupling with dielectric waveguide for excitation of operating WGMs in the non-uniform DR were carried out. Dielectric waveguide was made from Teflon and has a cross-section 7.2x3.4 $mm^2$. There was an inhomogeneity, such as, a passive slot in the metal mirror. Its dimensions and dimensions of the coupling slot are the same. The results of the studies showed, that at locating the passive slot outside the DR with the capillary, filled with various water samples, the resonant frequencies $f$ and Q-factors $Q$ are close and the differences between them do not exceed the measurement errors. Similar results were obtained at placing the dielectric semi-cylinder on a uniform mirror. At shifting the passive slot inside the resonator, the differences in resonant frequencies $f$ and Q-factors $Q$ of the DR with different water filling the capillary (distilled or sea water) become more significant. At the same time, they are close to the differences that were obtained at using the coupling slot (inside the resonator) for the excitation of operating WGMs in the semi-cylindrical DR. The greatest differences in frequency (15 MHz) and Q-factor (10.6%) are achieved at placing the passive slot inside the resonator, at a distance $\Delta\rho_s$ = 3 mm from its edge - at the point of maximum E-field intensity.

Thus, the location of the slot (passive or coupling) at the point of maximum strength of the eigenmode field (in the considered case $EH_{36\,1\delta}$) of the DR increases the resolution of measuring cell used for determination of the permittivity of studied liquid. The slot in the DR mirror can be considered as a regulator of the perturbation of the resonant field It makes it possible to detect distinctive features of liquids with similar dielectric properties. Thus, the coupling slot in the same time is a source of excitation of operating WGMs in the DR and an inhomogeneity, making it possible to improve the resolution of the measuring cell.

It is necessary to note a feature of the Q-factor behavior of the resonator with the capillary, filled by different samples of water, at changing the location of the source of the operating WGMs excitation in the DR. As can be seen from figure 6 in a DR with air-filled capillary, the displacement of the coupling slot inside the resonator is accompanied first by decreasing the Q-factor, and then by its increasing. Obviously, such a behavior of the energy characteristic of the DR is related with the changing the value of a diffraction loss at the coupling slot [22]. The resonator has a minimum Q-factor at locating the coupling slot at a distance $\Delta\rho_s$ = 3 mm from the flay side base of the semi-disk. At water filling the capillary, the opposite effect is observed. The displacement of the coupling slot inside the DR is accompanied first by increasing and then by decreasing the Q-factor of the resonator. It is obvious that such a behavior of the Q-factor is related with the redistribution of the energy of the resonant field because of influence of diffraction loss at the coupling slot and dielectric loss in the water filling of the capillary. At locating the source of excitation of oscillations at the point corresponding to the displacement $\Delta\rho_s$ = 3 мм, the resonator has a maximum Q-factor. It is revealing that at such a position of the slot, the Q-factor of the DR with distilled water filling of the capillary is higher than Q-factor of the DR with air-filled capillary.

Firstly, as shown by the results of studies of the distribution of the E-field intensity along the azimuthal coordinate in the plane $z = 0$ or $z = L$ , filling the capillary with water leads first to shifting the maximum of E-field intensity from the capillary region. This is determined by the significant dielectric loss in water [14, 15, 18]. Secondly, the E-field distribution along the azimuthal coordinate is significantly changed. Its intensity in the region of the coupling slot is reduced. In the considered task, the E-field intensity at air filling the capillary decreased by more than 2 times compared with water filling the capillary. As a result, the diffraction loss at the coupling slot at air filling the capillary is higher in comparison with its water filling.

It is necessary to pay attention to the radial distributions of the E-field intensity of $EH_{36\,1\delta}$-mode in the DR at air or distilled water filling the capillary. Figure 7 shows the dependence of the relative amplitude $A/A_{max}$ (A is a current value of an amplitude, $A_{max}$ is its maximum value) of the resonant response on the radial coordinate $\rho_t$ of a passive probe – passive slot in the mirror moveable in the plane $z = z_s = L/2$ .

The coupling slot was located in the region of the maximum E-field intensity ($\Delta\rho_s$ = 3 mm). It can be seen that air filling the capillary leads to decreasing the E-field intensity outside the DR and on its cylindrical surface $\rho = \rho_0$ . Therefore, the radiative loss of resonant field energy in the DR with an air-filled capillary is greater than at water-filled capillary. In addition, at filling the capillary with water, the maximum E-field intensity shifts deep into the resonator.

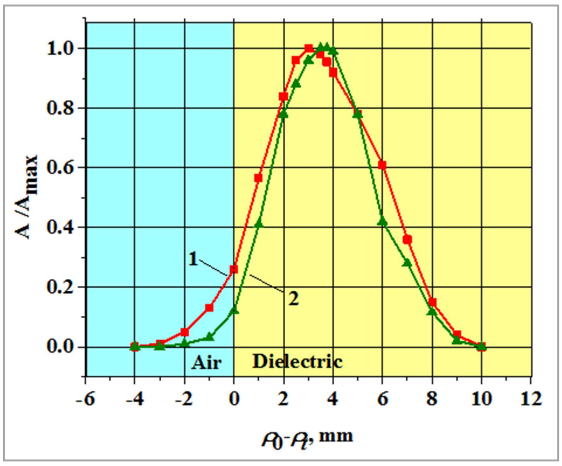


***Figure 7.*** *Radial distribution of E-field intensity of* $EH_{36\,1\delta}$*-mode in the DR with air filling the capillary (red curve 1) or distilled water filling it (green curve 2).*

Thus, the distributions of resonant fields along the azimuthal and radial coordinates determine the higher Q-factor of the DR with the distilled water-filled capillary compared to the DR with air-filled capillary.

The studies of the radial distribution of the E-field intensity of $EH_{36\,1\delta}$-mode in the DR with water-filled capillary at different location of the coupling slot along the radial coordinate $\rho$ were carried out additionally. The minimum relative amplitude $A/A_{max}$ on the border of the resonator $\rho = \rho_0$ in the plane $z = z_s = L/2$, equal to 0.12 rel. units, is observed at $\Delta\rho_s$ = 3 мм, and its maximum value, equal to 0.3 rel. units, — at $\Delta\rho_s$ = 0 мм. At $\Delta\rho_s$ = 7 мм the value of a relative amplitude reaches $A/A_{max}$ = 0.16 rel. units on the border $\rho = \rho_0$ of the DR. From the standpoint of the fundamental electromagnetism of uniform DRs, this result is unexpected. As shown in [22], decreasing the radial coordinate $\rho_s$ of the coupling slot (increasing the $\Delta\rho_s$) leads to a monotonic decreasing the E-field intensity on the border $\rho = \rho_0$ of a uniform DR. It follows from above, that the most effective interaction of the resonant field of the operating $EH_{n1\delta}$-mode with the liquid contained to the DR capillary is reached at locating the source of mode excitation in the region of the maximum E-field intensity along the radial coordinate. In this case, the radiative loss are minimal and the energy of the electromagnetic field, stored in the resonator. is the largest. This provides the highest field energy density in the region of the capillary with liquid.

# 6. Conclusion

A broadband resonant method for determining or controlling the dielectric properties of liquids (including ones with high losses) has been developed. The small volume of the tested liquid is one of its important advantages. The method is based on using an inhomogeneity semi-cylindrical DR with capillary, located in the region of one field variation of operating whispering gallery modes. The capillary is intended for filling the studied or tested liquid. Numerically studied the semi-cylindrical Teflon DR with water- and air-filled capillary, excited on the operating $TM_{36\,1\,0}$-mode. A technique for determining the complex permittivity of liquid using the dispersion equation of a similar resonator is presented.

The results of experimental studies of the eigenmode parameters of the resonator with whispering gallery $EH_{n1\delta}$-modes with azimuthal indexes $30 \le n \le 36$ in the frequency range of 31–37.5 GHz. The capillary was located inside DR in the region of maximum intensity of $EH_{36\,1\delta}$-mode and filled by distilled water, sea water, mineralized water or air. The excitation of the operating modes was carried out by the coupling slot in the resonator mirror or dielectric waveguide by the weak distributed coupling. It is shown; the most effective interaction between the resonant field of $EH_{36\,1\delta}$-mode and the water sample is achieved at locating the coupling slot in the region of maximum E-field intensity (along the radial coordinate). This leads to improving the resolution of the measuring cell, based on the semi-cylindrical DR, and accuracy of measurements. It is found, Q-factor of DR with water-filled capillary is higher than at air filling the capillary. The represented effect is determined by redistributing the resonant field energy because of the diffraction loss at the coupling slot and dielectric loss in aqueous medium of the capillary.

# Conflicts of Interest

The authors declare no conflicts of interest.

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