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Research and Demonstration of the Refraction Problems

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Abstract: The introduction of function extremum promotes the development of calculus, which is the prerequisite and important condition for the development and development of many mathematical ideas. Except for extreme problems in social life or science and technology, the issue of cost in economic issues, the shortest distance in mathematics problem can be solved by the function extreme value thought. However, because the extreme value problem is not well described, learners cannot easily observe and learn. This paper takes the uniform speed and uniform acceleration and descent in the descent model as an example. In the uniform acceleration, the dichotomy is introduced to solve the function extremum problem. Through MATLAB simulation, the position of the steepest descent point is found intuitively and relevant conclusions are obtained. It is hoped that the research process of the problem can be helpful to the study of refractive problems, and it is hoped that the visually intuitive simulation results can enable learners to understand the descent model more objectively.

Keywords: Refractive Problems, Function Extremum, Dichotomy, MATLAB Simulation

1. Introduction

In mathematical analysis, the maximum and minimum values of a function within a given range are collectively referred to as extreme values [1]. Pierre de Fermat was one of the first mathematicians to find the function's maximum and minimum. The mathematical ideas and methods based on the one-element extremum problem are applied in many aspects and can be combined with optimization methods to solve optimization problems [2], solve precipitation problems [3] [4], rocket orbit problems [5], artificial intelligence problems [6], especially on the issue of the shortest time, this article can discuss the shortest time path problem of transportation [7] [8] [9], traffic problem [10], artificial intelligence problem [11], etc. Extreme values can be used in life, and all aspects of learning. Extremum problems are also more extensive solutions, the use of physical methods [12], mathematical methods [13] and so on. MATLAB is one of the key tools for the analysis of modern mathematics and physics problems. It has a very important supporting role in physics teaching [14] [15] [16] and mathematics teaching [17] [18].

This paper starts from the concept and definition of the one-variable function extremum, and discusses the shortest time based on uniform acceleration and descent. Gradually derive the calculation method of time and distance, at the same time, organically combine dichotomy ideas, and finally launch relevant conclusions. Finally, using MATLAB tools for simulation, the description of the problem and the final result are all more intuitive, which will greatly help to understand the refraction problem in the shortest time.

2. One-Way Function Extreme Value Theory

2.1. Definition

Let function $f(x)$ be defined in interval (a,b), $x_0 \in (a,b)$, if in x_0, a certain neighborhood of the mind has: $f(x) \le f(x_0)$ (or $f(x) \ge f(x_0)$), then $f(x_0)$ is a maximum (or a minimum) of function $f(x)$, and x_0 is a maximum (or minimum) point of $f(x)$. The maxima and minima are collectively referred to as the extremum of the function, and the maxima and minima are collectively referred to as extremum points of the function.

2.2. Steps

The steps for finding the extremum of a one-way function are:

(1) Determine the domain of the function;

(2) Let $f'(x)=0$, find all the stagnation points, examine the symbols around each stagnation point, and take the opposite extreme, otherwise do not take the extreme value;

(3) If $f'(x_0)=0$, $f''(x_0)\neq 0$. When $f''(x_0)<0$, $f(x)$ takes a maximum value in x_0, when $f''(x_0)>0$, $f(x)$ takes a minimum value in x_0;

(4) Find the function value at each extreme point to get the corresponding extreme value using the above steps; you can solve extreme problems in real life.

3. The Shortest Time Problem of Constant Speed Drop

3.1. Problem Description

The particle moves from point $B(0,b)$ to point $A(a,0)$, the speed of line BD is v_1, and the speed of line DA is v_2. Find the horizontal axis x of the point $D(x,y_0)$ on the horizontal line $y=y_0$, so that the time from B to D reaches A is the shortest, where the speeds v_1 and v_2 are known quantities, the specific problem is shown in the figure:

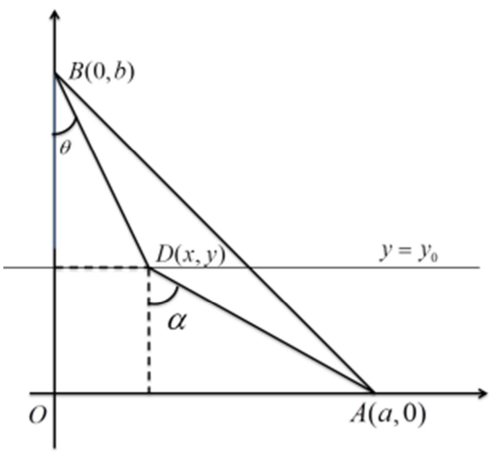


Figure 1. The description of shortest time problem of constant speed drop.

3.2. Basic Solution

3.2.1. Time

In order to obtain the coordinates of the most advantage D, the whole process is divided into two sections: $y_0\le y\le b$ and $0\le y<y_0$. From the physical principles and the Pythagorean theorem, this article can see that in paragraph BD, the movement time is:

$$t_{BD}=\frac{\sqrt{x^2+(b-y_0)^2}}{v_1} \tag{1}$$

In paragraph DA, the movement time is:

$$t_{DA}=\frac{\sqrt{(a-x)^2+y_0^2}}{v_2} \tag{2}$$

From (1) (2), the total exercise time is:

$$t(x)=\frac{\sqrt{x^2+(b-y_0)^2}}{v_1}+\frac{\sqrt{(a-x)^2+y_0^2}}{v_2} \tag{3}$$

To solve the problem of the minimum time required for a derivative function, and make it a value of 0, the conditions to obtain the shortest time. Which is:

$$t'=t'(x)=\frac{1}{v_1}\frac{x}{\sqrt{x^2+(b-y_0)^2}}-\frac{1}{v_2}\frac{(a-x)}{\sqrt{(a-x)^2+y_0^2}}=0 \tag{4}$$

The x_0 obtained at this time is the abscissa of the most advantage D.

Through the above-mentioned problem-solving method, it can be found that the procedure for obtaining the value simply by calculation is complicated, and if it encounters a special situation, it will not be solved. Therefore, it is feasible and effective to use MATLAB to find the coordinates of the best position.

3.2.2. Distance

In paragraph BD, the movement distance is:

$$s_1=s_1(x)=|BD|=\sqrt{x^2+(b-y_0)^2},(0\le x\le a) \tag{5}$$

$$s_2=s_2(x)=|DA|=\sqrt{(a-x)^2+y_0^2},(0\le x\le a) \tag{6}$$

$$s=s(x)=|BD|+|DA|=\sqrt{x^2+(b-y_0)^2}+\sqrt{(a-x)^2+y_0^2},(0\le x\le a) \tag{7}$$

$$s'=s'(x)=x\times\left(x^2+(b-y_0)^2\right)^{-\frac{1}{2}}-(a-x)\times\left((a-x)^2+y_0^2\right)^{-\frac{1}{2}}=0,(0\le x\le a) \tag{8}$$

When $s'(x)=0$, s has the shortest distance.

3.3. Simulation

3.3.1. Simulation Process

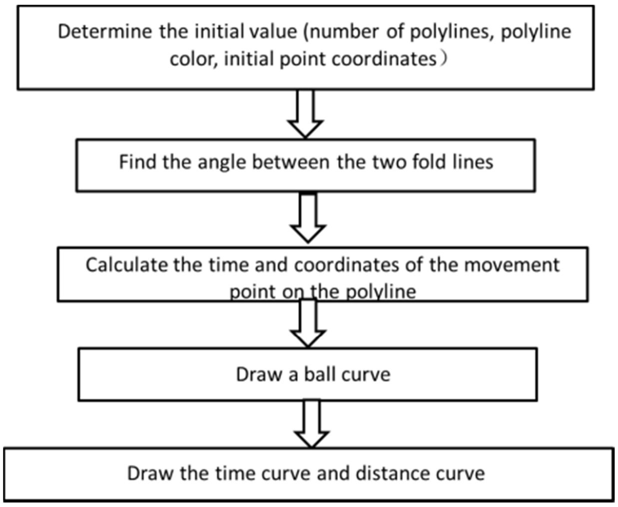


Figure 2. *Simulated flow chart for the shortest time problem of constant speed drop.*

3.3.2. Polyline

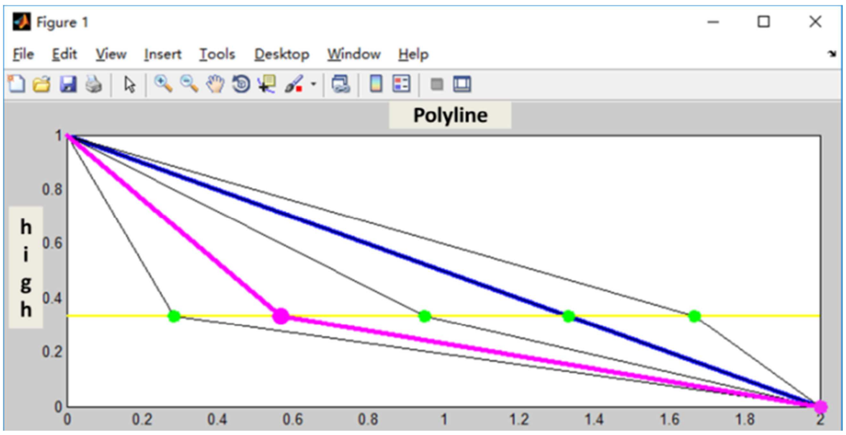
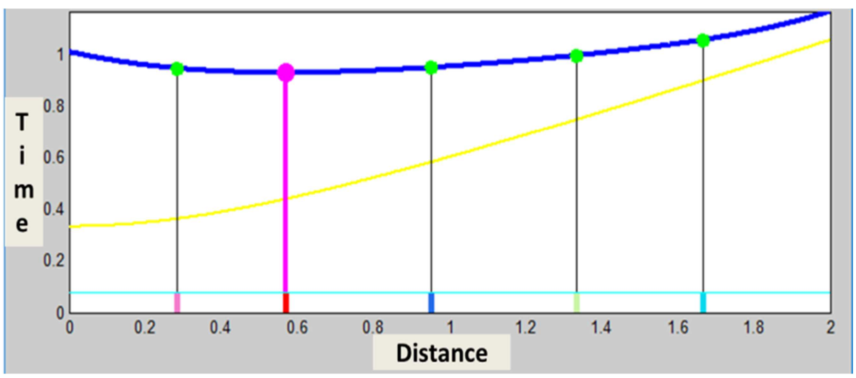


Figure 3. *Polyline.*

The five points in Figure 3 are taken as follows: sequentially named D_1, D_2, D_3, D_4, D_5, D_4 is not refracting. Follow a straight line from B to A, D_2 spends the shortest time, D_5 and D_2 are symmetrical about the line of motion, D_3 is the midpoint of the D_2 and D_4, D_1 and D_2, D_3, D_4 are equally spaced on the left side of.

3.3.3. Timing Schematic



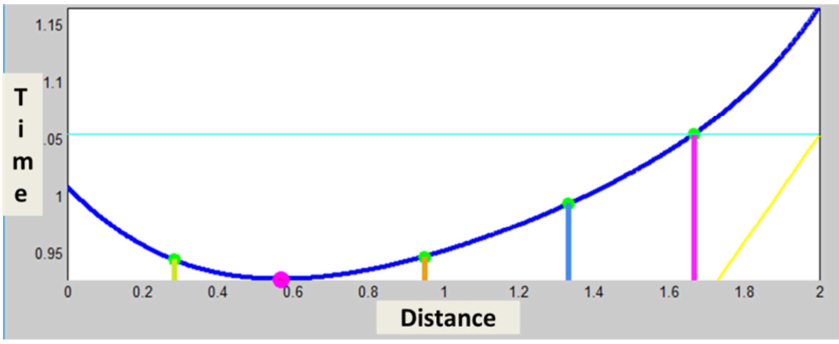


Figure 4. *Motion Time Curve.*

It can be seen from Figure 4 that the time-shifted ordinate corresponding to point D_2 is the smallest, indicating that the movement time of D_2 is the shortest. At the same time, based on the continuous change curve of the exercise time, the conclusion is that $t'(x)=0$ is the point of D_2.

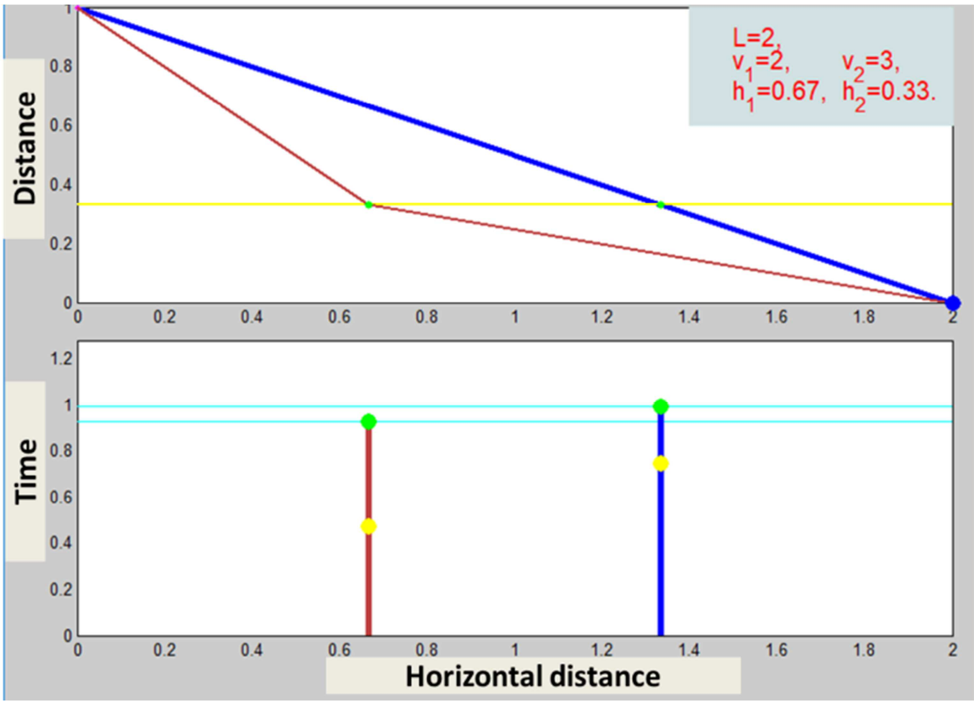


Figure 5. *Sport Time Comparison Curve.*

As can be seen from Figure 5, the arrival time of different points is different.

It can be seen from the results of Figure 3、Figure 4、and Figure 5 that due to the different angles between falling and refraction and the vertical direction, the total motion time of each scheme is different. Therefore, it can be concluded tentatively that the total movement time after B reaches A is related to two tilt angles θ_1 and θ_2.

3.3.4. Distance Schematic

It can be seen from Figure 6 that the shortest solution D_2 does not necessarily take the shortest distance, because in section DA, the movement time of D_2 is longer than that of the shortest solution. The principle can also be easily concluded.

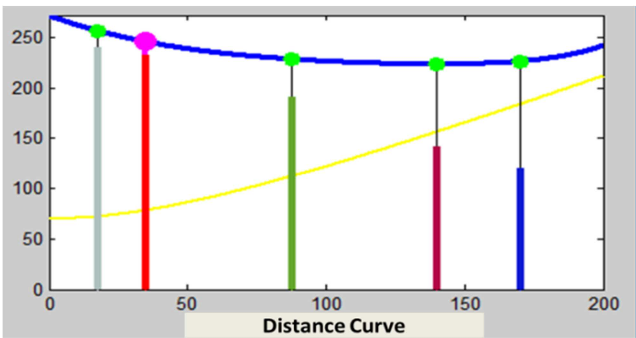


Figure 6. *Sport Distance Comparison Curve.*

3.3.5. Principle Expansion

From Figure 3、Figure 4, and Figure 5, it can be seen that the total time of movement through B to point A is related to two tilt angles θ_1 and θ_2. After dealing with equation (1) (2), this article get:

$$t' = t'(x) = \frac{1}{v_1}\frac{x}{\sqrt{x^2+(b-y_0)^2}} - \frac{1}{v_2}\frac{(a-x)}{\sqrt{(a-x)^2+y_0^2}} = 0 \quad (9)$$

$$\frac{\sin\theta}{v_1} = sv_1(x) = \frac{x}{v_1\sqrt{x^2+(b-y_0)^2}} \quad (10)$$

$$\frac{\sin\alpha}{v_2} = sv_2(x) = \frac{a-x}{v_2\sqrt{(a-x)^2+y_0^2}} \quad (11)$$

$$\therefore\ t' = t'(x) = \Leftrightarrow \frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0 \Leftrightarrow \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \quad (12)$$

This article can see that when $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$, this article can get the shortest time. The following is the simulation visualization result:

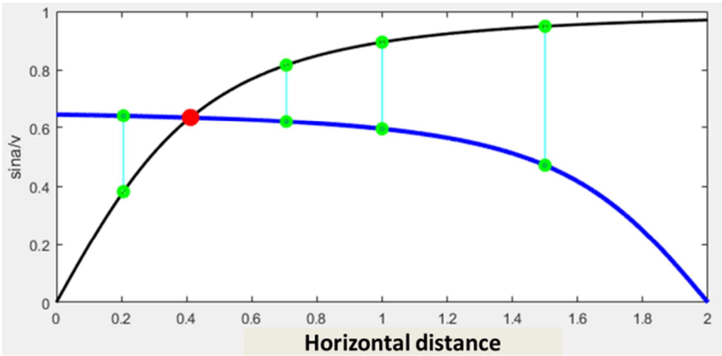


Figure 7. *Ratio Curve.*

Figure 7 shows that at $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$, the corresponding route has the shortest movement time. At the point on the left side of D_2, when it is at segment BD, the exercise time is shorter than D_2, and when it is at segment DA, the exercise time is longer than D_2; At the point on the right side of D_2, when it is at a section BD, the exercise time is shorter than that at case D_2, and when it is at a section DA, the exercise time is shorter than D_2.

In addition, find the second derivative of $t(x)$:

$$t''(x) = \frac{1}{v_1}\left[\frac{(b-y_0)^2}{\left(x^2+(b-y_0)^2\right)^{\frac{3}{2}}}\right] + \frac{1}{v_2}\left[\frac{y_0^{\,2}}{\left((a-x)^2+y_0^{\,2}\right)^{\frac{3}{2}}}\right] > 0 \quad (13)$$

It's proving that $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$ is the only minimum point.

3.3.6. Dynamic Presentation Process

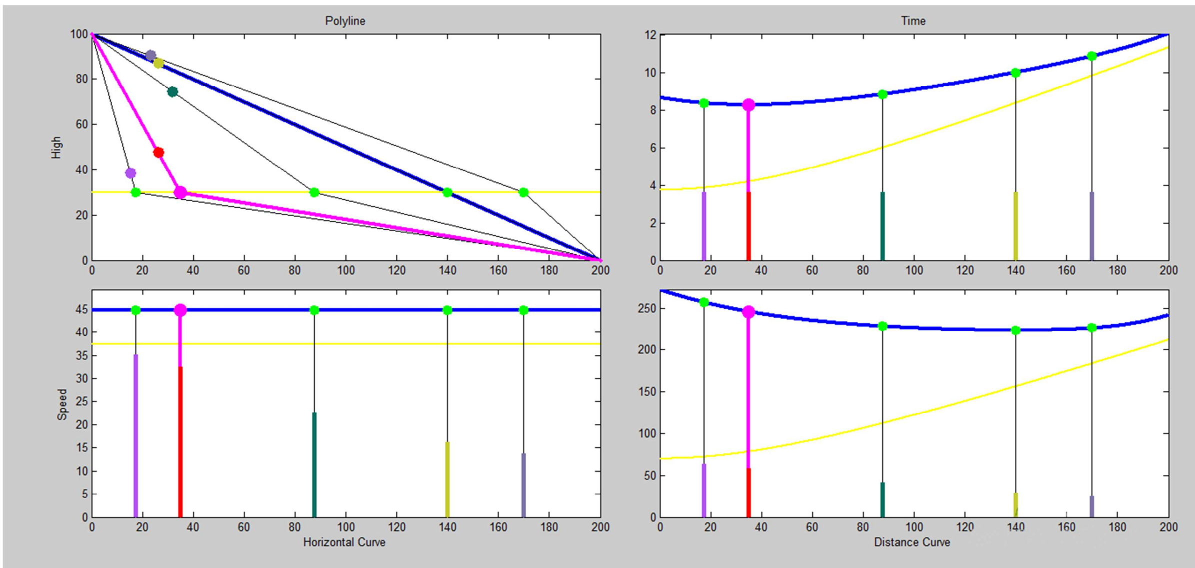


Figure 8. *The Movement of the Refraction Plane 1.*

As can be seen from Figure 8, the movement distance of D_1, D_2, D_3, D_4, D_5 increases sequentially in the BD segment, and the time required to pass BD increases in the same speed. It can be seen from Figure 8 that due to the segment BD, the movement distance in D_1, D_2, D_3 is shorter than D_4, D_5, so D_1, D_2, D_3 leads into refraction first. In paragraph DA, the distance of D_1, D_2, D_3, D_4, D_5 travelled in descending order. Therefore, compared to D_2, the required time of D_1 for exercise increases, which explains why the following D_2 is shorter than D_1.

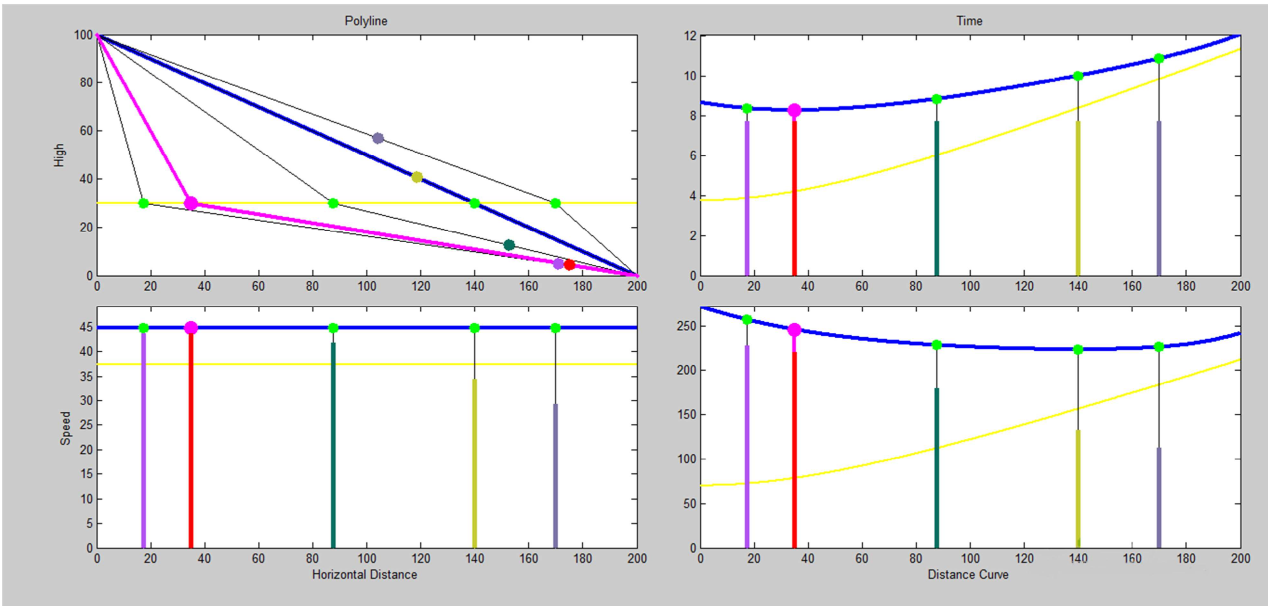


Figure 9. *The Movement of the Refraction Plane 2.*

It can be seen from Figure 9 that in the segment DA, the movement distance decreases in turn and the required movement time of D_1, D_2, D_3, D_4, D_5 decreases in turn. Therefore, D_2 reaches the end point earlier than D_1. Through rigorous theoretical calculations, it can also be shown that

When $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$, a total exercise time is indeed the shortest. This dynamic simulation solves this problem well.

In the dynamic simulation, the movement of each point can be easily observed. You can see the movement time of each broken line and the comparison chart of the distance traveled. Compared with the simple theoretical calculations, simulations achieve better observation and actual results.

4. The Shortest Time for Uniform Acceleration and Descent

4.1. Problem Description

The particle moves from point $B(0,b)$ to point $A(a,0)$, the speed of line BD is v_1, and the speed of line DA is v_2. Find the horizontal axis x of the point $D(x, y_0)$ on the horizontal line $y = y_0$, so that the time from B to D reaches A is the shortest. The difference between them and the uniform motion is that the two speeds are different. In the acceleration movement, the speed is treated as the average speed at both ends of the free fall. As shown in Figure 1.

4.2. Solution Analysis

First of all, according to the physical formula of free fall, the velocity of the particle whose initial velocity is zero falls to the point D (slope direction) is $v_D = \sqrt{2g(b-y_0)}$, and the average velocity from point B to point D can be obtained as:

$$v_1 = \frac{v_D}{2} = \frac{\sqrt{2g(b-y_0)}}{2} \tag{14}$$

The time from point B to point D is:

$$t_1 = t_1(x) = \frac{|BD|}{v_1} = \frac{\sqrt{2}\sqrt{x^2+(b-y_0)^2}}{\sqrt{g(b-y_0)}} \tag{15}$$

Similarly, the speed at which the particle drops to point A is $v_A = \sqrt{2gb}$, so the average speed from point D to point A is:

$$v_2 = \frac{v_D + v_A}{2} = \frac{\sqrt{2g(b-y_0)} + \sqrt{2gb}}{2} \tag{16}$$

The time from point D to point A is:

$$t_2 = t_2(x) = \frac{|DA|}{v_2} = \frac{\sqrt{2}\sqrt{(a-x)^2 + y_0^2}}{\sqrt{g(b-y_0)} + \sqrt{gb}} \tag{17}$$

After the above inference, it can be obtained that the entire period of time is:

$$t(x) = \frac{\sqrt{2}\sqrt{x^2 + (b-y_0)^2}}{\sqrt{g(b-y_0)}} + \frac{\sqrt{2}\sqrt{(a-x)^2 + y_0^2}}{\sqrt{g(b-y_0)} + \sqrt{gb}} \tag{18}$$

Since the average velocity v_1, v_2 of the two segments is only related to height, not related to point x on the horizontal line $y = y_0$, the derivatives of t are:

$$\begin{aligned} t' = t'(x) &= \frac{1}{v_1}\frac{x}{\sqrt{x^2 + (b-y_0)^2}} - \frac{1}{v_2}\frac{(a-x)}{\sqrt{(a-x)^2 + y_0^2}} \\ &= \frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0 \end{aligned} \tag{19}$$

It can be observed that when the angle and velocity satisfy the law of refraction, that is, when $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$ is satisfied, the time spent is the shortest.

It should be noted that the evaluation here can be obtained by using the dichotomy method to solve the problem of $t' = t'(x) = 0$, or the dichotomy method can be used to solve the problem of $\frac{\sin\theta_1}{v_1} - \frac{\sin\theta_2}{v_2} = 0$. There are many kinds of solving methods, and the former is used for simulation.

4.3. Simulation

In the uniform motion, the extreme points can be directly obtained, but because the evaluation method to be used in the uniform acceleration motion is more complicated, this paper considers using the dichotomy idea to solve.

The dichotomy idea mainly consists of: for a function that is continuous and $f(a) \times f(b) < 0$ on the interval $[a, b]$, by continuously dividing the interval where the zero of the function $f(x)$ is in two, so that the two endpoints of the interval gradually approach zero, and then the method of obtaining the zero point approximation is obtained.

4.3.1. Simulation Process

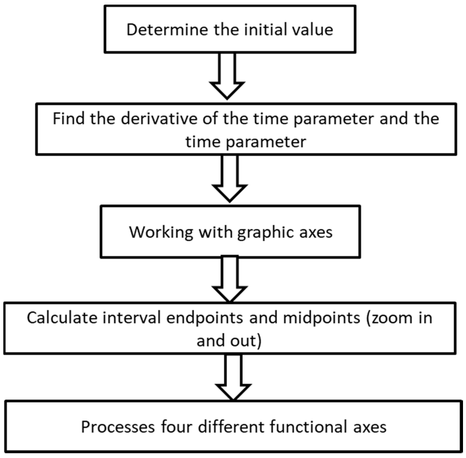


Figure 10. *Simulated flow chart for the shortest time problem of uniform acceleration and descent.*

4.3.2. Result Analysis

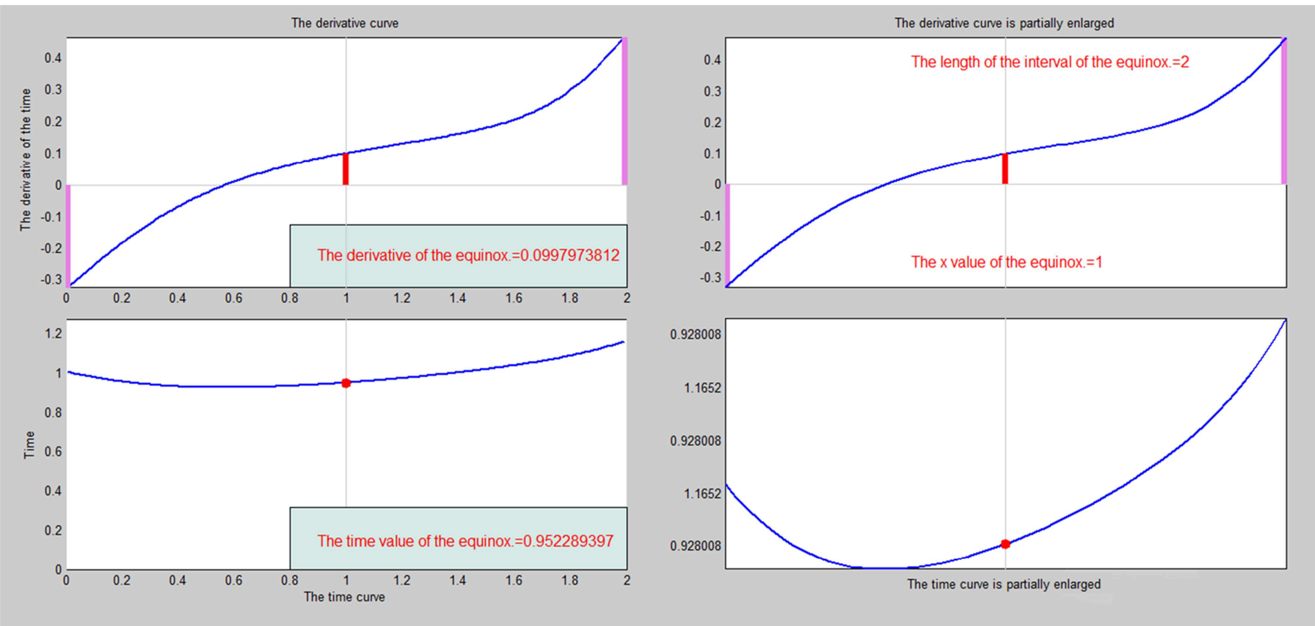


Figure 11. *Dynamic simulation of the shortest time problem of uniform acceleration and deceleration 1.*

Figure 11 is to draw the time derivative curve, select endpoints, use the nature of the bifurcation point to find the next point coordinates; In the second figure to enlarge the observation point; The third figure can be seen in the time value changes; The four figures can be seen as an enlarged view of the time value.

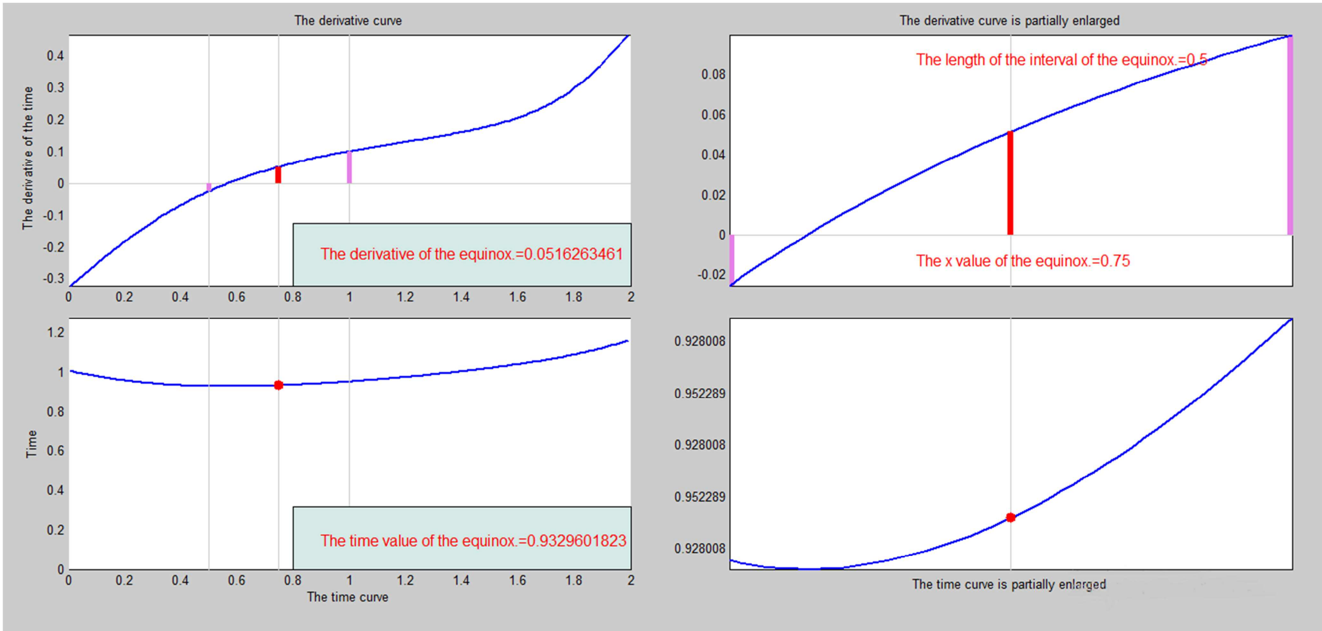


Figure 12. *Dynamic simulation of the shortest time problem of uniform acceleration and deceleration 2.*

Figure 12 shows the process of the third iteration. It can be seen that the derivative value is approaching zero and the time curve is continuously approaching the minimum point.

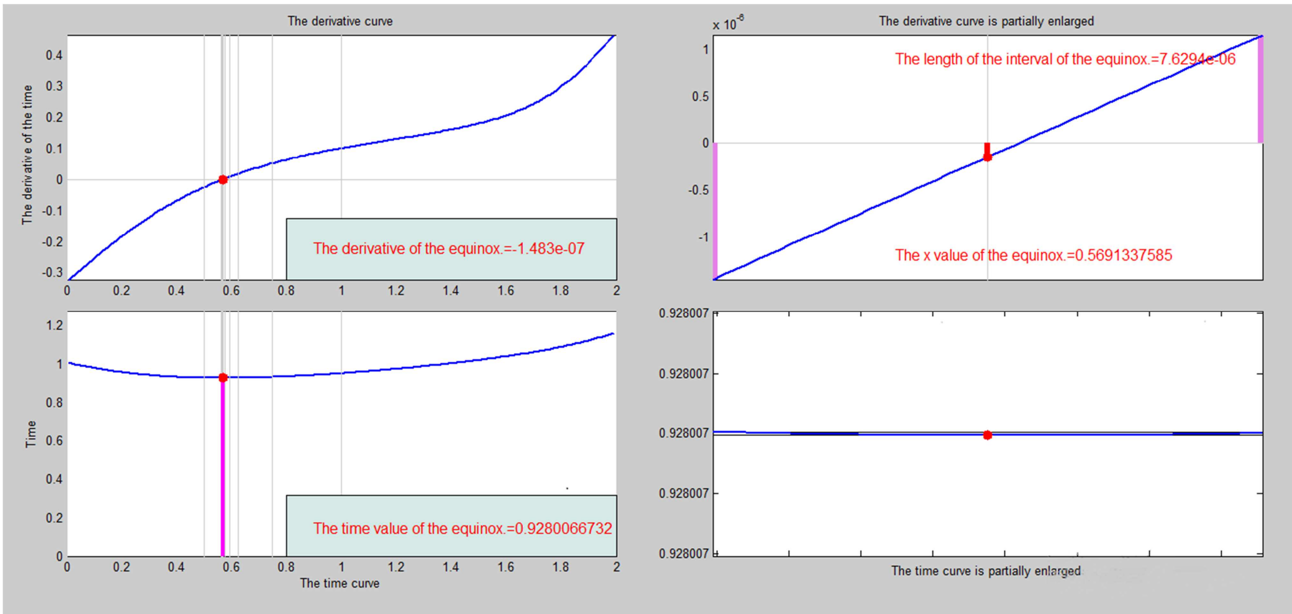


Figure 13. *Dynamic simulation of the shortest time problem of uniform acceleration and deceleration 3.*

Figure 13 shows the last iteration. It can be seen that the derivative of the final result is close to 0, and the last time value is very small.

4.4. Extension

It should be noted that the uniform acceleration motion can be generalized to any speed. If the speed is $V = V(x)$, in any interval $[z_1, z_2]$, the average value in that period of time can be calculated through integration. This provides a solid foundation for future complex calculations

5. The Comparison of the Uniform and Uniform Acceleration Movements

5.1. Spatial Thinking

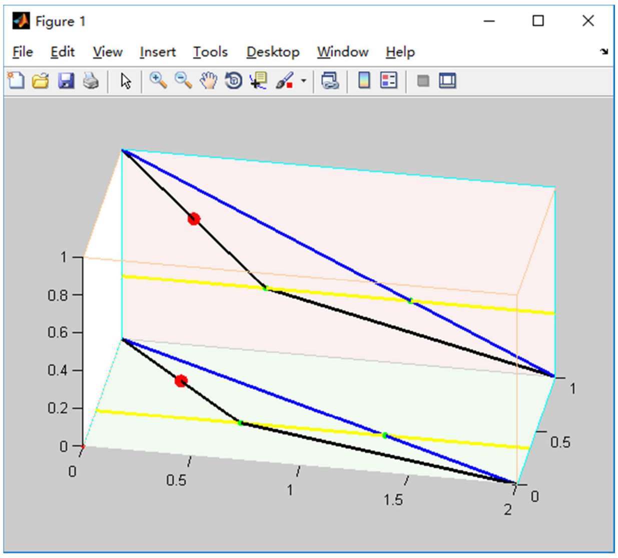


Figure 14. *The comparison of the Uniform and uniform acceleration movements in the space.*

The uniform motion and the uniform acceleration motion can be classified into different spatial conditions. The speed of uniform motion will not change, and the speed of uniform acceleration motion will change with the acceleration of free fall. Therefore, this paper will increase the space Z axis by uniformly accelerating motion. On the changes, the graph can more clearly distinguish the relationship between uniform motion and uniform acceleration.

5.2. Extension Issues

The problem of the shortest time for uniform and accelerated descent can be obtained:

When $\frac{\sin\theta_1}{v_1}=\frac{\sin\theta_2}{v_2}$ is satisfied, the time from B to A is the shortest. This problem can also be extended to the problem of uniform acceleration of multiple lines, that is, when $\frac{\sin\theta_1}{v_1}=\frac{\sin\theta_2}{v_2}=\ldots=\frac{\sin\theta_n}{v_n}$ is satisfied, the total motion time is the shortest. Therefore, the steepest descent line is a condition that satisfies $\frac{\sin\theta}{v}=C$ (C is a constant).

6. Conclusion

The MATLAB simulation results can clearly see the time and location of the steepest descent point. Whether it is uniform or uniform acceleration, the following conclusions can be drawn: Point $D(x, y_0)$ from left to right from the horizontal position, the total time used first decreases and then increases, there is a minimum value;

When the curve $\frac{\sin\theta_1}{v_1}$ intersects with the curve $\frac{\sin\theta_2}{v_2}$, it is the shortest point D for application; the best point D 's exercise plan is not the shortest route. The dichotomy is a better way to find the extremum problem. Although the declining problem is simple in description, it contains the shortest time and the shortest distance thoughts. It is also a subject worthy of study to combine various extreme values.

Acknowledgements

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