



# Unsteady Hydromagnetic Couette Flow with Magnetic Field Lines Fixed Relative to the Moving Upper Plate

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**Abstract:** This study concerns a magneto hydrodynamic flow between two parallel porous plates with injection and suction in the presence of a uniform transverse magnetic field with the magnetic field lines fixed relative to the moving plate with a constant pressure gradient. The study is aimed to determine the velocity profiles, the effects of permeability, pressure gradient and induced magnetic field on the flow. The nonlinear partial differential equation governing the flow are solved numerically using the finite difference method and implemented in MATLAB. The results obtained are presented in tables and graphs and the observations discussed on the effects of varying various parameters on the velocity profiles. A change is observed to either increase, decrease or to have no effect on the velocity profiles. The effect of magnetic field, time and suction /injection on the flow are discussed. The results provide useful information to the engineers to improve efficiency and performance of machines.

**Keywords:** Pressure Gradient, Suction and Injection, Magneto hydrodynamic (MHD)

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## 1. Introduction

A flow in a channel of a hydro magnetic fluid in which the motion of the fluid is due to movement of one of the plates of the channel, is called MHD Couette flow. MHD flows are characterized by a basic phenomenon which is the tendency of magnetic field to suppress vorticity that is perpendicular to itself which is in opposite to the tendency of viscosity to promote vorticity.

MHD Couette flow is studied by a number of researchers due its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering.

Researchers have studied unsteady channel or duct flows of a viscous and incompressible fluid with or without magnetic field analyzing different aspects of the problem.

Tao (1960) studied the Magneto hydrodynamic effects on the formation of Couette flow and Katagiri (1962) investigated unsteady hydro magnetic Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel.

Muhuri (1963) considered this fluid flow problem within a porous channel when fluid flow within the channel is induced

due to uniformly accelerated motion of one of the plates of the channel. Soundalgekar (1967) investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under transverse magnetic field. The effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel, studied by Muhuri (1963). The work by Muhuri (1969) was later analyzed by Govindrajulu (1969). Mishra and Muduli (1980) discussed effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion when one of the plates starts moving with a time dependent velocity. In the above mentioned investigations, magnetic field is fixed relative to the fluid.

Singh and Kumar (1983) studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when fluid flow within the channel is induced due to time dependent movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. Singh and Kumar (1983) considered two particular cases of interest in their study viz. (i) impulsive movement of one of the plates of the channel and (ii) uniformly accelerated movement of one of the plates of the channel and concluded that the magnetic field tends to

accelerate fluid velocity when there is impulsive movement of one of the plates of the channel and when there is uniformly accelerated movement of one of the plates of the channel.

Katagiri (1962) studied the problem when the flow was induced due to impulsive motion of one of the plates while Muhuri (1963) studied the problem with accelerated motion of one of the plates. Both had considered that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar (1983) considered the problem studied by Katagiri (1962) and Muhuri (1963) in a non-porous channel with the magnetic lines of force fixed relative to the moving plate. Khan *et al.* (2006) investigated MHD flow of a generalized Oldroyd-B fluid in a porous space taking Hall current into account while Khan *et al.* (2007), considered MHD transient flows of an Oldroyd-B fluid in a channel of rectangular cross-section in a porous medium. The influence of Hall current and heat transfer on the steady MHD flow of a generalized Burgers' fluid between two eccentric rotating infinite discs of different temperatures was studied by Hayat *et al.* (2008), in a case where the fluid flow was induced due to a pull with constant velocities of the discs.

Various aspect of the flow problems in porous channel have been studied, Bég *et al.* (2009), studied unsteady magneto hydrodynamic Hartmann-Couette flow and heat transfer in a Darcian channel with Hall current, ionslip, viscous and Joule heating effects. Makinde *et al.* (2012) studied unsteady hydro magnetic flow of a reactive variable viscosity third-grade fluid in a channel with convective cooling while Vieru *et al.* (2010) studied the Axial Flow of Several Non-Newtonian Fluids through a Circular Cylinder.

Seth *et al.* (2011), studied the problem considered by Singh and Kumar (1983) when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest, viz (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate. Seth *et al.* (2011) concluded that the suction exerted a retarding influence on the fluid velocity whereas injection has accelerating influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate. Jha and Apere (2011) investigated Hall and ion-slip effects on unsteady MHD Couette flow in a rotating system with suction and injection. Guchhait *et al.* (2011) studied the combined effects of Hall current and rotation on unsteady Couette flow in porous channel. Sheikholeslami *et al.* (2013) studied Heat transfer of Cu-water nanofluid flow between parallel plates while Prasad *et al.* (2012) considered unsteady hydro magnetic couette flow through a porous medium in a rotating system. Seth *et al.* (2012) studied the effects of Hall current and rotation on unsteady MHD Couette flow in the presence of an inclined magnetic field and also in the same year Seth *et al.* (2012).considered unsteady MHD Couette flow of class-II of a viscous incompressible electrically conducting fluid in a rotating system. More researchers, Ahmed and Kalita (2013) considered a sinusoidal fluid injection/suction on MHD three dimensional Couette flow through a porous medium in the presence of thermal radiation and also Ahmed and Kalita (2013)

studied Magneto hydrodynamic transient flow through a porous medium bounded by a hot vertical plate in presence of radiation.

Extensive researches have been done, including those cited above, on the flow between parallel plates. However, no emphasis has been given to the problems analyzed by Seth *et al.* (2011) with consideration when motion is on the upper plate. This work presents findings of studies on MHD couette flow problem between porous plates with magnetic field lines fixed relative to the moving upper plate with suction and injection on the plates.

## 2. Formulation

This study considers the flow of unsteady viscous incompressible electrically conducting fluid between two parallel porous plates  $y = 0$  and  $y = h$  of infinite length in  $x$  and  $z$  directions with a constant pressure gradient in the presence of a uniform transverse magnetic field  $H_0$  applied parallel to the  $y$  axis.

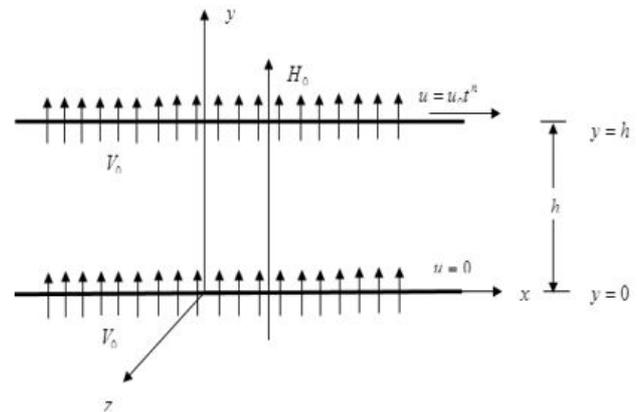


Figure 1. Physical model of the problem.

Initially (when time  $t \leq 0$ ), fluid and the porous plates of the channel are assumed to be at rest. When time  $t > 0$ , the upper plate ( $y = h$ ) starts moving with time dependent velocity  $u_0 t^n$  (where  $u_0$  is a constant and  $n$  a positive integer) in the  $x$  direction while the lower plate is fixed, the fluid suction/injection takes place through the walls of the channel with uniform velocity  $V_0$  where  $V_0 > 0$  for suction and  $V_0 < 0$  for injection. A constant pressure gradient is applied in the direction of the flow.

### 2.1. The Governing Equations

#### 2.1.1. Equation of Continuity

The equation of continuity is derived from the law of conservation of mass which states that mass can neither be created nor destroyed. It is derived by taking a mass balance on the fluid entering and leaving a volume element in the flow field. The general equation of continuity of a fluid flow is given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \tag{2.0}$$

Where  $\vec{q}$  is the velocity in x, y and z directions

$$(\vec{q} = u\hat{i} + v\hat{j} + w\hat{k} )$$

**2.1.2. Equation of Motion**

This equation is also known as the momentum equation and is derived from the Newton’s second law of motion. The law requires that the sum of all the forces acting on a control volume must be equal to the rate of change of fluid momentum within the control volume.

$$\frac{\partial \vec{q}}{\partial t} + \vec{q}(\nabla \vec{q}) = -\frac{1}{\rho} \nabla P + \nu \nabla^2 \vec{q} + \vec{F} \tag{2.1}$$

Where  $\frac{\partial \vec{q}}{\partial t}$  is the temporal acceleration,  $\vec{q}(\nabla \vec{q})$  is the convective acceleration,  $\nabla P$  is the pressure gradient,  $\nu \nabla^2 \vec{q}$  is the force due to viscosity and  $\vec{F}$  represents the body forces vector in x, y and z directions.

**2.1.3. Electromagnetic Equations**

The electromagnetic equations give the relationship between E the electric field intensity, B the magnetic induction vector, D the electric displacement, H the magnetic field intensity, J the induction current density vector and the charge density  $e_e$ , these are; Griffiths (1999).

$$\nabla \times H = J \tag{2.2}$$

$$\nabla \cdot B = 0 \tag{2.3}$$

$$\nabla \times E = \frac{\partial B}{\partial t} \tag{2.4}$$

**2.2. Non - Dimensional Numbers**

The dimensionless parameters allow the application of the results obtained in a model to any other dynamically similar case. In this work there are two non-dimensional numbers that are used. These are;

- Reynolds number
- Hartmann number

**2.2.1. The Reynolds Number, Re**

The Reynolds number is the ratio of inertial forces to viscous forces and is important in analyzing any type of flow where there is substantial velocity gradient shear. It is expressed as

$$Re = \frac{\rho VL}{\mu}$$

The Reynolds number indicates the relative significance of the viscous effects compared to the inertia effect. If the Reynolds number of the system is small, the viscous force is predominant and the effect of viscosity is important in the whole flow field otherwise if the Reynolds number is large,

the inertia force is predominant and the effect of viscosity is important only in the thin layer of the region near solid boundary.

**2.2.2. Hartmann Number, M**

The Hartmann number is the ratio of the magnetic force to viscous force and is defined as

$$M^2 = \frac{\sigma \mu_e^2 H_0^2 \nu}{U^2}$$

The Hartmann number gives a measure of the relative importance of drag forces resulting from magnetic induction and viscous forces in Hartmann flow, and determines the velocity profile for such flow.

**2.3. Problem Modelling**

Initially (when time  $t \leq 0$ ), fluid and the porous plates of the channel are assumed to be at rest. When time  $t > 0$ , the upper plate ( $y = h$ ) starts moving with time dependent velocity  $u_0 t^n$  (where  $u_0$  is a constant and  $n$  a positive integer) in the  $x$  direction while the lower plate is fixed, the fluid suction/injection takes place through the walls of the channel with uniform velocity  $V_0$  where  $V_0 > 0$  for suction and  $V_0 < 0$  for injection. A constant pressure gradient is applied in the direction of the flow.

The velocity and the magnetic fields are given as  $q = (u, v_0, 0)$  and  $\vec{H} \equiv (0, H_0, 0)$  respectively.

The magnetic forces =  $\sigma \mu_e^2 H_0 \times Velocity$

$$J \times B = \begin{vmatrix} i & j & k \\ 0 & 0 & \sigma \mu_e^2 H_0 u \\ 0 & H_0 & 0 \end{vmatrix} = (-\sigma \mu_e^2 H_0^2 u) \hat{i} \tag{2.5}$$

From the Navier stokes equation

$$\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla P + \mu \nabla^2 u + F \tag{2.6}$$

$$\rho \frac{\partial u}{\partial t} + \rho u \nabla u = -\nabla P + \mu \nabla^2 u + J \times B \tag{2.7}$$

The flow is incompressible (the density  $\rho$ , is considered a constant) and is considered in one dimension along the x- axis hence the Navier stokes equation along the x-axis is given as

$$\rho \frac{\partial u}{\partial t} + \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + J \times B \tag{2.8}$$

For a couette flow  $-\frac{\partial P}{\partial x} = 0$  but for our analysis  $-\frac{\partial P}{\partial x} =$  a constant  $\beta^*$ . The two plates are infinite in length hence

$\frac{\partial u}{\partial x} = 0$ . The fluid is injected on the lower plate with a constant velocity  $V_0$  and is also sucked from the upper plate at the same constant velocity  $V_0$ . The general equation governing the flow reduces to

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{\beta^*}{\rho} + \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{(-\sigma \mu_e^2 H_0^2 u)}{\rho} \tag{2.9}$$

where  $\beta = \frac{\beta^*}{\rho}$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \beta + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2 u}{\rho} \tag{2.10}$$

Where  $v = \frac{\mu}{\rho}$

The magnetic field lines are fixed relative the moving upper plate (The upper plate is accelerating –a function of time) hence the velocity is considered as a relative velocity and reflects how fast the fluid is moving relative to the plate.  $\rho$  -Density,  $\sigma$  -electric conductivity,  $\mu_e$  -magnetic permeability

$$\frac{v^2}{h^3} \frac{\partial u^*}{\partial t^*} + V_0 \frac{v}{h^2} \frac{\partial u^*}{\partial y^*} = \beta + v \frac{v}{h^3} \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma \mu_e^2 H_0^2}{\rho} \left( \frac{vu^*}{h} - u_0 \frac{t^* h^2}{v} \right) \tag{2.13}$$

and multiplying the equation by  $\frac{h^3}{v^2}$

$$\frac{\partial u^*}{\partial t^*} + \frac{V_0 h}{v} \frac{\partial u^*}{\partial y^*} = \frac{h^3}{v^2} \beta + \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma \mu_e^2 H_0^2 h^2}{\rho v} \left( u^* - u_0 \frac{t^* h^3}{v^2} \right) \tag{2.14}$$

But  $\frac{\sigma \mu_e^2 H_0^2 h^2}{\rho v} = M^2$ , the Hartmann number squared and

$\frac{u_0 h}{v}$ , the Reynolds number  $Re$  hence

$$\frac{\partial u^*}{\partial t^*} + \frac{V_0 h}{v} \frac{\partial u^*}{\partial y^*} = \frac{h^3}{v^2} \beta + \frac{\partial^2 u^*}{\partial y^{*2}} - M^2 \left( u^* - u_0 \frac{t^* h^3}{v^2} \right) \tag{2.15}$$

$$\frac{\partial u^*}{\partial t^*} + \frac{V_0 h}{v} \frac{\partial u^*}{\partial y^*} = \frac{h^3}{v^2} \beta + \frac{\partial^2 u^*}{\partial y^{*2}} - M^2 \left( u^* - \frac{Re h}{v} t^* \right) \tag{2.16}$$

which is the governing equation in non-dimensional form.

The non-dimensional boundary conditions from using the non-dimensional parameters are obtained as

$$u^* = 0 \quad 0 \leq y \leq 1 \text{ and } t^* \leq 0$$

$$u^* = \frac{t^* h}{v} Re \text{ at } y^* = 1; t^* > 0$$

and  $v$  -kinematic viscosity, where  $v = \frac{\mu}{\rho}$ . The general equation governing the flow

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \beta + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2 (u - u_0 t^n)}{\rho} \tag{2.11}$$

With the initial boundary conditions

$$u = 0 \quad 0 \leq y \leq h \quad t \leq 0$$

$$u = u_0 t^n \text{ at } y = h \quad t > 0$$

$$u = 0 \text{ at } y = 0 \quad t > 0$$

Taking  $n = 1$ , the governing equation for the flow becomes

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \beta + v \frac{\partial^2 u}{\partial y^2} - \frac{\sigma \mu_e^2 H_0^2 (u - u_0 t)}{\rho} \tag{2.12}$$

The terms of the governing equation are non dimensionalized according to the following definitions The non dimensionless numbers used in non dimensionalizing the equation are  $y^* = \frac{y}{h}$ ,  $u^* = \frac{uh}{v}$  and  $t^* = \frac{tv}{h^2}$  to obtain

#### 2.4. Finite Difference Technique

The finite difference approximations for derivatives are one of the simplest methods to solve differential equations. The principle of finite difference methods is close to the numerical schemes used to solve ordinary and partial differential equations. It consists in approximating the differential operator by replacing the derivatives in the equation using difference quotients. The domain is partitioned in space and in time and approximations of the solution are computed at the space or time points.

The governing equation together with the boundary conditions are solved numerically because of the nonlinear nature of the equations that are obtained. The finite difference analogues of the PDEs arising from the equation governing this flow are obtained by replacing the derivatives in the governing equations by their corresponding difference approximation taking into account the initial values and boundary values set. The following substitutions are done for the derivatives using the Crank Nicolson proposed averages

as

$$u^* = \frac{u_{i,j+1} + u_{i,j}}{2} \tag{2.17}$$

$$\frac{\partial u^*}{\partial y^*} = \frac{u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}}{4(\Delta y)} \tag{2.19}$$

$$\frac{\partial u^*}{\partial t^*} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \tag{2.18}$$

$$\frac{\partial^2 u^*}{\partial y^{*2}} = \frac{1}{2} \left( \left[ \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{(\Delta y)^2} \right] + \left[ \frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2} \right] \right) \tag{2.20}$$

Replacing in the governing equation, simplifying and rearranging

$$\begin{aligned} (u_{i,j+1} - u_{i,j}) + \frac{V_0 h \Delta t}{4v(\Delta y)} (u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}) &= \frac{h^3 \Delta t}{v^2} \beta + \\ \frac{1 \Delta t}{2(\Delta y)^2} (u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) - \frac{M^2 \Delta t}{2} (u_{i,j+1} + u_{i,j}) + M^2 \Delta t \frac{Re h}{v} t_j \end{aligned} \tag{2.21}$$

Letting  $A = -\frac{V_0 h \Delta t}{4v(\Delta y)}$   $B = \frac{h^3 \Delta t}{v^2} \beta$   $C = \frac{1 \Delta t}{2(\Delta y)^2}$   $D = \frac{M^2 \Delta t}{2}$   $E = M^2 \Delta t \frac{Re h}{v}$  with  $S = \frac{V_0 h}{v}$  representing the suction/ injection parameter and then substituting we have

$$\begin{aligned} (u_{i,j+1} - u_{i,j}) - A(u_{i+1,j+1} - u_{i-1,j+1} + u_{i+1,j} - u_{i-1,j}) &= B + \\ C(u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1} + u_{i+1,j} - 2u_{i,j} + u_{i-1,j}) - D(u_{i,j+1} + u_{i,j}) + Et_j \end{aligned} \tag{2.22}$$

Rearranging

$$\begin{aligned} u_{i,j+1} - u_{i,j} - Au_{i+1,j+1} + Au_{i-1,j+1} - Au_{i+1,j} + Au_{i-1,j} &= B + \\ Cu_{i+1,j+1} - 2Cu_{i,j+1} + Cu_{i-1,j+1} + Cu_{i+1,j} - 2Cu_{i,j} + Cu_{i-1,j} - Du_{i,j+1} - Du_{i,j} + Et_j \end{aligned} \tag{2.23}$$

Rearranging

$$\begin{aligned} u_{i,j+1} - Au_{i+1,j+1} + Au_{i-1,j+1} - Cu_{i+1,j+1} + Cu_{i-1,j+1} + 2Cu_{i,j+1} + Du_{i,j+1} &= B + \\ u_{i,j} - Au_{i-1,j} + Au_{i+1,j} + Cu_{i+1,j} - 2Cu_{i,j} + Cu_{i-1,j} - Du_{i,j} + Et_j \end{aligned} \tag{2.24}$$

Collecting the like terms

$$(1 + 2C + D)u_{i,j+1} - (A + C)u_{i+1,j+1} + (A + C)u_{i-1,j+1} = B + (1 - 2C + D)u_{i,j} + Cu_{i+1,j} + (C - A)u_{i-1,j} + Et_j \tag{2.25}$$

Rearranging

$$-(A + C)u_{i+1,j+1} + (1 + 2C + D)u_{i,j+1} + (A + C)u_{i-1,j+1} = Cu_{i+1,j} + (1 - 2C + D)u_{i,j} + (C - A)u_{i-1,j} + Et_j + B \tag{2.26}$$

The finite difference equations obtained at any space node, say  $i$  and at the time level  $t_{j+1}$  has only three unknown coefficients involving space nodes at  $i - 1, i$  and  $i + i$  at  $t_{j+1}$ .

In matrix notation, these equations are expressed as  $AU = B$  where  $U$  is the unknown vector of order  $(N - 1)$  at any time level  $t_{j+1}$ .  $B$  is the known vector of order  $(N - 1)$  which has the value of  $U$  at the  $n^{th}$  time level and  $A$  is the coefficient square matrix of order  $(N - 1) \times (N - 1)$  which is a tridiagonal structure.

If we let the coefficients of the interior nodes to be

$$\begin{aligned}
 a_j &= -(A + C) & d_j &= (C - A)u_{i-1,j} & g_j &= Et_j \\
 b_j &= (1 + 2C + D) & e_j &= (1 - 2C - D)u_{i,j} & h &= B \\
 c_j &= (A + C) & f_j &= Cu_{i+1,j} & &
 \end{aligned}
 \tag{2.27}$$

For  $j = 2, 3, 4, \dots, (N - 1)$ , then equation (4.6) becomes

$$a_j u_{i+1,j+1} + b_j u_{i,j+1} + c_j u_{i-1,j+1} = d_j + e_j + f_j + g_j + h
 \tag{2.28}$$

Equation (4.8) can be represented in system of equations in a tridiagonal matrix form as

$$\begin{bmatrix}
 a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 \\
 0 & a_3 & b_3 & c_3 & \ddots & 0 & 0 \\
 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\
 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots \\
 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1}
 \end{bmatrix}
 \begin{bmatrix}
 u_{1,j+1} \\
 u_{2,j+1} \\
 \vdots \\
 u_{3,j+1}
 \end{bmatrix}
 =
 \begin{bmatrix}
 d_2 \\
 d_3 \\
 \vdots \\
 d_{N-1}
 \end{bmatrix}
 +
 \begin{bmatrix}
 e_2 \\
 e_3 \\
 \vdots \\
 e_{N-1}
 \end{bmatrix}
 +
 \begin{bmatrix}
 f_2 \\
 f_3 \\
 \vdots \\
 f_{N-1}
 \end{bmatrix}
 +
 \begin{bmatrix}
 g_2 \\
 g_3 \\
 \vdots \\
 g_{N-1}
 \end{bmatrix}
 +
 \begin{bmatrix}
 h \\
 h \\
 \vdots \\
 h
 \end{bmatrix}
 \tag{2.29}$$

### 3. Results and Discussions

The physical situation of the problem and the effects of various flow parameters on the flow regime are depicted graphically and discussed. The simulations are carried out using ISO FLUIDS 3448 which are industrial oils whose kinematic viscosities range between 2 and 10. A constant pressure gradients between 1 and 5 and Reynolds numbers 1. The results are as follows;

From figure 2. The velocity increases with increase in the pressure gradient for both the cases of injection and suction. Pressure gradient is applied in the direction of the flow hence an increase in pressure gradient results in an increase in the force in the fluid in the direction of the flow which results in increased velocity of the fluid. The velocities for the injection case, ( $S > 0$ ) are greater than for the suction case ( $S < 0$ ). Injection increases the pressure which increases the force in the fluid hence an increase in the velocities. Injection increases the pressure which increase the force in the fluid hence an increase in the velocities while suction reduces the pressure which reduces the force in the fluid hence a decrease in the velocities which explains why the injection velocities are greater than the suction velocities.

From figure 3. The velocity increases with increase in the pressure gradient for varying kinematic viscosity.

There is rapid increase in velocity of the fluid with increase in pressure gradient for small kinematic viscosity as compared to large kinematic viscosity. The effect of pressure gradient decreases with increase in kinematic viscosity. The increase in the kinematic viscosity leads to increase in the frictional forces which oppose the fluid motion

From figure 4. The velocity increases with the increase in the Hartman number. The Hartmann number gives a measure of the relative importance of drag forces resulting from magnetic induction and viscous forces hence an increase in the Hartmann number reduces the drag forces hence increased velocities.

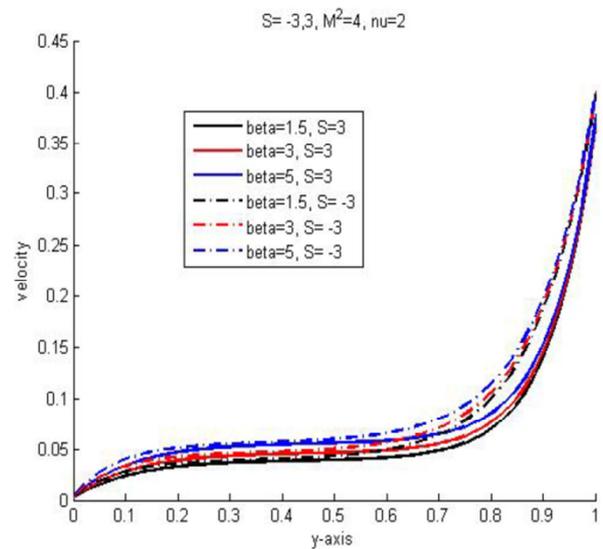


Figure 2. Varying the pressure gradient.

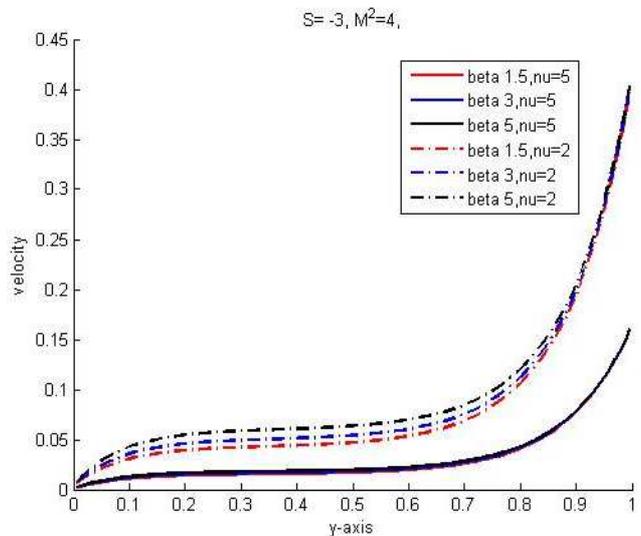


Figure 3. Varying the kinematic viscosity and pressure gradient.

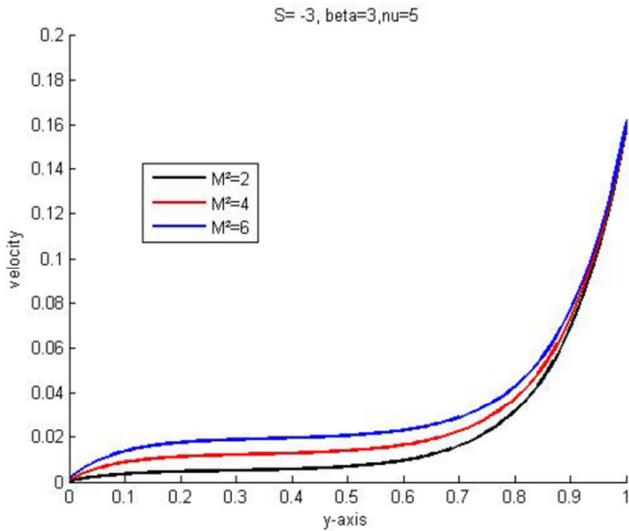


Figure 4. Varying the Hartman number,  $M^2$ .

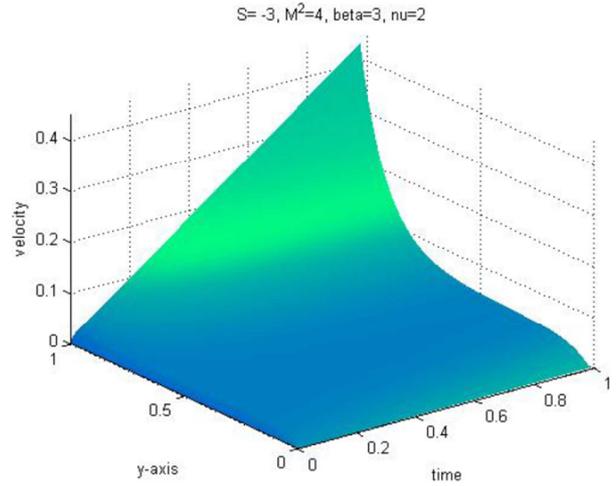


Figure 7. Surface plots.

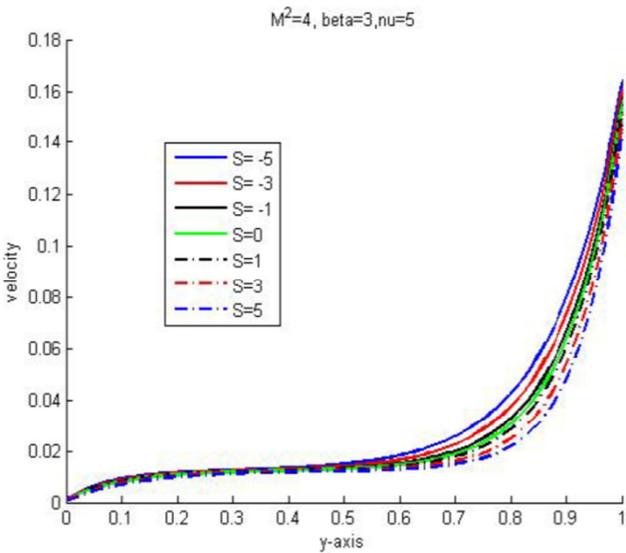


Figure 5. Varying the Injection/Suction,  $S$ .

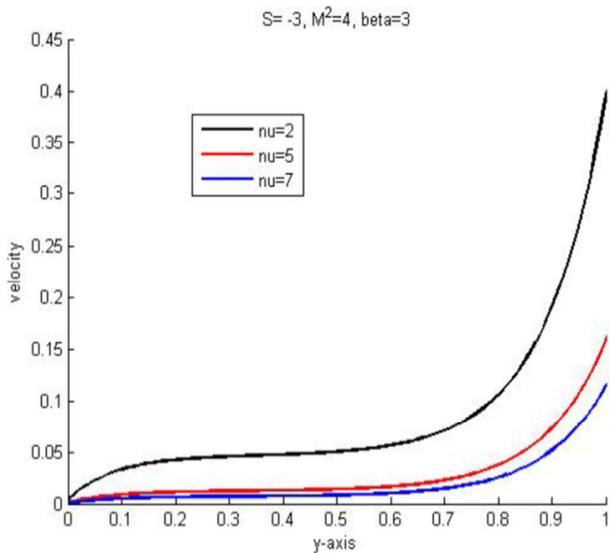
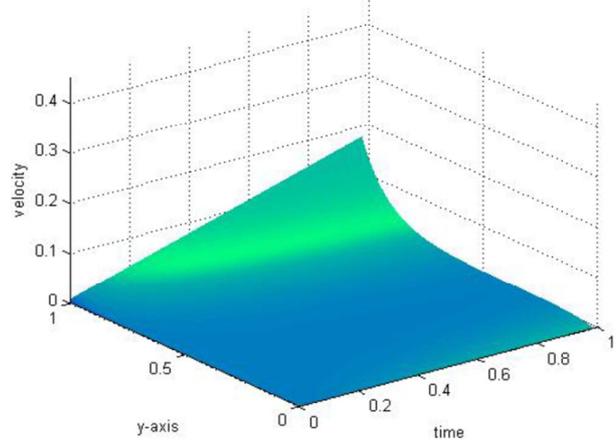


Figure 6. Varying the kinematic viscosity.

From figure 6. The velocity of the fluid decreases with the increase in the viscosity of the fluid. Increase in the viscosity of the fluid leads to increase in the viscous forces in the fluid hence decrease in the velocity of the fluid.

From figure 5. An increase in the suction parameter ( $S > 0$ ) leads to a decrease in the velocity of the fluid. An increase in the injection parameter ( $S < 0$ ) leads to an increase in the velocity of the fluid. An increase in the suction parameter ( $S > 0$ ) reduces the pressure which reduces the force hence decrease in the velocity with increase in the suction parameter while an increase in the injection parameter ( $S < 0$ ) increases the pressure which increases the force in the fluid hence increased velocities. Thus suction exerts a retarding influence on the fluid velocity whereas injection has an accelerating influence on it.

#### 4. Conclusion

The results leads to a conclusion that the magnetic field, pressure gradient, time and injection have an accelerating influence on the fluid flow with a constant pressure gradient in the direction of the flow on both cases of suction and injection while viscosity and suction exert a retarding influence. Fluid velocity in both the cases of suction and injection decreases

with increase in the suction parameter and increases with the increase in the injection parameter. Suction exerts retarding influence on the fluid velocity whereas injection has an accelerating influence on the fluid velocity. Viscosity exerts a retarding influence on the fluid velocity

## Nomenclature

|                |   |
|----------------|---|
| B              | Magnetic field strength vector, [wbm <sup>-2</sup> ]                      |
| H <sub>0</sub> | Magnetic flux identity along the y- axis [wbm <sup>-2</sup> ]             |
| g              | Acceleration due to gravity vector, [ms <sup>-2</sup> ]                   |
| H              | Magnetic field intensity vector in Amperes per meter, [Am <sup>-1</sup> ] |
| J              | Current density, [AM <sup>-2</sup> ]                                      |
| e              | Unit electric charge, [C]   |
| E              | Electric field, [v]   |
| S              | Suction/ Injection  |
| M              | Magnetic parameter  |
| Xi             | Permeability parameter  |
| Re             | Reynolds number   |
| P              | Pressure force, [nm <sup>-2</sup> ]                                       |
| P*             | Dimensionless pressure force.   |
| q              | Velocity vector, [ms <sup>-1</sup> ]                                      |
| i, j, k        | Unit vector is the x, y, z directions respectively                        |
| u, v, w        | Component of velocity vector q, [ms <sup>-1</sup> ]                       |
| u*, v*, w*     | Dimensionless velocity components   |
| x*, y*, z*     | Dimensionless Cartesian co-ordinates                                      |
| x, y, z        | Dimensional Cartesian co-ordinates  |
| F <sub>i</sub> | Body forces tensor, [N]   |
| U <sub>i</sub> | Velocity tensor, [ms <sup>-1</sup> ]                                      |
| x <sub>j</sub> | Space tensor, [m]   |
| Ha             | Hartman number  |
| V <sub>0</sub> | Suction velocity, [ms <sup>-1</sup> ]                                     |
| Greek symbol   | Meaning   |
| ε <sup>+</sup> | Porosity  |
| ρ              | Fluid density, [kgm <sup>-3</sup> ]                                       |
| μ              | Coefficient of viscosity, [kgm <sup>-1</sup> s]                           |
| σ              | Electrical conductivity, [Ω <sup>-1</sup> m <sup>-1</sup> ]               |
| μ <sub>e</sub> | Magnetic permeability, [Hm <sup>-1</sup> ]                                |

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